doi: 10.5862/MCE.63.4

Surrogate modeling for initial rotational stiffness of welded tubular joints

Суррогатное моделирование для определения начальной жесткости вращения сварных трубчатых соединений

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Key words: surrogate modeling; kriging; square hollow section; plane bending; finite element analysis

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Ключевые слова: суррогатное моделирование; кригинг; квадратный полый профиль; плоскость изгиба; анализ методом конечных элементов

Abstract. Recently, buildings and structures erected in Russia and abroad have to comply with stringent economic requirements. Buildings should not only be reliable and safe, have a beautiful architectural design, but also meet the criteria of rationality and energy efficiency. In practice, this usually means the need for additional comparative analysis in order to determine the optimal solution to the engineering task. Usually such an analysis is time-consuming and requires huge computational efforts. In this regard, surrogate modeling can be an effective tool for solving such problems. This article provides a brief description of surrogate models and the basic techniques of their construction, describes the construction process of a surrogate model to calculate initial rotational stiffness of welded RHS joints made of high strength steel (HSS).

Аннотация. В последнее время сильно возросли экономические требования, предъявляемые к зданиям и сооружениям, возводимым в России и за границей. Здания должны быть не только надежными и безопасными, иметь красивый внешний вид, но также удовлетворять критериям рациональности и энергоэффективности. На практике это означает необходимость дополнительного сравнительного анализа для определения оптимального решения инженерной задачи. Обычно такой анализ трудоемок и требует огромных вычислительных затрат. В связи с этим суррогатное моделирование может быть эффективным инструментом для решения таких проблем. В данной статье приводится краткое описание суррогатных моделей и основных методов их построения, описывается процесс строительства суррогатной модели для расчета начальной жесткости вращения сварных соединений квадратного полого профиля из высокопрочной стали.

Introduction

Tubular structures with welded joints are used in a wide range of structural applications. The most typical application is tubular truss. The structural analysis model is frequently constructed using beam finite elements, and the braces are connected to the chords using hinges. Actually, a welded joint does not behave as a hinge when it is loaded by a moment. The joint has resistance against the moment, but in the joint area deformations may occur both at the brace and at the chord, so the stiffness against the moment has to be taken into account in the global analysis of the structure. In [1] only the moment

resistance is given for the joint where the angle between the brace and the chord is 90 degrees. In [2] there is the equation, which can be used to calculate the initial rotational stiffness for the same case, angle 90 degrees.

When aiming to economic and environmental friendly design stiffness of the joints must be taken into account. This is especially true when using high strength steel in structures, because then buckling at the ultimate limit state and deflections and vibrations in the serviceability limit state are often critical. Besides, the impact of the installation process and operation of the construction on its strength properties should be considered [3–8]. In [9] and [10] it has been shown that the rotational stiffness of the welded tubular joint is the main parameter when considering buckling of members of tubular trusses.

In the design it is possible to define the rotational stiffness for the joint using comprehensive finite element analysis (FEA). In practice, this is impossible, especially when performing optimization of structures when the structural analysis must be done thousands of times. In order to avoid computationally heavy calculations, surrogate models (or meta models) have been developed. The surrogate model is the basis of a new direction in the simulation engineering. It is a mathematical method of drawing up a model based on the test results and/or computational experiments carried out with a variety of objects of the class in question [11]. Surrogate models have been used widely in aerospace [12–16], civil engineering [17]. Methods of using surrogate models for optimizing steel structures are described in [19–27]. The optimum design of steel frames is presented using semi-rigid joints and surrogate models [18]. This article shows construction of the surrogate model in the case of initial rotational stiffness of welded tubular structures.

This article describes the construction process of a surrogate model for calculating initial rotational stiffness of welded RHS joints made of high strength steel (HSS). Only joints with butt welds are considered. This assumption was made to simplify the surrogate and finite element modeling. The effect of fillet welds is considered in [28].

Theoretical background

Surrogate model construction

In the surrogate model construction, we replace the computationally expensive function f(x) with a sum of two other functions, where (x) is the vector of the variables [16]), which has the same dimension of input and output parameters as the original function [29, 30]:

$$f(x) = s(x) + \varepsilon(x), \tag{1}$$

where s(x) is the surrogate model at the point x and $\varepsilon(x)$ is the difference between the two.

The idea is to use the function s(x) during calculations or optimization instead of the function f(x). The function s(x) is chosen so that it should be cheap to evaluate, and hereby the computation time can be reduced considerably.

We can start with a quadratic regression model:

$$s_p(x) = \beta_0 + \sum_{i=1}^k \beta_i x_i + \sum_{i=1}^k \beta_{ii} x_i^2 + \sum_{i=1}^{k-1} \sum_{j=i+1}^k \beta_{ij} x_i x_j$$
(2)

or with linear regression ($\beta_{ij} = 0$) or with the constant, only $\beta_0 \neq 0$. If this gives good results (see later criteria), we can add to the regression a *predictor* Z(x) (stochastic process) and end up to Kriging.

Kriging is the most popular method for creating surrogate models. In [31-48] it was made wellknown in the context of modeling, and optimization of deterministic functions, respectively.

The Kriging models consist of two components. The first component is some simple model that captures the trend in the data, and the second component measures deviation between the simple model and the true function. An example of the surrogate model $\overline{f}(x)$ using Kriging with one variable *x* with *n* sample points is:

$$\overline{f}(x) = \frac{1}{n} \sum_{j=1}^{n} x_{j} + Z(x),$$
(3)

where the zero order regression is used and the predicted value f(x) is given scaled to [0;1].

The real values f(x) can be calculated from the normalized data f(x). In the construction of Z(x) we need a *correlation function* between points. Define *R* as the matrix *R* of stochastic-process correlations between the sample points x_i and x_j .

$$R_{ij} = R(\theta, x_i, x_j), \quad i, j, = 1, ..., n$$
 (4)

and let $\bar{r}(x)$ be a vector of correlation between sample points and untried points *x*:

$$\overline{r}(x) = \left[R(\theta, x_1, x) \dots R(\theta, x_n, x) \right]^T$$
(5)

The mostly preferred correlation function is the Gaussian correlation:

$$R(x_i, x_j) = \exp\left[-\sum_{k=1}^m \theta_k \left|x_i^k - x_j^k\right|^2\right],\tag{6}$$

where θ_k are unknown correlation parameters, k = 1, ..., m; *m* is the number of design variables;

 x_{i}^{k} and x_{i}^{k} are the components of samples x_{i} and x_{i} .

After this the surrogate model can be defined, see e.g. [16].

The article [29] indicates the main problems which arise in the construction of surrogate models and their optimization. For instance, the task of reducing the dimension [49], the task of building a multidimensional nonlinear approximating dependencies problem of clustering and classifying data.

The surrogate model can be used with data of both low and high accuracy. These data of low accuracy can be obtained by the analytical method and the data of high accuracy - during the "field" test [50].

The obtained approximate dependence is necessary not only to predict the result, but also to determine the accuracy of the calculation [51]. One of the basic principles to ensure greater accuracy of calculations in constructing the surrogate model is the removal of a set of superfluous or redundant parameters [52]. In particular, the task of reducing the dimension [53] not only greatly simplifies the calculations, but also allows the surrogate model to meet the required conditions [54]. In addition, the method of reducing the dimension of predicates (of explanatory parameters) is practiced [55].

For certain areas, where it is necessary to build a model, which gives the most accurate information, it is necessary to build several approximating dependencies. Thus, the final approximation procedure will include a classifier that determines which private approximator needs to be taken for the given input variable [30].

Some special cases of surrogate model construction are also considered in [56-61].

Surrogate model validation

The validation process uses a new sample size approximately equal to one third of the sample size used to build the surrogate model [62]. The validation process consists of comparing the results of the surrogate model with those of the real response. This is a specific problem which depends on the accuracy required of the fitted model. If this accuracy is too low, the surrogate model must be modified by the introduction of more sample points or by the modification of the surrogate model variables.

The validation process consists of testing a new set of sample points, but excluding the original sample point set. The accuracy of the surrogate model can be checked using R^2 value [18]. No single rule exists that specifies a minimum R^2 value which guarantees a good fitting surrogate model. In [18]

only the surrogate models with R^2 values larger than 0.85 are considered. We realized that R^2 was not proper in our case; all our models had $R^2 \ge 0.95$, so we had to refuse to consider it.

For validation we applied the relative error, Eq. (0.1). Usually, the optimization procedure for tubular trusses requires no more than 10% average relative error, so it is accepted as our main criterion.

$$Error = \frac{\left|C_{FEM} - C_{SURR}\right|}{C_{FEM}} \tag{7}$$

Design of Experiment

The method for determining the sample points to carry out an analysis is called the Design of Experiments (DOE). The location of the sample points is very important for generating an accurate surrogate model. It consists of a compromise between the usage of a reasonable number of sample points to build an accurate model. Several DOE methods are described in [63–65]. The Latin Hypercube Sampling (LHS) proposed by [63] is the most popular space filling sampling technique. In this research engineering justification is used to define the sample points.

Variables

At the first step of sampling the list of variables should be determined. Every Y joint can be described completely by the following variables (Fig. 1): chord dimensions b_0 , t_0 ; brace dimensions b_1 , t_1 ; angle φ between the brace and the chord; material properties f_{y_0} , f_{y_1} .



Figure 1. Dimensions of Y joint

The number of variables has a strong effect on the process of surrogate modeling: the lesser the number of variables is, the easier it is to create a reasonable surrogate model, so it was reasonable to reduce the number of variables.

First of all, we realized that the influence of t_1 on rotational stiffness was rather weak and excluded it from the surrogate modeling. Besides, in this research we considered joints with only butt welds. In the case of butt welds material properties have no effect on the initial rotational stiffness of joints, so we also excluded f_{v0} and f_{v1} from the list of variables.

We also replaced b_1 with its relative analogy $\beta = b_1/b_0$ so that this variable had similar values for all chords. It should be noted that due to low values of β , compared to other variables, it was important to input β with at least four characters after the decimal point to avoid the loss of precision.

After all, we presented the function of the initial rotational stiffness as a function of four variables:

$$C = f(b_0, t_0, \beta, \varphi) \tag{8}$$

Sampling

At the next step sample points should be determined. The sample points should be defined so that:

- they cover the whole range of our interest;
- they meet requirements of building codes
- the failure of the brace is not critical.

of cross-sections.

In this research the Eurocodes were used for joints [1] and extension for steel grades up to S700 [66]. The main requirement restricted the range of sections: $0.25 \le \beta \le 0.85$.

The ratio b_1/t_1 was limited by $b_1/t_1 \le 35$ and to cross-section class 1 or 2. The ratio b_0/t_0 was limited by $10 \le b_0/t_0 \le 35$ and, moreover, to the cross-section class 1 or 2.

The angle φ between the brace and the chord is due to welding in the range of 30 degrees $\leq \varphi \leq$ 90 degrees.

Moreover, we chose the sample points in such a way that for every variable there were 3 different values (minimum, middle and maximum), while others remained constant. Exceptions were made only for cases with the maximum t_0 and β , because they had complicated modeling in Abaqus. For those cases only two values (minimum and middle) of t_0 were considered.

Fig. 2 presents the distribution of sample points in respect to chord dimensions. Black crosses are the possible sections of Ruukki catalogue; blue points are the chosen sample points. Eurocode limits are marked with green lines. Similarly, Fig. 3 presents the relative distributions of sample points in respect to brace dimensions.



Figure 2. Sample points: b₀-t₀



Figure 3. Sample points: β-b₀

As a whole, we chose 285 sample points. To calculate the values of their initial rotational stiffness, the comprehensive FEA was exploited.

Finite element analysis

To calculate values of rotational stiffness in sample points, we conducted the Finite Element Analysis (FEA) in Abaqus. The model was made using C3D8 brick elements. All sections were modeled with round corners, according to [67]. Two-layered mesh was created with solid hexahedral elements, being refined near the joints, as shown in Fig. 4. The butt welds were modeled as "no weld" by using TIE constraints (Fig. 4).



Figure 4. FE model for Y-joint

The material does not influence the stiffness of joints with butt welds, so we applied the elastic material with the modulus of elasticity 210 GPa and Poisson's ratio 0.3.

The analyses were force controlled, and the load step was calculated with "Static, General" procedure. The joint rotation *C* was calculated from FEA by extracting the frame behavior from the FEA results, as given in [68].

The FEA models were validated with the tests of LUT [69] in [68]. The verification was done in three steps [70]: moment load in two opposite directions, use of shell elements instead of brick ones and varying the type of brick elements from 8 to 20 nodes. The proposed FEA model seemed to work well and was used for surrogate modeling.

Finally, the list of sample points is presented in Table 1.

Table 1. Sample points

				· = 30°	· = 60°	• = 90°		· = 30°	· = 60°	= 90°		• = 30°	· = 60°	°00 =
h _o	ß	t,	to	9- 0-	জ [kNm/rac	41 9-	to	9	জ ∑[kNm/ra	чı Э-	to	9	S- C[kNm	e /radl
100	0 400	4	4	55	27	23	6	174	85	72	10	1082	406	345
100	0.600	4	4	215	86	68	6	634	262	211	10	4007	1229	1013
100	0.800	4	4	1135	442	343	6	2847	1107	891				
110	0.364	4	4	44	23	20	5	83	43	37	6	140	72	62
110	0.545	4	4	150	63	50	5	272	116	94	6	450	193	158
110	0.818	4	4	1457	568	439	5	2349	948	751	6	3536	1389	1117
120	0.333	4	5	70	37	33	7.1	203	106	92	10	638	291	253
120	0.583	4	5	364	150	121	7.1	1009	422	345	10	3197	1155	953
120	0.833	5	5	2923	1170	923	7.1	6637	2532	2047				
140	0.286	4	5	53	30	27	7.1	152	85	76	10	453	231	205
140	0.571	5	5	353	143	117	7.1	944	399	328	10	2646	1075	891
140	0.786	5	5	2097	794	618	7.1	4846	1954	1569				
150	0.267	4	6	81	47	42	8.8	262	146	130	12.5	1004	433	382
150	0.533	5	6	448	191	158	8.8	1372	593	494	12.5	5046	1897	1586
150	0.800	6	6	3785	1471	1149	8.8	9697	3742	3034				
160	0.250	4	6	73	44	39	8.8	236	135	122	12.5	862	397	353
160	0.563	5	6	559	232	190	8.8	1677	714	590	12.5	5884	2242	1866
160	0.750	6	6	2493	943	735	8.8	6531	2617	2111				
180	0.278	4	7.1	148	85	76	8.8	280	158	141	12.5	946	467	415
180	0.556	6	7.1	896	378	309	8.8	1617	693	571	12.5	5165	2134	1774
180	0.833	7.1	7.1	8286	3312	2600	8.8	13374	5401	4322				
200	0.250	4	7.1	123	74	66	8.8	233	138	124	12.5	750	402	360
200	0.550	5	7.1	869	367	295	8.8	1566	673	546	12.5	4910	2040	1672
200	0.800	7.1	7.1	6690	2589	2020	8.8	10763	4362	3457				
220	0.273	4	8	203	118	105	10	395	226	203	12.5	840	454	405
220	0.545	7.1	8	1157	490	402	10	2131	922	762	12.5	4347	1899	1580
220	0.818	7.1	8	11157	4415	3474	10	18372	7443	5944				
250	0.280	4	8.8	285	164	146	10	416	237	212	12.5	859	472	422
250	0.560	7.1	8.8	1718	721	589	10	2429	1034	848	12.5	4815	2094	1731
250	0.800	7.1	8.8	12549	4943	3876	10	16880	6768	5384				
260	0.269	4	8.8	267	155	139	10	389	226	203	12.5	798	448	402
260	0.538	7.1	8.8	1496	639	524	10	2118	916	755	12.5	4175	1842	1527
260	0.846	8.8	8.8	20765	8271	6441	10	26909	10941	8628				
300	0.267	5	10	390	227	205	12.5	774	445	401				
300	0.533	8	10	2136	908	747	12.5	4114	1795	1486				
300	0.833	10	10	27313	10664	8367	12.5	45620	18624	14858				

Surrogate model construction

Attempt I

Today there are a number of methods for surrogate modeling. We started the construction of the surrogate model with a linear regression, but the error term R^2 was rather low. Next we exploited Kriging, as a surrogate model type to approximate deterministic noise-free data. Firstly, we used the DACE toolbox for Matlab [71] with zero, linear and second order regression [72] but we did not manage to construct a reasonable model. For our next surrogate modeling we exploited the ooDACE toolbox for Matlab (hereinafter – ooDACE) [73–74] and those results are reported in the article.

METHODS

We constructed surrogate models of two types: single model (one model for all sample points) and multi-model (with an independent model for every b_0). The idea of implementing the second approach was that the variable b_0 is discrete, getting its values from the Ruukki catalogue, with no intermediate values among them. Both types gave rather close results to each other and were used for our final model. It is worth saying that the multi-model requires much less computational time than a single model approach.

Our first validation gave us the following results: $R^2 = 0.8876$, average error about 56% and maximum error 678%. To explore the behavior of the model in detail, we plotted the graphs with rotational stiffness in respect to the different variables. Graphical validation demonstrated that the model behaved very unpredictably (Figs. 5–7).



Figure 6. C-t₀ response

Garifullin M.R., Barabash A.V., Naumova E.A., Zhuvak O.V., Jokinen T., Heinisuo M. Surrogate modeling for initial rotational stiffness of welded tubular joints. *Magazine of Civil Engineering*. 2016. No. 3. Pp. 53–76. doi: 10.5862/MCE.63.4



Pseudo points

During the validation of our model we came to conclusion that we needed more sample points to make it work properly. Calculating new sample points in Abaqus represented a complicated task and required time, so we decided to implement new points by other means.

To improve the behavior of the model, we decided to add certain boundary conditions for the model. It is obvious that for the angle close to 90 degrees (87...89 degrees) the *C*- φ curve must have a zero slope (very close to a horizontal line), see Fig 8. Analytically this means that the partial derivative $dC/d\varphi$ must equal to zero. Practically, to apply this boundary condition to a discrete function, we added some sample points for 95 and 100 degrees angles with the same stiffness value as for 90 degrees. To add the boundary conditions for low angles, we extrapolated stiffness for 20 and 25 degrees using the 4th order polynomial regression. Moreover, we decided to add the points between the existed sample ones. Graphically it is shown in Fig. 8 (for *C*- φ response).

We called these additional points that were determined not by Abaqus, but by other means, as "pseudo" points.



After applying pseudo points, we managed to improve the *C*- ϕ graph (Fig. 9).



To improve the behavior of the surrogate model in respect to other variables (t_0 and β), we introduced additional pseudo points using the idea that for zero values of these variables stiffness responses got zero values as well. Pseudo points were also added for two thicknesses (one between the lowest and the middle and one between the middle and the highest) and three betas (one between the lowest and the middle and two between the middle and the highest).

Overall, we added 1869 pseudo points (both extrapolated and interpolated), resulting with 285 sample points the total number of 2154 points. The whole range of pseudo points is presented in Table 2 (for 100 mm chord only).

Nº	<i>b</i> ₀ [mm]	<i>t</i> ₀ [mm]	β	φ [deg]	C [kNm/rad]
Pseudo φ int	100	4	0.40	45	36
Pseudo φ int	100	4	0.40	75	24
Pseudo φ ext	100	4	0.40	20	75
Pseudo φ ext	100	4	0.40	25	64
Pseudo φ ext	100	4	0.40	95	23
Pseudo φ ext	100	4	0.40	100	23
Pseudo φ int	100	6	0.40	45	113
Pseudo φ int	100	6	0.40	75	74
Pseudo φ ext	100	6	0.40	20	239
Pseudo φ ext	100	6	0.40	25	203
Pseudo φ ext	100	6	0.40	95	72
Pseudo φ ext	100	6	0.40	100	72
Pseudo φ int	100	10	0.40	45	602
Pseudo φ int	100	10	0.40	75	352
Pseudo φ ext	100	10	0.40	20	1654
Pseudo φ ext	100	10	0.40	25	1338
Pseudo φ ext	100	10	0.40	95	345
Pseudo φ ext	100	10	0.40	100	345

Table 2. Pseudo points

Nº	<i>b</i> ₀ [mm]	<i>t</i> ₀ [mm]	β	φ [deg]	C [kNm/rad]
Pseudo φ int	100	4	0.60	45	128
Pseudo φ int	100	4	0.60	75	71
Pseudo φ ext	100	4	0.60	20	310
Pseudo φ ext	100	4	0.60	25	258
Pseudo φ ext	100	4	0.60	95	68
Pseudo φ ext	100	4	0.60	100	68
Pseudo φ int	100	6	0.60	45	381
Pseudo φ int	100	6	0.60	75	219
Pseudo φ ext	100	6	0.60	20	911
Pseudo φ ext	100	6	0.60	25	759
Pseudo φ ext	100	6	0.60	95	211
Pseudo φ ext	100	6	0.60	100	211
Pseudo φ int	100	10	0.60	45	2010
Pseudo φ int	100	10	0.60	75	1030
Pseudo φ ext	100	10	0.60	20	6435
Pseudo φ ext	100	10	0.60	25	5087
Pseudo φ ext	100	10	0.60	95	1013
Pseudo φ ext	100	10	0.60	100	1013
Pseudo φ int	100	4	0.80	45	669
Pseudo φ int	100	4	0.80	75	359
Pseudo φ ext	100	4	0.80	20	1637
Pseudo φ ext	100	4	0.80	25	1364
Pseudo φ ext	100	4	0.80	95	343
Pseudo φ ext	100	4	0.80	100	343
Pseudo φ int	100	6	0.80	45	1653
Pseudo φ int	100	6	0.80	75	922
Pseudo φ ext	100	6	0.80	20	4184
Pseudo φ ext	100	6	0.80	25	3451
Pseudo φ ext	100	6	0.80	95	891
Pseudo φ ext	100	6	0.80	100	891
Pseudo t0 int	100	5	0.40	30	100
Pseudo t0 int	100	8	0.40	30	466
Pseudo t0 ext	100	0	0.40	30	0
Pseudo t0 ext	100	0.1	0.40	30	0
Pseudo t0 ext	100	12	0.40	30	2225
Pseudo t0 ext	100	12.5	0.40	30	2622
Pseudo t0 int	100	5	0.60	30	374
Pseudo t0 int	100	8	0.60	30	1694
Pseudo t0 ext	100	0	0.60	30	0
Pseudo t0 ext	100	0.1	0.60	30	0
Pseudo t0 ext	100	12	0.60	30	8418
Pseudo t0 ext	100	12.5	0.60	30	9969
Pseudo t0 int	100	5	0.80	30	1896
Pseudo t0 int	100	8	0.80	30	5324
Pseudo t0 ext	100	0	0.80	30	0
Pseudo t0 ext	100	0.1	0.80	30	0
Pseudo t0 ext	100	10	0.80	30	8564
Pseudo t0 ext	100	12	0.80	30	12570
Pseudo t0 ext	100	12.5	0.80	30	13691

Nº	<i>b</i> ₀ [mm]	<i>t</i> ₀ [mm]	β	φ [deg]	C [kNm/rad]
Pseudo t0 int	100	5	0.40	60	50
Pseudo t0 int	100	8	0.40	60	201
Pseudo t0 ext	100	0	0.40	60	0
Pseudo t0 ext	100	0.1	0.40	60	0
Pseudo t0 ext	100	12	0.40	60	736
Pseudo t0 ext	100	12.5	0.40	60	843
Pseudo t0 int	100	5	0.60	60	157
Pseudo t0 int	100	8	0.60	60	612
Pseudo t0 ext	100	0	0.60	60	0
Pseudo t0 ext	100	0.1	0.60	60	0
Pseudo t0 ext	100	12	0.60	60	2225
Pseudo t0 ext	100	12.5	0.60	60	2548
Pseudo t0 int	100	5	0.80	60	738
Pseudo t0 int	100	8	0.80	60	2068
Pseudo t0 ext	100	0	0.80	60	0
Pseudo t0 ext	100	0.1	0.80	60	0
Pseudo t0 ext	100	10	0.80	60	3325
Pseudo t0 ext	100	12	0.80	60	4878
Pseudo t0 ext	100	12.5	0.80	60	5312
Pseudo t0 int	100	5	0.40	90	43
Pseudo t0 int	100	8	0.40	90	172
Pseudo t0 ext	100	0	0.40	90	0
Pseudo t0 ext	100	0.1	0.40	90	0
Pseudo t0 ext	100	12	0.40	90	622
Pseudo t0 ext	100	12.5	0.40	90	712
Pseudo t0 int	100	5	0.60	90	126
Pseudo t0 int	100	8	0.60	90	501
Pseudo t0 ext	100	0	0.60	90	0
Pseudo t0 ext	100	0.1	0.60	90	0
Pseudo t0 ext	100	12	0.60	90	1842
Pseudo t0 ext	100	12.5	0.60	90	2112
Pseudo t0 int	100	5	0.80	90	586
Pseudo t0 int	100	8	0.80	90	1690
Pseudo t0 ext	100	0	0.80	90	0
Pseudo t0 ext	100	0.1	0.80	90	0
Pseudo t0 ext	100	10	0.80	90	2740
Pseudo t0 ext	100	12	0.80	90	4041
Pseudo t0 ext	100	12.5	0.80	90	4406
Pseudo β int	100	4	0.50	30	89
Pseudo β int	100	4	0.70	30	524
Pseudo β ext	100	4	0.00	30	0
Pseudo β ext	100	4	0.01	30	0
Pseudo β ext	100	4	0.90	30	2194
Pseudo β ext	100	4	0.95	30	2945
Pseudo β int	100	6	0.50	30	293
Pseudo β int	100	6	0.70	30	1397
Pseudo β ext	100	6	0.00	30	0
Pseudo β ext	100	6	0.01	30	0
Pseudo β ext	100	6	0.90	30	5308
Pseudo β ext	100	6	0.95	30	7035

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Nº	<i>b</i> ₀ [mm]	<i>t</i> ₀ [mm]	β	<i>φ</i> [deg]	C [kNm/rad]
Pseudo β int	100	10	0.50	30	2303
Pseudo β int	100	10	0.70	30	6136
Pseudo β ext	100	10	0.00	30	0
Pseudo β ext	100	10	0.01	30	0
Pseudo β ext	100	10	0.90	30	11102
Pseudo β ext	100	10	0.95	30	12334
Pseudo β int	100	4	0.50	60	39
Pseudo β int	100	4	0.70	60	205
Pseudo β ext	100	4	0.00	60	0
Pseudo β ext	100	4	0.01	60	0
Pseudo β ext	100	4	0.90	60	859
Pseudo β ext	100	4	0.95	60	1156
Pseudo β int	100	6	0.50	60	132
Pseudo β int	100	6	0.70	60	552
Pseudo β ext	100	6	0.00	60	0
Pseudo β ext	100	6	0.01	60	0
Pseudo β ext	100	6	0.90	60	2054
Pseudo β ext	100	6	0.95	60	2721
Pseudo β int	100	10	0.50	60	716
Pseudo β int	100	10	0.70	60	2053
Pseudo β ext	100	10	0.00	60	0
Pseudo β ext	100	10	0.01	60	0
Pseudo β ext	100	10	0.90	60	5207
Pseudo β ext	100	10	0.95	60	6437
Pseudo β int	100	4	0.50	90	32
Pseudo β int	100	4	0.70	90	160
Pseudo β ext	100	4	0.00	90	0
Pseudo β ext	100	4	0.01	90	0
Pseudo β ext	100	4	0.90	90	666
Pseudo β ext	100	4	0.95	90	897
Pseudo β int	100	6	0.50	90	109
Pseudo β int	100	6	0.70	90	444
Pseudo β ext	100	6	0.00	90	0
Pseudo β ext	100	6	0.01	90	0
Pseudo β ext	100	6	0.90	90	1659
Pseudo β ext	100	6	0.95	90	2201
Pseudo β int	100	10	0.50	90	596
Pseudo β int	100	10	0.70	90	1688
Pseudo β ext	100	10	0.00	90	0
Pseudo β ext	100	10	0.01	90	0
Pseudo β ext	100	10	0.90	90	4312
Pseudo β ext	100	10	0.95	90	5344

Attempt II

Using pseudo points for φ , t_0 and β , we managed to construct a new surrogate model with the following parameters: $R^2 = 0.9645$, average error 8% and maximum error 28%,16 points with errors higher than 10% (red points). The same results were observed using a multi-model approach. The graphical validation (Figs. 10, 11) showed that the model behaved properly but its accuracy should be significantly improved.





3D plots in Fig. 12-14 illustrate how the model behaves in respect to several variables simulteneously.





Accuracy improvements

Analyzing the validation points, we came to conclusion that all inaccurate cases were related to the points for which β was predicted (only β or together with other variables). As it was mentioned before, when choosing sample points for every case we calculated three betas: minimum (0.25...0.4), middle (0.5...0.55) and maximum (0.8...0.85), let call them β_1 , β_2 and β_3 respectively. We have noticed that for the points with $\beta_1 < \beta < \beta_2$ the predicted values were lower, whereas for the points with $\beta_2 < \beta < \beta_3$ the opposite trend was observed. Graphically it is shown in Fig. 15.



Figure 15. Differences in C-β curves before (left) and after (right) improvements

As it can be seen from the graph, the curve predicted in Excel using three sample points (blue line) did not suit accurately the actual curve obtained by using additional points (red line). This discrepancy led to inaccurate values of pseudo points used later in surrogate modeling and, eventually, caused

considerable errors in predicted values, up to 28% (green rectangular). The same difference was observed for all cases.

To tackle this discrepancy, we modified beta curves by changing pseudo points. It was done manually and had no scientific basis, but it allowed us to receive more accurate curves (Fig. 15).

Having improved β curves, we constructed a new surrogate model. Validation of the models showed that this time the number of red points decreased twice (from 16 to 8), the average error reduced a little (from 8% to 7%). However, for some points extremely high errors were observed (up to 75%). Validation for a multi-model approach brought slightly different results: 11 red points, 6% average error and 28% maximum.

We also noticed a similar difference in C- φ curves. For the points with $\varphi = 45^{\circ}$ the stiffness values were about 6% higher, whereas for the points with $\varphi = 745^{\circ}$ they were 2% lower. This trend repeated also for the multi-model approach.

To tackle this discrepancy, we applied the same technique as before for β curves and constructed a new surrogate model. We did not manage to receive a working surrogate for the single-model approach, although we did it for the multi-model. The results were rather questionable, although this model predicted the best results for 110 mm chords among the models created before.

Despite the fact that none of improved models met the required criteria of accuracy, they proved that it was possible to modify the model by changing the pseudo points and all these models were used to construct the complex surrogate model.

Final model

For that moment we had 5 surrogate models:

- Two without any improvements (single model and multi-model);
- Two with improved betas (single model and multi-model);
- A multi-model with improved betas and angles.

None of them had satisfactory results. However, their performance was different for various chords. It seemed logical to create a complex model which contained a surrogate model for every chord, which suited it best. Solving this task, every chord was analyzed separately to choose a surrogate model with the best performance. The analysis is given in 0. Here number 1 relates to a model without any improvements, number 2 to the model with improved betas and number 3 to the model with improved betas and angles, SM relates to single model, MM to a multi model.

Table 3. Complex model

<i>b</i> ₀ [mm]	1SM	1MM	2SM	2MM	3MM
100		х			
110					х
120			х		
140			х		
150					х
160				х	
180			х		
200	х				
220			х		
250				х	
260			х		
300			х		

All the models were collected combined in the final model. Its validation is presented in 0. The average error was 4%, maximum 16%.

Chord		Brace		FEM	Surro	Surrogate model		Chord		Brace		FEM Surrogate mod	
b ₀	t ₀	β	φ	С	С	Error [%]	b ₀	t ₀	β	φ	С	С	Error [%]
100	8	0.800	79	1838	1735	5.6	180	10	0.389	59	405	391	3.4
100	6	0.400	42	115	122	6.2	180	12.5	0.667	33	8504	9029	6.2
100	10	0.400	80	349	348	0.4	180	10	0.500	40	1085	1096	1.1
100	8	0.500	89	300	285	5.0	180	10	0.611	60	1415	1470	3.9
110	5	0.364	34	71	72	1.4	200	8.8	0.250	69	130	129	1.0
110	6	0.364	55	76	75	1.2	200	12.5	0.400	62	855	811	5.1
110	6	0.364	50	82	81	0.9	200	8	0.300	57	138	131	4.8
110	5	0.818	88	746	748	0.3	200	7.1	0.600	66	448	518	15.5
120	5.6	0.583	80	170	171	0.8	220	8.8	0.545	71	569	547	3.9
120	7.1	0.750	83	1098	1111	1.2	220	12.5	0.455	58	1195	1247	4.4
120	8.8	0.833	42	6030	6726	11.6	220	10	0.409	37	703	819	16.4
120	7.1	0.500	39	399	387	3.0	220	8	0.727	62	1757	1974	12.4
140	7.1	0.357	90	106	113	6.5	250	12.5	0.720	60	6397	6163	3.7
140	6	0.786	89	977	1017	4.1	250	10	0.600	90	1073	1086	1.2
140	7.1	0.500	40	393	400	1.7	250	12.5	0.600	87	2199	2170	1.3
140	6	0.500	30	349	366	4.8	250	8.8	0.280	72	151	151	0.2
150	7.1	0.533	47	405	400	1.2	260	10	0.846	79	8820	8802	0.2
150	7.1	0.800	89	1643	1783	8.5	260	12.5	0.577	67	2135	2115	0.9
150	6	0.400	71	84	77	8.0	260	12.5	0.308	73	498	522	4.8
150	7.1	0.267	39	104	105	0.9	260	12.5	0.692	51	6208	6222	0.2
160	8.8	0.750	86	1946	2102	8.0	300	12.5	0.833	33	39163	40996	4.7
160	10	0.313	76	244	243	0.3	300	12.5	0.467	56	1290	1396	8.2
160	8.8	0.500	85	416	438	5.3	300	12.5	0.833	85	14946	14908	0.3
160	8.8	0.438	81	303	321	6.0	300	12.5	0.600	86	2247	2338	4.1

Table 4. Final validation

This is the best surrogate model we managed to construct. Since it met the requirements of accuracy introduced above, we accepted the model to optimize the procedure. The detailed description of the surrogate modeling process is provided in [75]. The model is available for free download at [76].

Alternative methods

We also tried alternative approaches for constructing the surrogate model.

First a linear regression model [77] using existing Matlab tools was tried. It was found out that the results were not satisfactory. Therefore, the model was restricted to considering only the nearest sampling space to every validation point. It presents a validation point as:

$$x^{*} = \begin{bmatrix} b_{0}^{*} & t_{0}^{*} & \beta^{*} & \varphi^{*} \end{bmatrix}$$
(9)

The nearest points were chosen to meet the following conditions:

$$\begin{cases}
\left| b_{0}^{*} - b_{0i} \right| < 30 \\
\left| \beta_{0} - \beta_{i} \right| < 0.08 \\
\left| \varphi_{0} - \varphi_{i} \right| < 30
\end{cases}$$
(10)

Then the chosen points were sorted by their normalized distances from the validation point:

$$dist_{i} = \sqrt{\sum_{k=1}^{4} \left[\frac{\max x_{k} - x_{k}^{*}}{\max x_{k} - \min x_{k}} \right]^{2}},$$
(11)

where x_1 , x_2 , x_3 , x_4 relate to b_0 , t_0 , β and φ respectively.

Then 6 nearest points were taken to form the local linear model. Exploiting this procedure, we managed to construct a model with the following results: $R^2 = 0.9552$, average error 21 % and maximum error 115 %.

We tried also an approach using the Matlab toolbox called Polyfitn. It constructs a polynomial regression model using traditional linear least squares techniques. Using this toolbox, we managed to construct a model with the following results: $R^2 = 0.9552$, average error 37 % and maximum error 454 %.

We could be satisfied with none of these models and had to reject both.

Conclusions

1. There exists no analytical method to calculate the initial rotational stiffness for welded tubular Y joints for different angles φ . Surrogate modeling based on the comprehensive FEA might be a reasonable solution to this task. The developed model can be utilized to optimize tubular frames and trusses, as it avoids resorting to time-consuming FE analyses.

2. In this article the Kriging method, realized in ooDACE toolbox for Matlab, was exploited for surrogate modeling. It was shown that the original number of 285 sample points was not enough to construct a physically reasonable surrogate model. To make the model behave properly, the additional sample (pseudo) points were applied without exploiting the comprehensive FEA. Besides, utilizing Latin Hypercube Sampling (LHS), instead of engineering justification, could avoid the problem of sampling.

3. Interpolated pseudo points can cause a considerable loss of accuracy which can be avoided through several iterations. Main attention should be paid to C- β curves, as the variable β plays a dominating role in the surrogate model behavior.

4. The idea of a multi-model approach might be very effective in surrogate modeling in case some of the variables are of discrete type. In this article the final surrogate model was constructed from several others which had the best performance for every chord width.

5. The surrogate model is constructed for Y joints with butt welds loaded by an in-plane bending moment. However, its application might be expanded to consider other joints and loadings by taking into account the effect of fillet welds, the effect of axial forces, the effect of residual stresses, etc. Moreover, the similar model can be constructed for other types of joints (N, K, KT joints) which are widely used in tubular trusses.

References

- European Committee for Standardisation, (CEN). Eurocode 3. Design of steel structures, Part 1–8: Design of joints (EN 1993-1-8:2005). Brussels, 2005.
- Grotmann D., Sedlacek G. Rotational stiffness of welded RHS beam-to-column joints. Cidect 5BB-8/98. RWTH-Aachen. Aachen. 1998.
- 3. Al Ali M., The Welding Process as a Local. Issue with Global Consequences. *Advanced Materials Research*. 2014. Vol. 969. Pp. 340–344.
- Al Ali M., Daneshjo N., Size and Distribution of Welding Stresses. *Procedia Engineering*. 2012. Vol. 40. Pp. 2–7.
- Garifullin M., Trubina D., Vatin N. Local buckling of coldformed steel members with edge stiffened holes. *Applied Mechanics and Materials*. 2015. Vols. 725-726. Pp. 697– 702.
- Vatin N., Havula J., Martikainen L., Sinelnikov A., Orlova A.V., Salamakhin S.V. Thin-walled cross-sections and their joints: Tests and FEM-Modelling. *Advanced Materials Research*. 2014 Vols. 945-949. Pp. 1211–1215.
- Vatin N., Sinelnikov A., Garifullin M., Trubina D. Simulation of cold-formed steel beams in global and distortional buckling. *Applied Mechanics and Materials*. 2014. Vols. 633-634. Pp. 1037–1041.
- Vostrov V.K., Vasilkin A.A. Optimizaciya visot poyasov stenki rezervuara [Optimization of heights of zones of a wall of the tank]. *Montazhnye i Spetsial'nye Raboty v Stroitel'stve.* 2005. No. 11. Pp. 37–39.

Литература

- European Committee for Standardisation, (CEN). Eurocode 3. Design of steel structures, Part 1–8: Design of joints (EN 1993-1-8:2005). Brussels, 2005.
- Grotmann D., Sedlacek G. Rotational stiffness of welded RHS beam-to-column joints. Cidect 5BB-8/98 // RWTH-Aachen. Aachen. 1998.
- Al Ali M., The Welding Process as a Local. Issue with Global Consequences // Advanced Materials Research. 2014. Vol. 969. Pp. 340–344.
- Al Ali M., Daneshjo N., Size and Distribution of Welding Stresses // Procedia Engineering. 2012. Vol. 40. Pp. 2–7.
- Garifullin M., Trubina D., Vatin N. Local buckling of coldformed steel members with edge stiffened holes // Applied Mechanics and Materials. 2015. Vols. 725-726. Pp. 697– 702.
- Vatin N., Havula J., Martikainen L., Sinelnikov A., Orlova A.V., Salamakhin S.V. Thin-walled cross-sections and their joints: Tests and FEM-Modelling // Advanced Materials Research. 2014 Vols. 945-949. Pp. 1211–1215.
- Vatin N., Sinelnikov A., Garifullin M., Trubina D. Simulation of cold-formed steel beams in global and distortional buckling // Applied Mechanics and Materials. 2014. Vols. 633-634. Pp. 1037–1041.
- Востров В.К., Василькин А.А. Оптимизация высот поясов стенки резервуара // Монтажные и специальные работы в строительстве. 2005. № 11. Рр. 37–39.

- Boel H. Buckling length factors of hollow section members in lattice girders. *Ms. Sci. thesis.* Eindhoven University of Technology. Eindhoven.2010.
- Snijder H.H., Boel H.D., Hoenderkamp J.C.D., Spoorenberg R.C. Buckling length factors for welded lattice girders with hollow section braces and chords. *Proceedings of Eurosteel.* 2011. Pp. 1881–1886.
- Prikhodko P.V. Primeneniye metodov agregatsii ekspertov i regressii na osnove gaussovskikh protsessov dlya postroyeniya metamodeley Candidate of physicomathematical sciences dissertation. Moscow institute of physics and technology. Moskow. 2013. Pp.1-26.
- Roux W.J., Stander N., Haftka R.T. Response surface approximations for structural optimization. *International Journal for Numerical Methods in Engineering*. 1998. Vol. 42. No. 3. Pp. 517–534.
- Jin R., Chen W., Simpson T.W. Comparative studies of metamodelling techniques under multiple modelling criteria. *Structural and Multidisciplinary Optimization*. 2001. Vol. 23. No. 1. Pp. 1–13.
- Queipo N.V., Haftka R.T., Shyy W., Goel T., Vaidyanathan R., Kevin Tucker P. Surrogate-based analysis and optimization. *Progress in Aerospace Sciences*. 2005. Vol. 41. Pp. 1–28.
- Kleijnen J.P.C. Simulation experiments in practice: statistical design and regression analysis. *Journal of Simulation.* 2008. Vol. 2. Pp. 19–27.
- Müller J. Surrogate Model Algorithms for Computationally Expensive Black-Box Global Optimization Problems. *Tampere University of Technology. Publication 1092*. 2012.
- Mukhopadhyay T., Dey T.K., Dey S., Chakrabarti A. Optimization of fiber reinforced polymer web core bridge deck – A hybrid approach. *Structural Engineering International.* 2015. Vol. 25. No. 2. Pp. 173–183.
- Díaz C., Victoria M., Querin O.M., Martí P. Optimum design of semi-rigid connections using metamodels. *Journal of Constructional Steel Research.* 2012. Vol. 78. Pp. 97–106.
- Yun G.J., Ghaboussi J., Elnashai A.S. Self-learning simulation method for inverse nonlinear modeling of cyclic behavior of connections. *Computer Methods in Applied Mechanics and Engineering.* 2008. Vol. 197. No. 33-40. Pp. 2836–2857.
- Jadid M.N., Fairbairn D.R. Neural-network applications in predicting moment-curvature parameters from experimental data. *Engineering Applications of Artificial Intelligence*. 1996. Vol. 9. No. 3. Pp. 309–319.
- Anderson D., Hines E.L., Arthur S.J., Eiap E.L. Application of artificial neural networks to the prediction of minor axis steel connections. *Computers & Structures*. 1997. Vol. 63, No. 4. Pp. 685–692.
- Stavroulakis G.E., Avdelas A.V., Abdalla K.M., Panagiotopoulos P.D. A neural network approach to the modelling, calculation and identification of semi-rigid connections in steel structures. *Journal of Constructional Steel Research*. 1997. Vol. 44. No. 1-2. Pp. 91–105.
- De Lima L.R.O., Vellasco P.C.G. da S., De Andrade S.A.L., Da Silva J.G.S., Vellasco M.M.B.R. Neural networks assessment of beam-to-column joints. *Journal of the Brazilian Society of Mechanical Sciences and Engineering*. 2005. Vol. 27. No. 3. Pp. 314–324.
- Guzelbey I.H., Cevik A., Gögüş M.T. Prediction of rotation capacity of wide flange beams using neural networks. *Journal of Constructional Steel Research.* 2006. Vol. 62. No. 10. Pp. 950–961.
- 25. Pirmoz A., Gholizadeh S. Predicting of moment--rotation behavior of bolted connections using neural networks. *3rd national congress on civil engineering.* 2007.

- Boel H. Buckling length factors of hollow section members in lattice girders. // Ms. Sci. thesis. Eindhoven University of Technology. Eindhoven.2010.
- Snijder H.H., Boel H.D., Hoenderkamp J.C.D., Spoorenberg R.C. Buckling length factors for welded lattice girders with hollow section braces and chords // Proceedings of Eurosteel. 2011. Pp. 1881–1886.
- 11. Приходько П.В. Применение методов агрегации экспертов и регрессии на основе гауссовских процессов для построения метамоделей: диссертация ... кандидата физико-математических наук. Московский физико-технический институт. Москва. 2013. С.1-26.
- Roux W.J., Stander N., Haftka R.T. Response surface approximations for structural optimization // International Journal for Numerical Methods in Engineering. 1998. Vol. 42. No. 3. Pp. 517–534.
- Jin R., Chen W., Simpson T.W. Comparative studies of metamodelling techniques under multiple modelling criteria // Structural and Multidisciplinary Optimization. 2001. Vol. 23. No. 1. Pp. 1–13.
- Queipo N.V., Haftka R.T., Shyy W., Goel T., Vaidyanathan R., Kevin Tucker P. Surrogate-based analysis and optimization // Progress in Aerospace Sciences. 2005. Vol. 41. Pp. 1–28.
- Kleijnen J.P.C. Simulation experiments in practice: statistical design and regression analysis // Journal of Simulation. 2008. Vol. 2. Pp. 19–27.
- Müller J. Surrogate Model Algorithms for Computationally Expensive Black-Box Global Optimization Problems // Tampere University of Technology. Publication 1092. 2012.
- Mukhopadhyay T., Dey T.K., Dey S., Chakrabarti A. Optimization of fiber reinforced polymer web core bridge deck – A hybrid approach // Structural Engineering International. 2015. Vol. 25. No. 2. Pp. 173–183.
- Díaz C., Victoria M., Querin O.M., Martí P. Optimum design of semi-rigid connections using metamodels // Journal of Constructional Steel Research. 2012. Vol. 78. Pp. 97–106.
- Yun G.J., Ghaboussi J., Elnashai A.S. Self-learning simulation method for inverse nonlinear modeling of cyclic behavior of connections // Computer Methods in Applied Mechanics and Engineering. 2008. Vol. 197. No. 33-40. Pp. 2836–2857.
- Jadid M.N., Fairbairn D.R. Neural-network applications in predicting moment-curvature parameters from experimental data // Engineering Applications of Artificial Intelligence. 1996. Vol. 9. No. 3. Pp. 309–319.
- Anderson D., Hines E.L., Arthur S.J., Eiap E.L. Application of artificial neural networks to the prediction of minor axis steel connections // Computers & Structures. 1997. Vol. 63, No. 4. Pp. 685–692.
- 22. Stavroulakis G.E., Avdelas A.V., Abdalla K.M., Panagiotopoulos P.D. A neural network approach to the modelling, calculation and identification of semi-rigid connections in steel structures // Journal of Constructional Steel Research. 1997. Vol. 44. No. 1-2. Pp. 91–105.
- De Lima L.R.O., Vellasco P.C.G. da S., De Andrade S.A.L., Da Silva J.G.S., Vellasco M.M.B.R. Neural networks assessment of beam-to-column joints // Journal of the Brazilian Society of Mechanical Sciences and Engineering. 2005. Vol. 27. No. 3. Pp. 314–324.
- Guzelbey I.H., Cevik A., Gögüş M.T. Prediction of rotation capacity of wide flange beams using neural networks // Journal of Constructional Steel Research. 2006. Vol. 62. No. 10. Pp. 950–961.
- Pirmoz A., Gholizadeh S. Predicting of moment--rotation behavior of bolted connections using neural networks // 3rd national congress on civil engineering. 2007.

- Salajegheh E., Gholizadeh S., Pirmoz A. Self-organizing parallel back propagation neural networks for predicting the moment-rotation behavior of bolted connections. *Asian Journal of Civil Engineering.* 2008. Vol. 9. No. 6. Pp. 625– 640).
- Kim J., Ghaboussi J., Elnashai A.S. Mechanical and informational modeling of steel beam-to-column connections. *Engineering Structures*. 2010. Vol. 32. No. 2. Pp. 449–458.
- Heinisuo M., Garifullin M., Jokinen T., Tiainen T., Mela K. Surrogate modeling for rotational stiffness of welded tubular Y- joints. *Proceedings of the eighth international workshop on connection in steel structures (Connections VIII)*. 2016 (accepted manuscript)
- Burnayev Ye.V., Panov M., Kononenko D., Konovalenko I. Sravnitelnyy analiz protsedur optimizatsii na osnove gaussovskikh protsessov [Comparative analysis of optimization procedures based on Gaussian processes][Electronic resources]. Sistem requirements: AdobeAcrobatReader. URL: http://itas2012.iitp.ru/pdf/1569602385.pdf (date of application: 04.11.2015).(rus)
- Bernshteyn A. V. Intellektualnyy analiz dannykh v teorii nadezhnosti [Data mining in reliability theory] [Electronic resources]. Sistem requirements: AdobeAcrobatReader. URL: http://mmr.gubkin.ru/uploads/submitted_papers/bernstein

%20.pdf (date of application: 04.11.15).(rus)

- Matheron G. Principles of geostatistics. *Economic geology*. 1963. Vol. 58. No. 8. Pp. 1246–1266.
- Sacks J., Schiller S.B., Welch W.J. Designs for computer experiments. *Technometrics*. 1989. Vol. 31. No. 1. Pp. 41– 47.
- Sacks J., Welch W.J., Mitchell T.J., Wynn H.P. Design and analysis of computer experiments. *Statistical science*. 1989. Pp. 409–423.
- Jones D.R., Schonlau M., Welch W.J. Efficient global optimization of expensive black-box functions. *Journal of Global optimization*. 1998. Vol. 13. No. 4. Pp. 455–492.
- Halonen L., Applying metamodels to parametric structural analysis. *M. Sci. thesis.* Tampere University of Technology. 2012. Pp.84.
- Kwon H., Yi S., Choi S. Numerical investigation for erratic behavior of Kriging surrogate model. *Journal of Mechanical Science and Technology*. 2014. Vol. 28. No. 9.Pp. 3697–3707.
- Balesdent M., Morio J., Marzat J. Kriging-based Adaptive Importance Sampling Algorithms for Rare Event Estimation. *Structural Safety*. Vol. 44. 2013. Pp.1-10.
- Kaymaz I. Application of kriging method to structural reliability problems. *Structural Safety*. 2005. Vol. 27. Pp.133-151.
- Zhaoyan Lv, Zhenzhou Lu, Pan Wang. A new learning function for Kriging and its applications to solve reliability problems in engineering. *Computers and Mathematics with Applications*. 2015. Vol. 70. Issue 5. Pp.1182-1197.
- David J.J. Toal. Some considerations regarding the use of multi-fidelity Kriging in the construction of surrogate models. *Structural and Multidisciplinary Optimization.* 2015. Vol. 51. Issue 6. Pp.1223-1245.
- Huachao Dong, Baowei Song, Peng Wang, Shuai Huang. Multi-fidelity information fusion based on prediction of kriging. *Structural and Multidisciplinary Optimization*. Vol. 51. Issue 6. 2015. Pp.1267-1280.
- Echard B., Gayton N., Lemaire M. AK-MCS: An active learning reliability method combining Kriging and Monte Carlo Simulation. *Structural Safety*. 2011. Vol. 33. Pp.145-154.
- 43. Fauriat W., Gayton N. AK-SYS: An adaptation of the AK-

- Salajegheh E., Gholizadeh S., Pirmoz A. Self-organizing parallel back propagation neural networks for predicting the moment-rotation behavior of bolted connections // Asian Journal of Civil Engineering. 2008. Vol. 9. No. 6. Pp. 625–640).
- Kim J., Ghaboussi J., Elnashai A.S. Mechanical and informational modeling of steel beam-to-column connections // Engineering Structures. 2010. Vol. 32. No. 2. Pp. 449–458.
- Heinisuo M., Garifullin M., Jokinen T., Tiainen T., Mela K. Surrogate modeling for rotational stiffness of welded tubular Y- joints // Proceedings of the eighth international workshop on connection in steel structures (Connections VIII). 2016 (accepted manuscript)
- 29. Бурнаев Е.В., Панов М., Кононенко Д., Коноваленко И. Сравнительный анализ процедур оптимизации на основе гауссовских процессов [Электронный ресурс]. Систем. требования: AdobeAcrobatReader. URL: http://itas2012.iitp.ru/pdf/1569602385.pdf (дата обращения: 04.11.2015).
- Бернштейн А.В. Интеллектуальный анализ данных в теории надёжности [Электронный ресурс]. Систем. требования: AdobeAcrobatReader. URL: http://mmr.gubkin.ru/uploads/submitted_papers/bernstein %20.pdf (дата обращения: 04.11.15).
- Matheron G. Principles of geostatistics // Economic geology. 1963. Vol. 58. No. 8. Pp. 1246–1266.
- Sacks J., Schiller S.B., Welch W.J. Designs for computer experiments // Technometrics. 1989. Vol. 31. No. 1. Pp. 41–47.
- Sacks J., Welch W.J., Mitchell T.J., Wynn H.P. Design and analysis of computer experiments // Statistical science. 1989. Pp. 409–423.
- Jones D.R., Schonlau M., Welch W.J. Efficient global optimization of expensive black-box functions // Journal of Global optimization. 1998. Vol. 13. No. 4. Pp. 455–492.
- Halonen L., Applying metamodels to parametric structural analysis // M. Sci. thesis. Tampere University of Technology. 2012. Pp.84.
- Kwon H., Yi S., Choi S. Numerical investigation for erratic behavior of Kriging surrogate model // Journal of Mechanical Science and Technology. 2014. Vol. 28. No. 9. Pp. 3697–3707.
- Balesdent M., Morio J., Marzat J. Kriging-based Adaptive Importance Sampling Algorithms for Rare Event Estimation // Structural Safety. Vol. 44. 2013. Pp.1-10.
- Kaymaz I. Application of kriging method to structural reliability problems // Structural Safety. 2005. Vol. 27. Pp.133-151.
- Zhaoyan Lv, Zhenzhou Lu, Pan Wang. A new learning function for Kriging and its applications to solve reliability problems in engineering // Computers and Mathematics with Applications. 2015. Vol. 70. Issue 5. Pp.1182-1197.
- David J.J. Toal. Some considerations regarding the use of multi-fidelity Kriging in the construction of surrogate models // Structural and Multidisciplinary Optimization. 2015. Vol. 51. Issue 6. Pp.1223-1245.
- Huachao Dong, Baowei Song, Peng Wang, Shuai Huang. Multi-fidelity information fusion based on prediction of kriging // Structural and Multidisciplinary Optimization. 2015. Vol. 51. Issue 6. Pp.1267-1280.
- Echard B., Gayton N., Lemaire M. AK-MCS: An active learning reliability method combining Kriging and Monte Carlo Simulation // Structural Safety. 2011. Vol. 33. Pp.145-154.
- Fauriat W., Gayton N. AK-SYS: An adaptation of the AK-MCS method for system reliability // Reliability Engineering and System Safety. 2014. Vol. 123. Pp.137-144.
- 44. Xufeng Yang, Yongshou Liu, Yi Gao, Yishang Zhang,

MCS method for system reliability. *Reliability Engineering and System Safety.* 2014. Vol. 123. Pp.137-144.

- 44. Xufeng Yang, Yongshou Liu, Yi Gao, Yishang Zhang, Zongzhan Gao. An active learning kriging model for hybrid reliability analysis with both random and interval variables. *Struct Multidisc Optim.* 2015. No. 51. Pp. 1003-1016.
- 45. Yongkai An, Wenxi Lu, Weiguo Cheng. Surrogate Model Application to the Identification of Optimal Groundwater Exploitation Scheme Based on Regression Kriging Method—A Case Study of Western Jilin Province. International Journal of Environmental Research and Public Health .2015. No. 12. Pp. 8897-8918.
- 46. Zhaoyan Lv, Zhenzhou Lu, Pan Wang. A new learning function for Kriging and its applications to solve reliability problems in engineering. *Computers and Mathematics with Applications*. 2015. No. 70. Pp. 1182-1197.
- 47. Marrel A., Marie N., De Lozzo M. Advance surrogate model and sensitivity analysis methods for sodium fastreactor accident assessment. *Reliability Engineering and System Safety.* 2015. No. 138. Pp. 232-241.
- Jiang Xiangwen, Zhao Qijun, Zhao Guoqing, Li Peng. Integrated optimization Analyses of aerodynamic/shealth characteristics of helicopter rotor based on surrogate model. *Chinese Journal of Aeronautics*. 2015. No. 28(3). Pp. 737–748.
- 49. Bernshteyn A.V., Burnayev Ye.V., Yerofeyev P.D. Eksperimentalnoye sravneniye podkhodov k zadache modelirovaniya mnogoobraziy [Experimental comparison of approaches to the problem of modeling varieties]. *Trudy* 55-y nauchnoy konferentsii MFTI. Tom 1. (Upravleniye i prikladnaya matematika). Moscow.: Izd-vo MFTI, 2012. 98 p. (rus)
- Burnayev Ye.V., Zaytsev A.A. Surrogatnoye modelirovaniye raznotochnykh dannykh v sluchaye vyborok bolshogo razmera [Surrogate modeling of mutlifidelity data for large samples]. *Information* processes. Vol.15. No. 1. 2015. Pp. 97–109.(rus)
- Burnayev Ye.V., Panov M.Ye. Ob otsenivanii tochnosti surrogatnykh modeley [About the estimation accuracy of surrogate models]. *Trudy 53-y nauchnoy konferentsii MFTI*. (Sektsiya problem peredachi i obrabotki informatsii). Moscow: Izd-vo MFTI. 2010. Pp. 105-106. (rus)
- Burnayev Ye.V., Prikhodko P.V. Metodologiya postroyeniya surrogatnykh modeley dlya approksimatsii prostranstvenno-neodnorodnykh funktsiy [Methodology of construction of surrogate models for the approximation of spatially inhomogeneous functions]. *Trudy MFTI*. Vol. 5. No. 4. (Informatika, matematika). Moscow: Izd-vo MFTI. 2013. Pp. 122-132.(rus)
- 53. Bernshteyn A.V., Kuleshov A.P. Kognitivnyye tekhnologii v probleme snizheniya razmernosti opisaniya geometricheskikh obyektov [Cognitive technologies in reducing the problem of dimensionality description of geometric objects]. *Informatsionnye tekhnologii I vychislitelnye sistemy*. 2008. No. 2/4. Pp. 6-19. (rus)
- 54. Yalaletdinov A.D., Chepyzhov V.V., Chernova S.S. Primeneniye protsedur snizheniya razmernosti k surrogatnoy modeli aerodinamiki kryla samoleta v zadachakh optimizatsii [The use of dimension reduction procedures for surrogate model of aircraft wing's aerodynamics in optimization problems] [Electronic resources]. Sistem requirements: AdobeAcrobatReader. URL: http://itas2011.iitp.ru/pdf/1569459067.pdf (date of application: 04.11.15). (rus)
- Bernshteyn A.V., Kuleshov A.P. Snizheniye razmernosti pri nalichiye predikatov [Reducing dimension in presence of predicates]. Information processes. 2008. Vol. 8. No. 1. Pp. 47-57. (rus)
- 56. Belyayev M.G. Uchet osobennostey dizayn eksperimenta pri reshenii zadach approksimatsii v surrogatnom

Zongzhan Gao. An active learning kriging model for hybrid reliability analysis with both random and interval variables // Struct Multidisc Optim. 2015. No. 51. Pp. 1003-1016.

- 45. Yongkai An, Wenxi Lu, Weiguo Cheng. Surrogate Model Application to the Identification of Optimal Groundwater Exploitation Scheme Based on Regression Kriging Method—A Case Study of Western Jilin Province // International Journal of Environmental Research and Public Health. 2015. No. 12. Pp. 8897-8918.
- 46. Zhaoyan Lv, Zhenzhou Lu, Pan Wang. A new learning function for Kriging and its applications to solve reliability problems in engineering // Computers and Mathematics with Applications. 2015. No. 70. Pp. 1182-1197.
- 47. Marrel A., Marie N., De Lozzo M. Advance surrogate model and sensitivity analysis methods for sodium fastreactor accident assessment // Reliability Engineering and System Safety. 2015. No. 138. Pp. 232-241.
- Jiang Xiangwen, Zhao Qijun, Zhao Guoqing, Li Peng. Integrated optimization Analyses of aerodynamic/shealth characteristics of helicopter rotor based on surrogate model // Chinese Journal of Aeronautics. 2015. No. 28(3). Pp. 737–748.
- 49. Бернштейн А.В., Бурнаев Е.В., Ерофеев П.Д. Экспериментальное сравнение подходов к задаче моделирования многообразий // Труды 55-й научной конференции МФТИ. Том. 1 (Управление и прикладная математика). Москва: Изд-во МФТИ, 2012. 98 с.
- 50. Бурнаев Е.В., Зайцев А.А. Суррогатное моделирование разноточных данных в случае выборок большого размера // Информационные процессы. Том 15. № 1. 2015. С. 97–109.
- Бурнаев Е.В.б Панов М.Е. Об оценивании точности суррогатных моделей // Труды 53-й научной конференции МФТИ (Секция проблем передачи и обработки информации). Москва: Изд-во МФТИ, 2010. С. 105-106.
- 52. Бурнаев Е.В., Приходько П.В. Методология построения суррогатных моделей для аппроксимации пространственно неоднородных функций // Труды МФТИ. Том 5 №4. (Информатика, математика). Москва: Изд-во МФТИ. 2013. С. 122-132.
- 53. Бернштейн А.В., Кулешов А.П. Когнитивные технологии в проблеме снижения размерности описания геометрических объектов // Информационные технологии и вычислительные системы. 2008 № 2/4. С. 6-19.
- 54. Ялалетдинов А.Д., Чепыжов В.В., Чернова С.С. Применение процедур снижения размерности к суррогатной модели аэродинамики крыла самолёта в задачах оптимизации [Электронный ресурс]. Систем. требования: AdobeAcrobatReader. URL: http://itas2011.iitp.ru/pdf/1569459067.pdf (дата обращения: 04.11.15).
- 55. Бернштейн А.В., Кулешов А.П. Снижение размерности при наличие предикатов // Информационные процессы. 2008. Vol. 8. № 1. Рр. 47-57.
- 56. Беляев М.Г. Учёт особенностей диазйн эксперимента при решении задач аппроксимации в суррогатном моделировании [Электронный ресурс]. Систем. требования: AdobeAcrobatReader. URL: http://itas2013.iitp.ru/disk/pdf/1569754979.pdf (дата обращения: 04.11.15).
- 57. Назаренко А.М. Эффективный алгоритм многокритериальной суррогатной оптимизации: выпускная квалификационная работа на степень магистра. ИППИ РАН. Москва. 2013. С. 1-48.
- 58. Бурнаев Е.В., Янович Ю.А.. Построение гладких суррогатных моделей // Труды 53-й научной конференции МФТИ (Секция проблем передачи и обработки информации). Москва: Изд-во МФТИ, 2010.

modelirovanii [Accounting features of experimental design for solving approximation tasks in surrogate modeling] [Electronic resources]. Sistem requirements: AdobeAcrobatReader. URL: http://itas2013.iitp.ru/disk/pdf/1569754979.pdf (date of application: 04.11.15). (rus)

- 57. Nazarenko A.M. Effektivnyy algoritm mnogokriterialnoy surrogatnoy optimizatsii: vypusknaya kvalifikatsionnaya rabota na stepen magistra [An efficient algorithm for multiobjective surrogate optimization: Final qualifying work on a master's degree]. IITP RAS. Moscow. 2013. Pp. 1-48. (rus)
- Burnayev Ye.V., Yanovich Yu.A. Postroyeniye gladkikh surrogatnykh modeley [Construction of smooth surrogate models]. *Trudy 53-y nauchnoy konferentsii MFTI*. (Sektsiya problem peredachi i obrabotki informatsii). Mosow. : Izd-vo MFTI, 2010. Pp. 103-104. (rus)
- 59. Burnayev Ye. V., Yerofeyev P., Zaytsev A., Kononenko D., Kapushev Ye. Surrogatnoye modelirovaniye i optimizatsiya profilya kryla samoleta na osnove gaussovskikh protsessov [Surrogate modeling and optimization of aircraft wing profile based on Gaussian processes] [Electronic resources]. Sistemrequirements: AdobeAcrobatReader. URL: http://itas2012.iitp.ru/pdf/1569602325.pdf (date of application: 04.11.2015). (rus)
- 60. Kornilov M.V., Sysoyev I.V., Bezruchko B.P. Optimalnyy podbor parametrov prognosticheskikh modeley v metode nelineynoy prichinnosti po Greyndzheru v prilozhenii k signalam, kharakterizuyemymi khorosho vyrazhennymi vremennymi masshtabami [Optimal selection of parameters of the forecasting models used for the nonlinear Granger causality method in application to the signals with a main time scales]. *Izhevsk: NITs «RKhD»*. 2014. Vol. 10. No. 3. Pp. 279-295. (rus)
- Kornilov M.V., Sysoyev I.V. Vliyaniye vybora struktury modeli na rabotosposobnost metoda nelineynoy prichinnosti po Greyndzheru [Influence of the choice of the modle structure for working capacity of ninlinear Granger causality approach]. *Izvestiya VUZ. Applied Nonlinear Dynamics.* 2013. Vol. 21. No. 2. Pp. 74-87. (rus)
- Lee T., Jung, J. Metamodel-based shape optimization of connecting rod considering fatigue life. *Key Engineering Materials*. 2006. Vol. 306-308. Pp. 211–216.
- 63. McKay M.D., Bechman R.J., Conover W.J. A comparison of three methods for selecting values of input variables in the analysis of output from a computer code. *Technometrics.* 1979. Vol. 21. No. 2. Pp. 239–245.
- 64. Fang K.T., Li R., Sudjianto A. Design and modeling for computer experiments. Chapman & Hall/CRC. 2006.
- 65. Montgomery D.C. *Design and Analysis of Experiments.* John Wiley & Sons. 2012.
- European Committee for Standardisation, (CEN). Eurocode 3. Design of steel structures, Part 1-12: Additional rules for the extension of EN 1993 up to steel grades S 700 (EN 1993-1-12: 2007). Brussels. 2007
- European Committee for Standardisation, (CEN). Cold formed welded structural hollow sections of non-alloy and fine grain steels. Part 2: Tolerances, dimensions and sectional properties (EN 10219-2:2006). Brussels. 2006
- Haakana Ä. In-Plane Buckling and Semi-Rigid Joints of Tubular High Strength Steel Trusses. *Ms. Sci. thesis.* Tampere University of Technology. 2014.
- Tuominen N., Björk T. Ultimate Capacity of Welded Joints Made of High Strength Steel CFRHS. *Proceedings of Eurosteel.* 2014. Pp. 83–84.
- Mela K., Heinisuo M. Weight and cost optimization of welded high strength steel beams. *Proceeding of the METNET Seminar 2013 in Lulea.* 2014. Pp 44-55
- 71. Lophaven S.N., Søndergaard J. and Nielsen H.B. DACE,

Гарифуллин М.Р., Барабаш А.В., Наумова Е.А., Жувак О.В., Йокинен Т., Хейнисуо М. Суррогатное моделирование для определения начальной жесткости вращения сварных трубчатых соединений // Инженерно-строительный журнал. 2016. № 3(63). С. 53–76.

C. 103-104.

- 59. Бурнаев Е.В., Ерофеев П.Д., Зайцев А.А., Кононенко Д., Капушев Е. Суррогатное моделирование и оптимизация профиля крыла самолёта на основе гауссовских процессов [Электронный ресурс]. Систем. требования: AdobeAcrobatReader. URL: http://itas2012.iitp.ru/pdf/1569602325.pdf (дата обращения: 04.11.2015).
- 60. Корнилов М.В., Сысоев И.В., Безручко Б.П. Оптимальный подбор параметров прогностических моделей в методе нелинейной причинности по Грейнджеру в приложении к сигналам, характеризуемым хорошо выраженными временными масштабами // НИЦ Регулярная и хаотическая динамика. 2014. Vol. 10. № 3. Рр. 279-295.
- 61. Корнилов М.В., Сысоев И.В. Влияние выбора структуры модели на работоспособность метода нелинейной причинности по Грейнджеру // Изв. вузов. ПНД. 2013. Том 21. № 2. С. 74-87.
- Lee T., Jung, J. Metamodel-based shape optimization of connecting rod considering fatigue life // Key Engineering Materials. 2006. Vol. 306-308. Pp. 211–216.
- McKay M.D., Bechman R.J., Conover W.J. A comparison of three methods for selecting values of input variables in the analysis of output from a computer code // Technometrics. 1979. Vol. 21. No. 2. Pp. 239–245.
- 64. Fang K.T., Li R., Sudjianto A. Design and modeling for computer experiments. Chapman & Hall/CRC. 2006.
- 65. Montgomery D.C.Design and Analysis of Experiments. John Wiley & Sons. 2012.
- European Committee for Standardisation, (CEN). Eurocode 3. Design of steel structures, Part 1-12: Additional rules for the extension of EN 1993 up to steel grades S 700 (EN 1993-1-12: 2007). Brussels. 2007
- European Committee for Standardisation, (CEN). Cold formed welded structural hollow sections of non-alloy and fine grain steels. Part 2: Tolerances, dimensions and sectional properties (EN 10219-2:2006). Brussels. 2006
- Haakana Ä. In-Plane Buckling and Semi-Rigid Joints of Tubular High Strength Steel Trusses // Ms. Sci. thesis. Tampere University of Technology. 2014.
- Tuominen N., Björk T. Ultimate Capacity of Welded Joints Made of High Strength Steel CFRHS // Proceedings of Eurosteel. 2014. Pp. 83–84.
- Mela K., Heinisuo M. Weight and cost optimization of welded high strength steel beams // Proceeding of the METNET Seminar 2013 in Lulea. 2014. Pp 44-55
- Lophaven S.N., Søndergaard J., Nielsen H.B. DACE, A MATLAB Kriging Toolbox Version 2.0. August 1. Technical University of Denmark, Copenhagen.2002.
- Heinisuo M., Mela K., Tiainen T., Jokinen T., Baczkiewicz J. and Garifullin M. Surrogate model for rotational stiffness of welded tubular Y-joints // Proceedings of the METNET Seminar 2015 in Budapest. 2015. Pp. 18–39.
- Couckuyt I., Dhaene T., Demeester P. ooDACE Toolbox: A Flexible Object-Oriented Kriging Implementation // Journal of Machine Learning Research. 2014. Vol. 15. Pp. 3183–3186.
- Ulaganathan S., Couckuyt I., Deschrijver D., Laermans E., Dhaene T. A Matlab Toolbox for Kriging Metamodelling // Procedia Computer Science. 2015. Vol. 51 Pp. 2708– 2713.
- 75. Garifullin M., Jokinen T. and Heinisuo M. Supporting document for surrogate model construction of welded HSS tubular Y-joints // Tampere University of Technology, Tampere. Publication 164. 2016.
- 76. Research Centre of Metal Structures. [Электронный ресурс]. Систем. требования: AdobeAcrobatReader. URL:http://metallirakentaminen.fi/ (дата обращения:

A MATLAB Kriging Toolbox Version 2.0. August 1. Technical University of Denmark, Copenhagen.2002.

- Heinisuo M., Mela K., Tiainen T., Jokinen T., Baczkiewicz J. and Garifullin M. Surrogate model for rotational stiffness of welded tubular Y-joints. *Proceedings of the METNET Seminar 2015 in Budapest.* 2015. Pp. 18–39.
- Couckuyt I., Dhaene T., Demeester P. ooDACE Toolbox: A Flexible Object-Oriented Kriging Implementation. *Journal of Machine Learning Research*. 2014. Vol. 15. Pp. 3183–3186.
- Ulaganathan S., Couckuyt I., Deschrijver D., Laermans E., Dhaene T. A Matlab Toolbox for Kriging Metamodelling. *Procedia Computer Science*. 2015. Vol. 51 Pp. 2708– 2713.
- Garifullin M., Jokinen T. and Heinisuo M. Supporting document for surrogate model construction of welded HSS tubular Y-joints. *Tampere University of Technology, Tampere. Publication* 164. 2016.
- Research Centre of Metal Structures. [Electronic resources]. Sistem requirements: AdobeAcrobatReader. URL:http://metallirakentaminen.fi/ (date of application: 29.01.16)
- Tiainen T., Heinisu M., Jokinen T. and Salminen M. Steel building optimization applying metamodel techniques. *Rakenteiden Mekaniikka (Journal of Structural Mechanics).* 2012. Vol. 45. No. 3. Pp. 152–161.

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 Tiainen T., Heinisu, M., Jokinen T. and Salminen M. Steel building optimization applying metamodel techniques // Rakenteiden Mekaniikka (Journal of Structural Mechanics). 2012. Vol. 45. No. 3. Pp. 152–161.

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