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## Damage identity in fatigue assessment of structures

## Идентичность повреждения в расчете ресурса конструкций

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**Ключевые слова:** долговечность конструкций; деформационный метод; суммирование повреждений; идентичность повреждения

**Abstract.** The modified strain criterion-based method for fatigue assessment of structures is discussed. The damage is estimated based on the specified parameters of the criterion and the damage summation procedure by employing the finite-element method. With a reasonably fine mesh of the finite-element model of the 'critical location' structure, the condition of the identity of damage in the material of the test specimen and the structure is provided and, respectively, the effect of uncertainty on the fatigue life assessment of the structure is reduced. The implementation of this version of the method is using the example of the fatigue life evaluation of a ship hull and superstructure detail at expansion joint. For comparison, the fatigue life of the detail is estimated using the standard S-N approach. The results are in approximate agreement; however, reducing the computational uncertainties with the help of the deformation criterion shows more physically reasonable fatigue properties of the detail.

**Аннотация.** Приводится развитие метода оценки ресурса конструкций, основанного на использовании деформационного критерия. Оценка повреждения в узле конструкции выполняется на основе уточнения параметров критерия и процедуры суммирования повреждений с использованием метода конечных элементов. При целесообразно мелкой сетке конечных элементов расчетной модели «критической области» узла конструкции обеспечивается условие идентичности повреждения материала образца и конструкции и соответственно снижается эффект неопределенности в оценке долговечности конструкции. Применение метода в таком представлении показано на примере оценки усталости узла конструкции корпуса и надстройки судна в районе выреза для расширительного соединения. Для сравнения выполнен расчет ресурса узла с помощью расчетной S-N кривой, характеризующей свойства сварных соединений. Получено примерное согласование результатов, однако снижение роли неопределенностей в расчете с помощью деформационного критерия дает более благоприятные показатели надежности узла.

### Introduction

Fatigue assessment of welded structures according to the current rules is based on applying the S-N criteria of fatigue failure at cyclic loading [1-6], etc. The test results implemented for determining the S-N curves include a crack initiation phase and crack growth until almost complete failure of specimens in two parts. Consequently, the methodology of the analysis and the particulars of the S-N curves do not allow determining the indications of damage of structural details and fatigue crack size; the occurrence of the latter is uncertain. Respectively, if the residual operational life of a structure should be estimated considering the safe period of the initiated crack propagation, and the crack extensions should be evaluated by applying the recommended approaches of the linear fracture mechanics, the necessary information on the initial crack size cannot be found. Apart from that, the designed S-N curves in Refs. [1-6], etc., are composed as a unified characterization of fatigue in a range of structural steels, Петинов С.В., Гучинский Р.В., Сидоренко В.Г. Идентичность повреждения в расчете ресурса конструкций // Инженерно-строительный журнал. 2016. №1(61). С. 82-88.

irrespective of the mechanical properties of steels. This also brings uncertainty into the results of fatigue evaluations of structures. Development of the approaches for numerically evaluating local stresses at critical locations for fatigue analysis (Hot-spot, Notch stress approaches [3–6]) introduces additional uncertainties since the analyses recommended have to be carried out based on the elastic behavior of the material, which contradicts the mechanics of fatigue. The mentioned factors cannot provide identity of damage between the test specimen and the structural detail; this fundamental principle is realized fairly approximately.

Understanding the problem of damage fitness in test pieces and structures aroused researchers' interest decades ago. V.P. Kogaev [7] suggested a statistical theory of fatigue similitude in which the leading role was given to the stress gradient at the critical location in a structure. Lately, attempts were made to establish the criteria of damage identity based on evaluating the «informative» crack extensions within the stress concentration areas [8, 9]. However, considering the mechanisms of damage development in the polycrystalline structure of structural materials [10, 11], the significance of the durability assessment of structures should be based on the damage identity between test specimen and structure material.

The influence of the above uncertainty factors in fatigue analyses may be substantially reduced by applying the strain-life technique in which the criterion for fatigue gives the dependence of fatigue life of the cyclic strain range. Cyclic strain characterizes cyclic elastic-plastic properties of a particular structural material; it is physically and mechanically more realistic than stress in determining the material damage at the stress concentration areas where the fatigue process develops. In a sense, in fatigue testing of specimens (under strain range control), the failure of material is determined by the manifestation of the early phase of macroscopic crack initiation, by the distortion of the ascending part of the elastic-plastic hysteresis loop. Applying the criterion together with finite-element modeling of the structure and the technique of fatigue damage accumulation in critical locations allows following the principle of identity of fatigue damage of test piece and structural detail the most closely.

In fatigue analyses, when the strain-life approach is applied, the cyclic elastic-plastic strain has to be assessed at the location where damage is expected to develop in the structure. Although the local strain range may be found by using the finite-element method (FEM), the current rules, e.g., Ref. [2], recommend the approach based on Neuber's heuristic formula [12]. The approach, as well as the FEM, do not provide an analytical description of the cyclic elastic-plastic strain; therefore, fatigue assessment at irregular service loading requires transforming the continuous probability distribution of the stress history into a block diagram, or a histogram, e.g., Ref. [1].

A brief description, the necessary improvement of the criterion and the illustration of applying the method using the example of fatigue analysis of a ship structure detail at the expansion joint cut in the superstructure are given below.

### *Strain-life approach and the necessary improvements*

The strain-life criterion for fatigue failure of materials at cyclic loading (strain range control testing) is obtained in the following form [13]:

$$\Delta\varepsilon = CN^{-\alpha} + BN^{-\beta}, \quad (1)$$

where  $\Delta\varepsilon$  is the cyclic elastic-plastic strain range,  $C$ ,  $B$ ,  $\alpha$  and  $\beta$  are the empirical (material) parameters of the criterion;  $N$  is the number of loading cycles prior to fatigue failure of material (early crack initiation).

It was observed long ago, e.g., in Ref. [10], that fatigue damages and microcracks develop well below the conventional fatigue limit stress. Even occasional stress cycles over this stress level provide the conditions for microcracks to extend into macroscopic and resulting in fatigue failure. Respectively, this effect of irregular loading must be accounted for by lowering the «minimum damaging» stress to  $0.55\sigma_e$  [2], where  $\sigma_e$  is the conventional fatigue limit stress (obtained at cyclic loading resting). The corresponding strain range, accordingly (1), is:

$$\Delta\varepsilon = 1.1\sigma_e / E = CN^{-\alpha} + BN^{-\beta} \quad (2)$$

at  $N = 10^7$  cycles [2]. Since fatigue damage in structural components is caused mostly by the moderate service stresses, the «high-cycle» parameter,  $B$ , should be corrected accordingly (2):

$$B^* = 1.1\sigma_e N^\beta / E - CN^{\beta-\alpha}, \quad (3)$$

where the number of cycles is  $N = 10^7$ .

Further correction of the criterion (1) is needed since it is applied for fatigue evaluation at stress concentration; the effects of stress concentration are most pronounced in high-cycle fatigue. For this reason, the high-cycle component,  $BN^{-\beta}$ , should be improved taking into account correction (3):

$$\Delta\varepsilon = CN^{-\alpha} + B * N^{-\beta} K_t / K_f, \quad (4)$$

where  $K_t$  is the stress concentration factor which depends on the loading type and the detail geometry,  $K_f$  is the respective notch factor.

The elastic-plastic strain  $\Delta\varepsilon$  at critical location, stress concentration area, is estimated by using Neuber's formula [12]:

$$\Delta\sigma \Delta\varepsilon = (K_t \Delta\sigma_n)^2 / E = S^2 / E, \quad (5)$$

where  $E$  is the elasticity modulus,  $\Delta\sigma_n$  is the nominal stress range,  $S = K_t \Delta\sigma_n$ , is the maximum stress range in the affected location of the structural detail. To solve Eq. (5) and find the strain  $\Delta\varepsilon$ , the generalized cyclic stress-strain curve obtained from the cyclic testing of specimens is applied.

Using Eq. (5) does not provide analytical solution: local strain is obtained at a discrete value of nominal stress. Consequently, the continuous probability distribution of stress at the detail location should be substituted, as mentioned above, by the equivalent (in the sense of fatigue damage) step-wise diagram, i.e., histogram. The rules, e.g., [1], do not indicate unambiguous recommendations for evaluating the characteristic stress ranges,  $S_i$ , and the respective number of load cycles,  $n_i$ , for each histogram class. These histogram parameters, *equivalent by fatigue damage* to the probability distribution of stress at the detail location, can be found by applying the technique developed in [14].

Furthermore, the fatigue analysis of the examined detail is carried out by using the linear damage summation procedure:

$$\sum_i n_i(S_i) / N_i(S_i) = D, \quad (6)$$

where  $D$  is the damage index of the accumulated damage;  $D = 1$  is the condition for fatigue failure of material at the stress concentration, namely, for macroscopic crack initiation according to criterion (1),  $n_i(S_i)$  is the number of load cycles in the  $i$ -th fragment of cyclic loading of the histogram at the stress  $S_i$ ;  $N_i(S_i)$  is the number of load cycles determined by the material failure criterion (4).

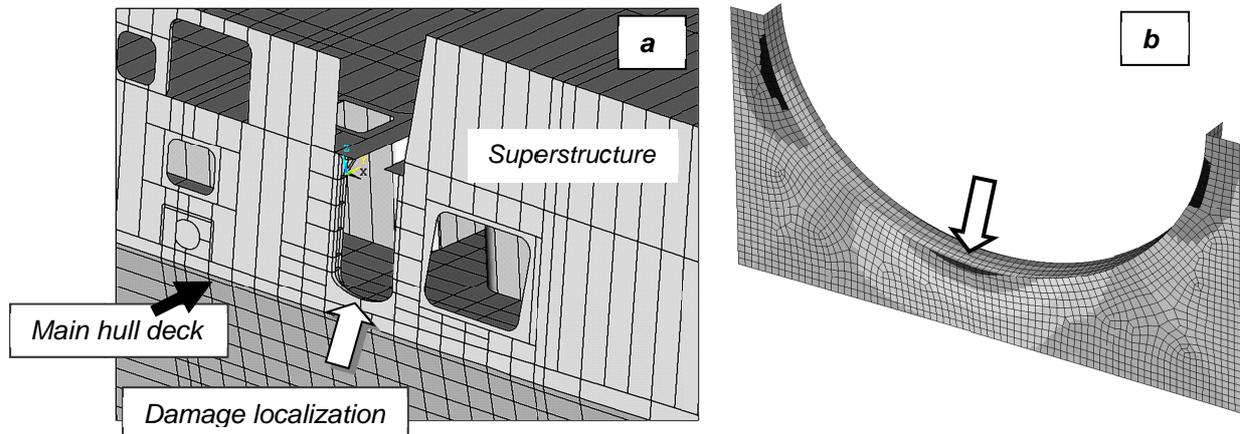
### *Example of applying the approach*

In a ship structure with a long superstructure, whose longitudinal walls are extending the side structure of the ship hull, the superstructure walls are transversally cut and fitted with expansion joints. Local stress increase at the cut endings is regarded as menacing the main hull integrity; fatigue analysis is necessary when designing the superstructure.

Dividing long superstructures and deck houses into separate blocks in order to retain the stress flow within the main hull and at the weight savings of superstructures has been long known and applied in shipbuilding. However, a sensible solution for the problem of reliability of the superstructure details at the expansion joints has not been found yet ([15], [16], etc.). Dividing superstructures and deck houses makes it necessary, apart from paying attention to designing the cut endings, to assess fatigue properties of the details.

The outline of a detail of a superstructure is shown in Fig. 1. Stress analysis of the ship structure in the examined area was carried out by the FEM and the respective software. Fig. 1 also shows the finite-element mesh at the cut ending and the localization of fatigue damage. Element sizes were selected so that the necessary precision of the local stress would be maintained and the stress gradient through the elements was insignificant enough to assume that damage accumulation in the elements was uniform.

The stress in the superstructure at the cut ending is caused by the hull (and superstructure) bending and shear deformation in the seaway, both in the vertical and the horizontal planes.



**Figure 1. Superstructure at the expansion joint: geometrical model (a); The FE mesh and stress distribution in the detail at the cut ending (b)**

However, for a semi-circular or a semi-elliptical shape of the cut ending in the superstructure, increases in local stress occur in different parts of the superstructure, as shown in Fig.1,b: the arrow indicates the area where a high stress is due to the bending deformation of the hull. Stress elevations caused by the shear deformation are shifted to the vertical edges of the superstructure at the cutout (Fig.1,b). For this reason, only the stresses caused by the effects of the hull (and the superstructure) bending in the vertical and the horizontal planes are considered further in fatigue analysis.

In the strength and reliability assessment of the ship and marine structures, the wave loads are characterized by the long-term probability distribution (of bending moments and respective stress) composed of the probability distribution at stationary loading conditions throughout the intended service life [1, 17], etc. The long-term probability distribution of the nominal stress caused by the hull bending is given by the two-parameter Weibull formula [17, 19]:

$$Q(S > S_1) = \exp(-(S_1 / a_s)^k), \tag{7}$$

which is understood as the probability that the stress range would exceed a provisional value of the stress range,  $S_1$ ;  $a_s, k$  are the scale and the share parameters of the distribution, respectively.

The bending moments in the mid-part of the ship hull can be estimated following the rules recommended in Ref. [17]. The nominal stresses with regard to the bending modes (double amplitudes, ranges) are obtained as follows: at the hull bending in the vertical plane,  $S_{v,n} = \Delta M_v / W_v$ , and at the bending in the horizontal plane,  $S_{h,n} = \Delta M_h / W_h$ ;  $\Delta M_v$  and  $\Delta M_h$  are the bending moments in the vertical and horizontal planes,  $W_v$  and  $W_h$  are the section modules of the hull at the sheerstrake and deck stringer joint, respectively.

The total nominal stress at the examined detail location is evaluated considering the statistical correlation of the bending moments [17, 19]. The results, the bending moments and the nominal stress ranges characterized by the exceedance  $Q = 1/N_s = 2.1 \cdot 10^{-8}$  (only the time in the seaway is considered) are given in Table 1.

**Table 1. Bending moments and nominal stresses in ship hull at the detail location**

The mode of bending	Bending moment range, $\kappa Nm$	$W_v, W_h, m^3$	Nominal stress range, $MPa$	Scale factor, $a_s, MPa$
Vertical	$4.719 \cdot 10^5$	1.9961	236.4	16.594
Horizontal	$2.098 \cdot 10^5$	2.8750	73.0	5.124
Total stress	-	-	254.6	17.872

The distribution shape parameter values according to [1],  $k = 2.21 - 0.54 \lg L = 1.081$ ,  $L$  is the ship length, molded.

The fatigue life of the detail is assessed for two shape versions of the cut ending characterized by the values of the stress concentration factor  $K_t = 1.85$  and  $2.28$ , and of the maximum total stress  $S_{\max} = 466.9$  and  $581.6$  MPa, respectively.

The material of the structure is a higher-strength steel (D40S) whose yield stress is  $\sigma_y = 390$  MPa, and the conventional fatigue limit stress amplitude is  $\sigma_e = 112$  MPa (loading along the superstructure side shell and flange joint, the weld thoroughly machined, 100% NDT). The parameters of criterion (4) of the steel are [20]:  $C = 0.400$ ,  $\alpha = 0.653$ ,  $B = 0.015$ ,  $\beta = 0.140$ ; corrected (3) parameter  $B^* = 0.0058$ .

Notch factor values of both detail versions are estimated by applying Peterson's [21] formula:  $K_f = 1 + (K_t - 1)/(1 + g/r)$ , where  $r$  is the notch root radius (cut ending),  $g$  is the «structural parameter», for hull structural steels  $g = 0.38(350/\sigma_u)^{1.16}$ ,  $\sigma_u$  is the ultimate strength of the steel. In the examined detail, the root radius is substantially larger than the structural parameter; approximately,  $K_f \approx 1 + (K_t - 1)/1.02$ .

The necessary cyclic stress-strain curve for evaluating the strain range values at the stress concentration is found in Ref. [20]. The appropriate technique of using Neuber's formula (5) and the cyclic curve for evaluating the local strain range values is also shown in Ref. [20], etc.

In order to estimate the fatigue life of the detail, the parameters of the histogram *equivalent by fatigue damage* to the distribution (7) have to be determined. For the version of the cut ending shape which is specified by  $K_t = 2.28$ ,  $S_{\max} = S_{n,\max} \cdot K_t = 254.6 \cdot 2.28 = 581.0$  MPa, the total stress range is arbitrarily subdivided into 7 sub-ranges, or classes (based on the recommendations in Ref. [1]), Table 2. Furthermore, the relative number of equivalent loading cycles (probability of the class in the ensemble)  $p_i$ , the partial damage  $d_i$  and the equivalent stress range  $S_i^{eq}$  are estimated for each stress class by applying the procedure described in Refs. [14, 18]. The results are given in Table 2.

**Table 2. Parameters of the equivalent stress histogram**

S, class, MPa	28 – 107	107 – 186	186 – 265	265 – 344	344 – 423	423 – 502	502 – 581
$p_i$	0.455	0.053	$5.270 \cdot 10^{-3}$	$4.802 \cdot 10^{-4}$	$4.120 \cdot 10^{-5}$	$3.370 \cdot 10^{-6}$	$2.650 \cdot 10^{-7}$
$d_i$	0.231	0.680	0.398	0.125	0.028	0.0049	0.000735
$S_{eq}$ , MPa	68.52	137.5	214.1	291.8	371.0	448.9	532.1

The values of the strain range for each class of the histogram are obtained through the equivalent stresses  $S_i^{eq}$  and the cyclic curve of the steel following (5); the strain ranges  $\Delta \varepsilon_i$  are applied to calculate fatigue lives  $N_i(\Delta \varepsilon_i)$ , and the damage is assessed accordingly (6):

$$\sum_i n_i(S_i^{eq}) / N_i(S_i^{eq}) = N^* \sum_i p_i(S_i^{eq}) / N_i(\Delta \varepsilon_i) = D. \quad (8)$$

The results of the damage evaluation are presented in Table 3 for the two versions of the cut ending shape, the semi-elliptical (1) and the semi-circular (2). For comparison, the results of the damage calculation following the standard scheme with the S-N criteria parameters recommended in Refs. [4, 5] are also given in Table 3.

**Table 3. Fatigue life (accumulated damage) of the detail**

Shape version	$S_{eq}^{\max}$ , MPa	$K_t^{(eq)}$	Damage, D, approach	
			Strain-life	Stress-life
1	466.9	1.85	0.325	0.581
2	581.6	2.28	1.280	1.467

As can be seen from the table data, the necessary fatigue life of the detail is not provided when the semi-circular shape of the cut ending (2) might be applied. If the cut ending is given a semi-elliptical shape, (1) fatigue life is provided with a notable factor of safety.

Evidently, the strain-life approach results in better reliability characteristics of the structure than the standard method based on applying the S-N criteria. It may be explained by using particular steel characteristics in the analysis (the designed S-N curve [1] presents generalized data on a range of steels, from low-carbon to higher-strength steels); strain is physically and mechanically more correct than stress in characterizing fatigue damage, the strain-life criterion defines damage at an early stage of macroscopic crack origination.

It should be noted that the damage estimated by the strain-life approach predicts the initiation of a macroscopic fatigue crack in the side shell of the superstructure at the cut ending within the limits of the finite-element size in Fig.1,b. This suggestion is based on the principle of terminating fatigue testing and defining the parameters of criterion (4) – as mentioned above – by transition of the microscopic crack into a macroscopic one in the gage part of the specimen.

Respectively, allowing for insignificant conservatism (the local cyclic strain is almost constant within the limits of the volume included into the finite-element size of the fine mesh) it may be concluded that the displayed approach provides the identity of fatigue damage between the test specimen and the critical location of the structural detail. This statement certainly does not extend to the effects of uncertainties in fatigue analyses of structures, where the most substantial source may be the variability of service loads in practice compared to those recommended by the rules.

## Conclusion

A modified strain-life approach for structural fatigue assessment is briefly discussed. In combination with the finite-element modeling of the structure, in particular, when the critical location area is modeled with the necessarily fine mesh, the most substantial principle of fatigue modeling is provided by the approach, i.e., the principle of damage identity between the test specimen and the fatigue-affected area of the structure.

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