doi: 10.18720/MCE.78.4

Dynamic factors in case of damaging continuous beam supports

Коэффициенты динамичности при повреждении опор неразрезных балок

A. Tusnin,
Moscow State University of Civil Engineering, Moscow, Russia

Д-р техн. наук, профессор А.Р. Туснин,
Московский государственный строительный университет, Москва, Россия

Key words: beam; support; liveness calculations; dynamic factor; dynamic calculations; frequency; time length

Ключевые слова: балка; опора; расчет живучести; коэффициент динамичности; динамический расчет; частота; временной промежуток

Abstract. The article provides the research results of the continuous beam operation in case of the support damage. When the support is damaged, the design model of the beam is changing, the spans are increasing, and the force in the beam is increasing. Moreover, a fast removal of the support at the effect of the unaltering during the destruction of the beam concentrated or distributed force will lead to the evolvement of the vibrations and increase of the beam force. The theoretical jurisdictions are provided for the determination of dynamic factors which might be used for the determination of dynamic force based on the results of the static calculations for the damaged construction. Theoretical dynamic factors are determined for the beams loaded by the concentrated loads. The numerical computations have been performed with the use of finite-element design models. By the example of the continuous beams loaded by the concentrated and distributed forces, the consequence of the dynamic calculations for damaged beams is shown taking to account the time of the support breakdown. It is set that the maximum force values appear in the beams with time for the support damage from 0.01 to 0.1 sec. The comparison of theoretical and numerical dynamic factors is conducted. It showed a good compliance of factor values determined by different methods. The recommendations are provided for practical applications of the dynamic factor at the calculation of continuous beams.

Аннотация. В статье представлены результаты исследования работы неразрезных балок при повреждении опор. При повреждении опор изменяется расчётная схема балки, возрастают пролёты и увеличиваются усилия в балке. Кроме того быстрое исключение опоры при действии на балку не меняющейся в момент разрушения сосредоточенной или распределённой нагрузки ведёт к развитию колебаний и увеличению усилий в балке. Представлены теоретические обоснования для определения коэффициентов динамичности, которые можно использовать для определения динамических усилий по результатам статических расчётов повреждённой конструкции. Теоретические коэффициенты динамичности определены для балок загруженных сосредоточенными нагрузками. Проведены численные расчёты с применением конечно-элементных расчётных схем. На примере неразрезных балок, загруженных сосредоточенными и распределёнными нагрузками показана последовательность динамических расчётов повреждённых балок с учётом времени выхода опор из строя. Установлено, что максимальные значения усилий возникают в балках при времени повреждения опоры от 0,01 до 0,1 сек. Выполнено сравнение теоретических и численных коэффициентов динамичности, показавшее хорошее совпадение значений коэффициентов, полученных разными методами. Даны рекомендации для практического применения коэффициента динамичности при расчетах неразрезных балок.

1. Introduction

In Russia and other countries, a scientific direction which studies the behavior of load-bearing structures when they are damaged is developing. In the design practice, the analysis of the bearing capacity of structures in the case of damage is called the calculation of stability against progressive collapse or the calculation of survivability. The problem of studying of the bearing capacity of damaged structures is very urgent due to the adverse consequences of the destruction of buildings [1, 2]. In

Туснин А.Р. Коэффициенты динамичности при повреждении опор неразрезных балок // Инженерно-строительный журнал. 2018. № 2(78). С. 47–64.

Russia, such a calculation is carried out, as a rule, when designing unique buildings and structures. However, the calculation of stability with progressive collapse is also carried out for buildings of mass construction [3–5]. The purpose of the calculation of survivability is to increase the reliability of structures and to prevent the destruction of the building in case of damage to certain elements of load-bearing structures.

In the article [6] there are represented an overview of the regulatory requirements of the United States, Canada, and Europe in carrying out calculations to prevent avalanche collapse, and methods to avoid progressive collapse are analyzed.

In article [7] the problem of providing load-bearing capacity in accidents leading to damage the structures is considered as a complex of preliminary and operational measures reducing possible adverse consequences from the destruction of individual elements. The formulation of the survivability of structures is presented in [8]. Great attention is paid to resistance to the progressive destruction of frame buildings [9–11]. The authors consider the destruction schemes, and the results of calculations of the damaged structures in static and dynamic formulations are presented. In the paper [12], a simplified approach to the calculation of damaged structures is considered, which ensures an acceptable accuracy of the results. The causes of the destruction of structures and constructive measures that reduce the possibility of progressive destruction are considered in [13]. The questions of the evaluation of the ability of structures to resist to progressive destruction have been investigated in [14], where new recommendations increasing the survivability of structures are shown. The design of the joints has a significant effect on ensuring the survivability of the load-bearing system [15]. The problems of progressive collapse of steel trusses and frames are being investigated. On the example of the destruction of the large-span bridge [16], the reasons for the collapse of the span are analyzed. For a multistory steel frame [17], questions of structural behavior in the case of damage of the lower floor column are investigated, as well as the influence of the damping coefficient, which varies from 0.01 to 0.1.

Static, dynamic, linear and nonlinear calculations are successfully used to analyze load-bearing capacity of the damaged structures [18–20]. The effect of an increase of the number of floors on the decrease in dynamic effects in the framework was noted [18].

The review of literature has shown that significant studies of the work of damaged structures have been carried out. There are normative requirements for the design of structures with regard to the destruction of individual elements. There are recommendations for the selection of damaged elements, the appointment of the load and the appointment of materials. However, up to the present time there are no recommendations for designers on the calculation of damaged structures taking into account dynamic processes. One way to calculate survivability is to apply a modified long-term load with a dynamic factor that is determined by the damaged element, its failure rate and the current load. The magnitude of the dynamic factor with a fast turn-off of the element can be taken as 2, assuming an instantaneous application of the load. However, for spatial structures, local damage, even with instantaneous failure of the element, does not always lead to dynamic effects corresponding to the dynamism factor 2. In addition, the dynamic effects on the structure depend on the failure rate of the structural element. Determination of the dynamic coefficients for typical structures with different damage schemes is of great practical importance.

The aim of the study is to determine the dynamic coefficients for the concentrated and distributed load on continuous beams in the case of damage of intermediate supports.

To achieve this goal, the following tasks are solved:

1. Theoretical justification of the dynamic coefficients under the action of concentrated loads on the beam.

2. Development of a numerical technique for the dynamic calculation of continuous beams in the case of damage of supports and the action of concentrated and distributed loads.

3. Numerical studies of the dynamic coefficients of continuous beams in the case of damage of supports, including beam systems contain main and secondary beams.

2. Methods

For the purpose of theoretical justification of the dynamic factor, the simple beam construction can be used [19, 20]. Let us consider the double-span beam with a hinged support. In the midspan, the beam was subjected to a concentrated load P. Damage to the construction includes midspan support damage. The basic and damaged beams are shown in Figure 1.

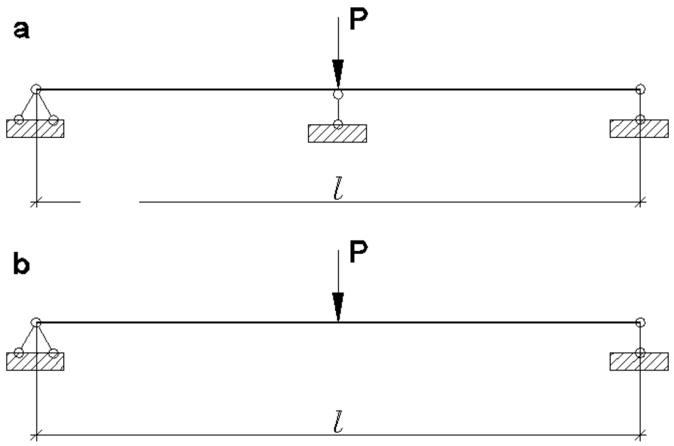

Figure1. Beam subjected to a concentrated load in the midspan:
a – basic construction; b – damaged construction.

Depending on the initial position of the beam and midspan support (deflection or hogging of the beam in the support area), several types of interaction between this support and the beam are possible:

Type 1. Supporting force is zero, i.e. at the static action of the load P the beam span *l* due to the behavior in bending flexed to the size of deflection w_c (w_c – beam deflection on two supports from the concentrated force P), and only then an intermediate support was underpinned.

Type 2. The support was pinned under a load P_o equal to force fraction P, at the moment of the contact of the beam with the midspan support the beam deflection is w_o and is equal to the proportional to the load part of deflection w_c of the beam on two supports from the normal load.

Type 3. The beam supports on the intermediate support before the load is applied, initial deflection is zero.

Type 4. The beam supports on the intermediate support, herein the beam is put into the shape of initial upward buckling equal to w_c of the beam span *l* on two hinged supports under the action of force P.

We shall find the dynamic factor for the concentrated load, acting in each mentioned type of structural damage. To determine the dynamic factor k_d known relationship may be used [21]:

$$k_d = 1 + \sqrt{1 + \frac{2H}{w_c}}, \tag{1}$$

where H – height, with which the weight P – falls down on the beam, w_c – deflection of the beam span l of hinge-supported at each end of the static action of force P. This dependence is valid when the weight falls down on the undeformed beam, design model of which isn't changed in the process of interaction between the beam and falling weight. If the design model is changed, additional studying of the behavior of the construction is needed.

For finding the connection between force and deflection, use the following relations [21]: $P = cw_c$ and $P_d = cw_d$, where c – control stiffness (the same for static and dynamic load action), P_d – dynamic force, w_d – dynamic deflection.

2.1. Type 1

In the first type, the beam originally has the deflection equal to the full static when it is supported on two seats at the ends, and the effect of force in the middle P. The absence of the intermediate support in case of its damaging does not affect the behavior of the beam. Therefore, the dynamic factor for the load P will be 1.

2.2. Type 2

In the first type of the damage of the construction, the part of the full load P_o is taken up through the beam deformations, and the remaining load is delivered on the intermediate support. At the initial deflection w_o prior to the beginning of interaction with the support, the fraction of the force taken up by the beam deformation is $P_o = \frac{P\,w_o}{w_c}$. After removing the support, the beam will be affected by the instantly superimposed unbalanced fraction of force, equal to $\Delta P = P\left(1-\frac{w_o}{w_c}\right)$. This part of the load will have a dynamic effect with a dynamic factor 2, as instantly superimposed to the beam. The aggregate load on the beam will be: $P_d = P_o + 2\Delta P = P\left(\frac{w_o}{w_c}+2-\frac{2w_o}{w_c}\right) = P\left(2-\frac{w_o}{w_c}\right)$. The dynamic factor of the aggregate load, in this case, will be:

$$k_d = \frac{P_d}{P} = 2-\frac{w_o}{w_c} \tag{2}$$

With the initial deflection over the midspan support, $w_o = 0.5w_c$ the dynamic factor calculated using the formula (2) will be 1.5.

2.3. Type 3

In the third type of the damage of the construction, the total load is delivered to the midspan support. After the instant removal of this support, the whole load is instantly applied to the beam, the initial height from which the weight falls (acts) P is zero, and the dynamic factor is $k_d = 2$. This result can be obtained using the formula (1), or the formula (2).

2.4. Type 4

In the fourth type of the damage of the construction (the initial deflection of the midspan support is w_c), in order to determine the dynamic factor let us consider the power U_d, which is accumulated in the system after its deformation, the strain power of preliminary hogging of the beam U_o and action A, performed by the load P after the removal of the support. The power balance of the system makes it possible to make up an equation:

$$U_o + A = U_d \tag{3}$$

where $U_o = \frac{Pw_c}{2}$ is the power accumulated in the beam when it is hogged over the midspan support by the amount w_c, $A = P(w_c + w_d)$, $U_d = \frac{P_d w_d}{2} = \frac{cw_d^2}{2} = \frac{Pw_d^2}{2w_c}$, w_d – the dynamic deflection of the beam, measured from the rectilinear axis of the beam.

After the simplest transformations, we will get the quadratic equation:

$$w_d{}^2 - 2w_c w_d - 3w_c^2 = 0 \tag{4}$$

The solution of the quadratic equation is as follows:

$$w_{d_{1,2}} = w_c \pm \sqrt{w_c^2 + 3w_c^2} \tag{5}$$

The roots of the quadratic equation are:

$$w_{d_1} = 3w_c\,,\; w_{d_2} = -2w_c \tag{6}$$

The second root is in contrast to the physical content of equation, therefore, $w_d = 3w_c$, and the dynamic factor is:

$$k_d = w_d / w_c = 3 \tag{7}$$

The same value of the dynamic factor was obtained using the formula (2) if the initial deflection is as follows: $w_o = -w_c$.

At the initial hogging over the midspan support $0.5\,w_c$ the power accumulated with the beam:

$$U_o = 0.25Pw_c\,, \tag{8}$$

In case of damaging the midspan support, the work of the force P is as follows:

$$A = P(0.5w_c + w_d)\,. \tag{9}$$

The power of the beam deformation is: $U_d = \dfrac{Pw_d^2}{2w_c}$

With allowance for the power balance (3), the quadratic equation may be made up:

$$w_d{}^2 - 2w_c w_d - 1.5w_c^2 = 0 \tag{10}$$

The solution of the quadratic equation is as follows:

$$w_{d_{1,2}} = w_c \pm \sqrt{w_c^2 + 1.5w_c^2} \tag{11}$$

The roots of the quadratic equation are:

$$w_{d_1} = 2.581w_c\,,\quad w_{d_2} = -0.581w_c \tag{12}$$

The second root is in contrast to the physical content of equation, therefore, $w_d = 2.581w_c$, and the dynamic factor is:

$$k_d = w_d / w_c = 2.581 \tag{13}$$

The dynamic factor obtained using the formula (2) at the initial deflection: $w_o = -0.5w_c$, is:

$$k_d = \frac{P_d}{P} = 2 - \frac{-0.5w_c}{w_c} = 2.5 \tag{14}$$

The difference between dynamic factors calculated using the formulas (13) and (14) is not too large and is approximately 3 %.

Thus, the dynamic factor depends on how the construction was formed during the construction. In the process of design and construction, the presence of the clearances between the supports and the construction should be taken into account, as well as the presence of the preliminary deflections in the process of assembling the supports. The presence of the preliminary deflection of the construction is considered to be more dangerous for design survivability.

Let us consider the behavior of the construction in the form of the continuous double-span beam, subjected to a concentrated load P in the middle of each span (Fig.2).

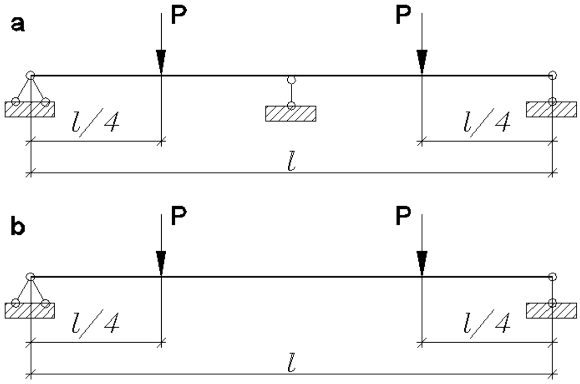

Figure 2. Double-span beam subjected to two concentrated loads in the middle of the span: a – basic construction; b – damaged construction

The support reactions for the basic construction are as follows:

– at the end supports:

$$F_1 = \frac{5}{16}P; \tag{15}$$

– at the midspan supports:

$$F_2 = \frac{22}{16}P \tag{16}$$

The deflection of the beam under each concentrated load in the basic construction is:

$$w_o = \frac{7P(0.5l)^3}{768EI} = \frac{0.001139Pl^3}{EI} \tag{17}$$

The deflection of the beam under each concentrated load under the static action of the load in the damaged construction (without the midspan support), measured from the rectilinear axis of the beam is as follows:

$$w_c = \frac{l^3}{48EI} = \frac{0.02083Pl^3}{EI} \tag{18}$$

To determine the dynamic factor, consider the power balance of the system (3).

The power accumulated in the basic construction during its loading *P* is:

$$U_o = \frac{2Pw_o}{2} = Pw_o \tag{19}$$

The work performed by the power *P* in case of damaging the support is:

$$A = 2P(w_d - w_c) \tag{20}$$

The energy accumulated in the beam in case of damaging the support is:

$$U_d = \frac{2P_d w_d}{2} = \frac{2cw_d^2}{2} = \frac{2Pw_d^2}{2w_c} = \frac{Pw_d^2}{w_c}, \tag{21}$$

where w_d – dynamic beam deflection in the points of load application, measured from the rectilinear axis of the beam.

After the simplest transformations, we will get the quadratic equation:

$$w_d^2 - 2w_c w_d + w_c w_o = 0 \tag{22}$$

The solution of the quadratic equation is as follows:

$$w_{d_{1,2}} = w_c \pm \sqrt{w_c^2 - w_o w_c} \quad (23)$$

Taking into account the formulas (17) and (18), write down:

$$w_o = \frac{0.001139}{0.02083} w_c = 0.05468 w_c \quad (24)$$

Then:

$$w_{d_{1,2}} = w_c \pm \sqrt{w_c^2 (1 - 0.05468)} = w_c \pm 0.9723 w_c \quad (25)$$

The roots of the quadratic equation are:

$$w_{d_1} = 1.9723 w_c, \quad w_{d_2} = 0.02773 w_c \quad (26)$$

If we do not consider the second root, that is in contrast to the statement, then $w_d = 1.9723 w_c$, and the dynamic factor is:

$$k_d = w_d / w_c = 1.9723 \quad (27)$$

2.5. Computational investigation

The computational investigation is carried out using the finite element models of the damaged construction in the static and dynamic setting [22–27]. In the process of dynamic designing, the damaged element is removed, and the internal forces occurring in the element being removed are impressed upon the construction. The internal forces are impressed so as to completely replace the failed component, and then these forces are decreased to zero during the time corresponding to the breakdown of the component [22–26]. It is recommended to take the value of decrease time of the equivalent forces as 0 to 0.1 sec. It is assumed, that if the vibration period of the damaged construction coincides with the decrease time of the forces, then the dynamic factor will be maximum.

The computational investigation was carried out using the computer system Nastran. The beam was simulated with the axial finite elements "beam". Each beam span was divided into 6 finite elements. For accounting the mass during the dynamic designing, the weight of which is equal to the actual loading, in the points of concentrated load application, the element of "mass" type is used.

The investigation of the dynamic factors is carried out for the continuous double-span beam from double-T iron 20B1 Corporate Standard of Association CHERMETSTANDART 20-93. The beam spans are 6 m, the support is hinged. The damage to the construction in the form of removing the midspan support and several types of loading are considered:

Type 1: concentrated force 48.94 kN is impressed above the midspan support upon the undeformed beam;

Type 2: concentrated force is impressed upon the beam with the span of 12 m, after the deformation of the beam by the amount of the half of deflection the midspan support is pinned under the beam, and the load is adjusted to 48.94 kN, the support reaction is 24.47 kN;

Type 3: the beam is hogged by the bearing of the midspan support by the amount equal to the half of deflection of the beam with the span of 12 m from the concentrated force of 48.94 kN, after that the concentrated force of 48.9 kN is impressed above the midspan support upon the beam, the support reaction is 73.41 kN.

The types of the constructions under investigation are presented in Figure 3. The initial position of the beam and the support being removed are stippled in Figure 3.

For each type of the damaged constructions, the frequency of the first vibration mode was calculated. When calculating the vibration mode for each type, the beam proper weight (21 kg/m) is taken into account as distributed mass, and the concentrated mass is 4894 kg (equivalent force is 48.94 kN). According to the results of the calculation, the frequency and the self-induced vibration period of the first form were: 0.712 Hz and 1.404 sec. The following time periods are considered, during which the support reactions are decreased, sec.: 0.05, 0.1, 0.14, 0.4, 0.7, 1.0, 1.404, 2.0, 3.0.

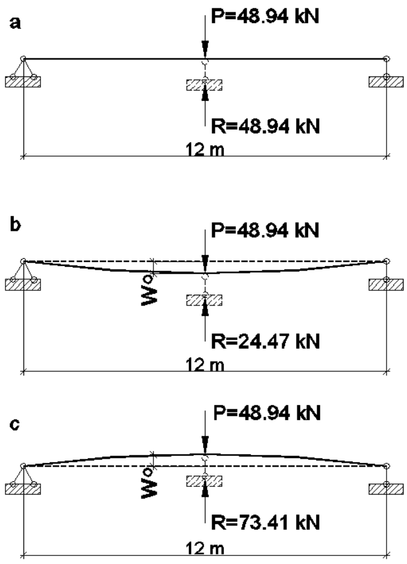

Figure 3. Diagrams of the constructions under investigation
a – type 1; b – type 2; c – type 3

When calculating the type 1 of the construction, the exterior load is constant, and the support reaction is decreasing for the predetermined time interval to zero. In types 2 and 3, the load and the support reaction vary according to the dependencies illustrated in Figure 4 (exterior load increases for 20 sec. from zero to the complete value, the support reaction increases to the complete value for the same time period and then decreases to zero for the selected time interval).

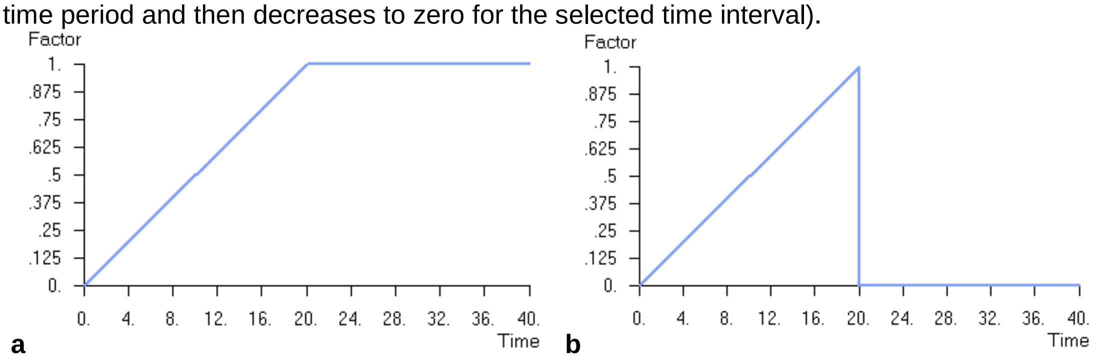

Figure 4. Change of load and support reaction during
a – load; b – support reaction

The work of beams on load at the spams of the continuous beam from the double-T iron 20B1 was numerically studied. The behavior of two constructions (fig. 5) was considered: the beam with the concentrated force in the middle of each span and beam with the uniformly distributed load.

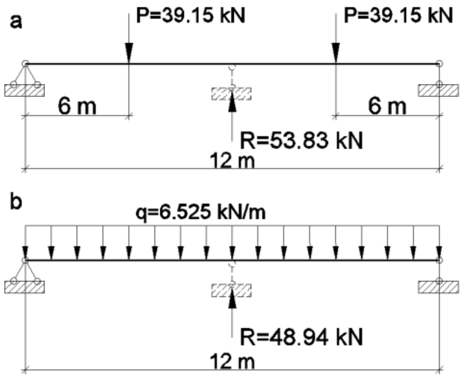

Fig.5. Diagram of the studied construction on load at the span
a – concentrated force; b – distributed force

The proper weight (21 kg/m) is taken into account at the places of the concentrated load action of the 3915 kg mass. For beams with loaded distributed force, the distributed mass which is equal to

652.5 kg/m together with the self-weight is considered. For the diagrams of the load at the spams, the frequency and the period of the first form of the self-induced vibration are: at the concentrated load – 0.794 Hz and 1.260 sec., at the distributed force – 0.812 Hz and 1.232 sec. The following time periods are considered, during which the support reactions are decreasing, sec.: 0.05, 0.1, 0.14, 0.4, 0.7, 1.0, 1.260 (for concentrated load), 1.232 (for distributed force), 2.0, 3.0.

When calculating the beams with load at the spam, the exterior load is recognized as permanent over time and the support reaction when removing the midsupport is changing during the above-mentioned time intervals in accordance with the diagram, shown in the Figure 6.

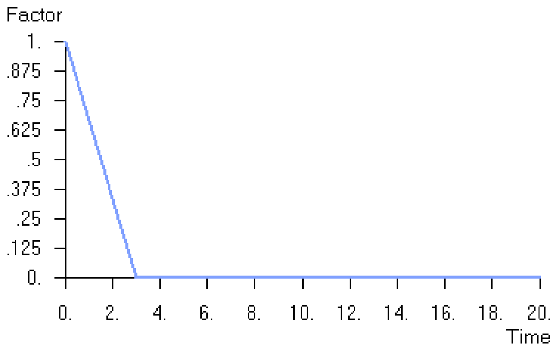

Figure 6. Change of the support reaction at the loaded beam in the spam and time interval of 3 sec

The work of three- and fourspan beams from the double-T iron 20B1 is considered in case of the damage of one of the midspans. At the Figure 7, a threespan beam from the double-T iron 20B1 is shown; a fourspan beam – at the Figure 8. At the figures, the removed supports are pointed with dot line. The support reaction value of the removed support:

for the threespan beam is 1.1*6.525*6=43.07 kN;

for the fourspan beam is 0.929*6.525*6=36.37 kN.

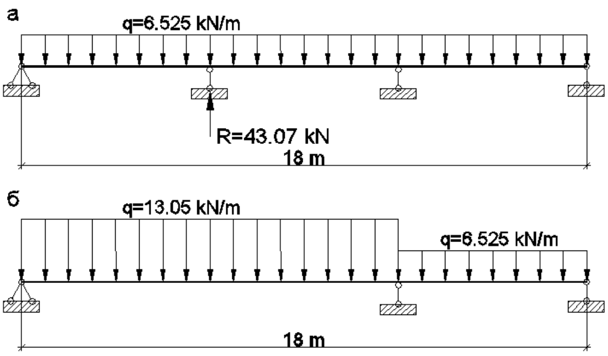

Fig.7. Design model at the destruction of the internal support of the threespan beam a – at the calculation in the dynamic setting; b – at the calculation in the static setting with k_d=2 for the load at the spans adjoining to the damaged support

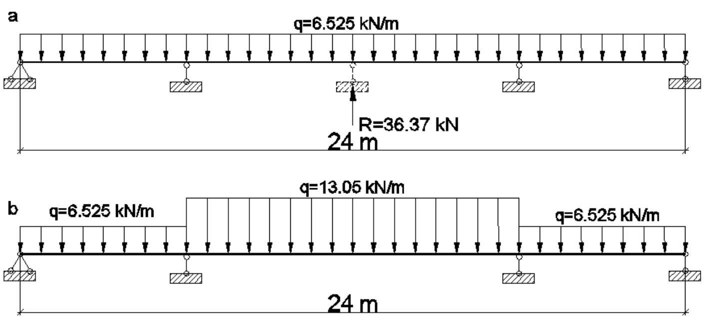

Figure 8. Design model at the destruction of internal support of the fourspan beam a – at the calculation in the dynamic setting; b – at the calculation in the static setting with k_d=2 for the load at the spans adjoining to the damaged support

The calculation in the dynamic setting was performed in the same manner as for the double-span beam; the time of the support reaction reduction during the removal of the midsupport is 0.05 sec.

At the static calculation, the design model with the corresponding removed support is taken into consideration. The load in the spans adjoining to the damaged span is admitted with the dynamic factor 2.

The behavior of the multispan beam which carries the secondary beams is studied at the removal of the midsupport. The main beam made of the double-T iron 20B1 is a fourspan beam, the length of each beam is 6 m. Secondary beams made of the double-T iron 10B1have span of 6 m length and are located at the 2 m pitch. Two types of construction are studied: type 1 – secondary beams are hinged to the main beam; type 2 – secondary beams are continuous. The load on the secondary beams is 1.088 kN/m. Fig. 9 shows the design model of the continuous beam with secondary beams attached to the main beam by the hinged fastening.

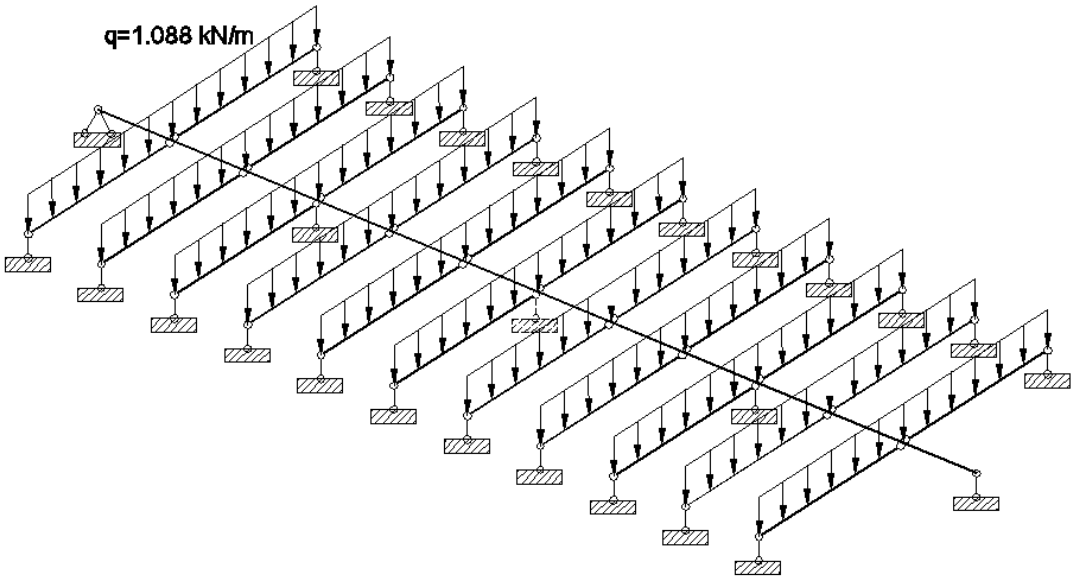

Figure 9. Design model with secondary beams attached by the hinged fastening

The midsupport reaction of the continuous beam is 40.41 kN. In case of construction damage, the midsupport is removed, and the first form frequency of the vibration is 0.706 Hz (simply supported secondary beams) and 0.759 Hz (continuous secondary beams), vibration periods are 1.416 and 1.318 sec. respectively. During the dynamic calculations, the following is taken into account: the distributed mass for the main beam – 21 kg/m; for the proper weight of the secondary beam – 8.1 kg/m, and the additional mass – 108.8 - 8.1=100.7 kg/m (for consideration of dynamic action of exterior load at the support damage).

3. Results and Discussion

Figure10 shows the relationship between the bending moment over the midspan (type 3) of doublespan beam and the action of the concentrated load over the midspan (Fig. 3) at the removal of this support within 0.05 sec.

Table 1 provides the forces at the doublespan beam depending on reduction time of support reaction at the damage of the midsupport. Except for the forces, Table 1 gives dynamic factors calculated as the ratio of final force in the damaged construction to the corresponding forces calculated at the static loading conditions to the damaged construction. The forces of the static load are:

maximum bending moment: M=48.94*12/4=146.82 kN·m,

maximum shear force: Q=48.94/2=24.47 kN.

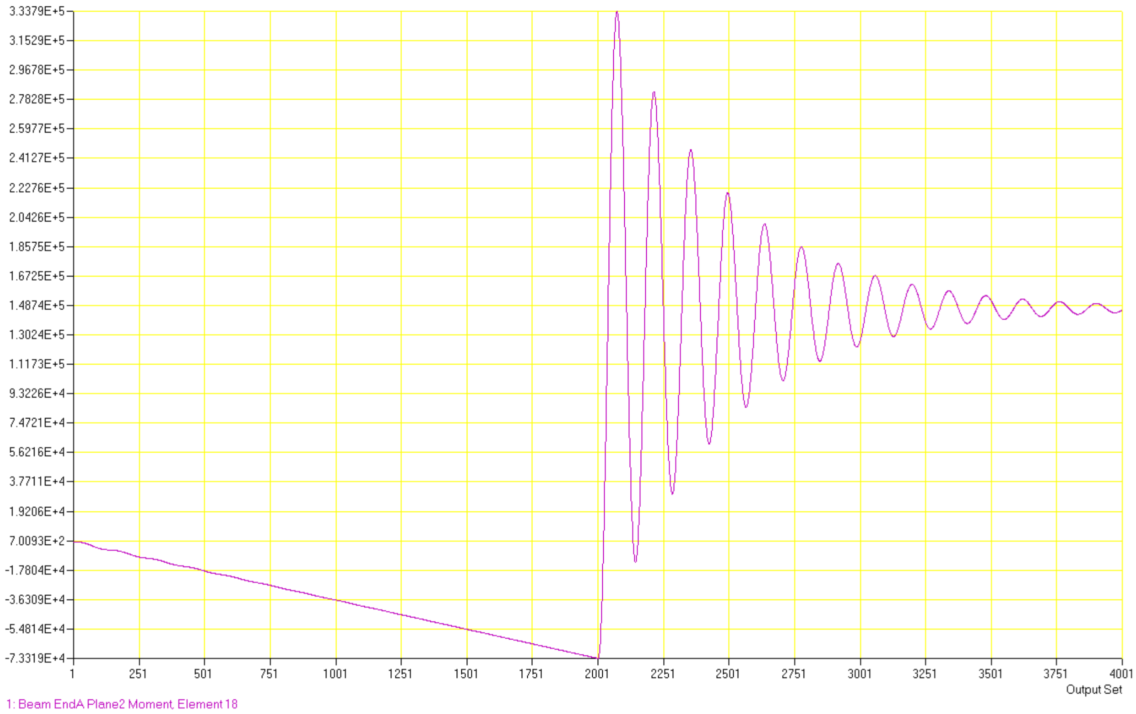

Figure 10. Change of bending moment over the midspan for type 3 of the construction

Table 1. Forces and dynamic factors at the double span beam when the middle support is damaged

Parameter	Forces and dynamic factors at the time of reaction reduction								
	0.05	0.10	0.14	0.40	0.70	1.00	1.401	2.00	3.00
1 type									
M, kN m	273.88	273.08	272.05	257.78	228.17	191.77	152.04	171.38	154.26
kd	1.87	1.86	1.85	1.76	1.55	1.31	1.04	1.17	1.05
Q, kN	45.95	45.82	45.65	43.24	38.23	32.07	25.35	28.62	25.35
kd	1.88	1.87	1.86	1.76	1.56	1.31	1.03	1.17	1.03
2 type									
M, kN m	210.25	209.95	209.43	202.14	187.49	169.30	149.43	159.10	150.54
kd	1.43	1.43	1.43	1.38	1.28	1.15	1.02	1.08	1.03
Q, kN	35.22	35.15	35.06	33.86	31.37	28.28	24.92	26.55	25.11
kdyn	1.44	1.43	1.43	1.38	1.28	1.15	1.02	1.08	1.02
3 type									
M, kN m	333.79	332.67	331.24	310.63	267.17	213.51	154.95	182.92	157.87
kd	2.27	2.27	2.26	2.12	1.82	1.45	1.06	1.25	1.08
Q, kN	56.12	55.94	55.70	52.20	44.85	35.76	25.85	30.58	26.34
kd	2.29	2.28	2.27	2.13	1.83	1.46	1.06	1.25	1.08

The computational investigation demonstrated that the less time for support reaction reduction the bigger is the dynamic factor. The studied simple constructions show that during the time of support reaction reduction equal to the period of the first frequency of self-induces vibration, the dynamic factor is close to 1. The maximum values of the dynamic factors received in numerical computation are different from the theoretical values – from 4 to 8–12 %. Theoretical dynamic factors are a bit bigger than the numerically calculated.

Carried out numerical calculations confirmed the effectiveness of the previously proposed [22, 26] method of numerical dynamic calculation with the replacement of the damaged element by internal forces, decreasing from time to zero for a time interval from 0 to 0.1 seconds. At carrying out the numerical researches the factor of damping 0.1 is accepted. Studies have shown that using such a damping factor reduces the damping time of the oscillations, but does not affect the level of peak values of forces and displacements of the structure. This was the results obtained earlier [17] in the calculation of multi-story steel frames.

Figure 11 shows the relation of the dynamic factors to the time of support reaction reduction for the considered types of construction damage.

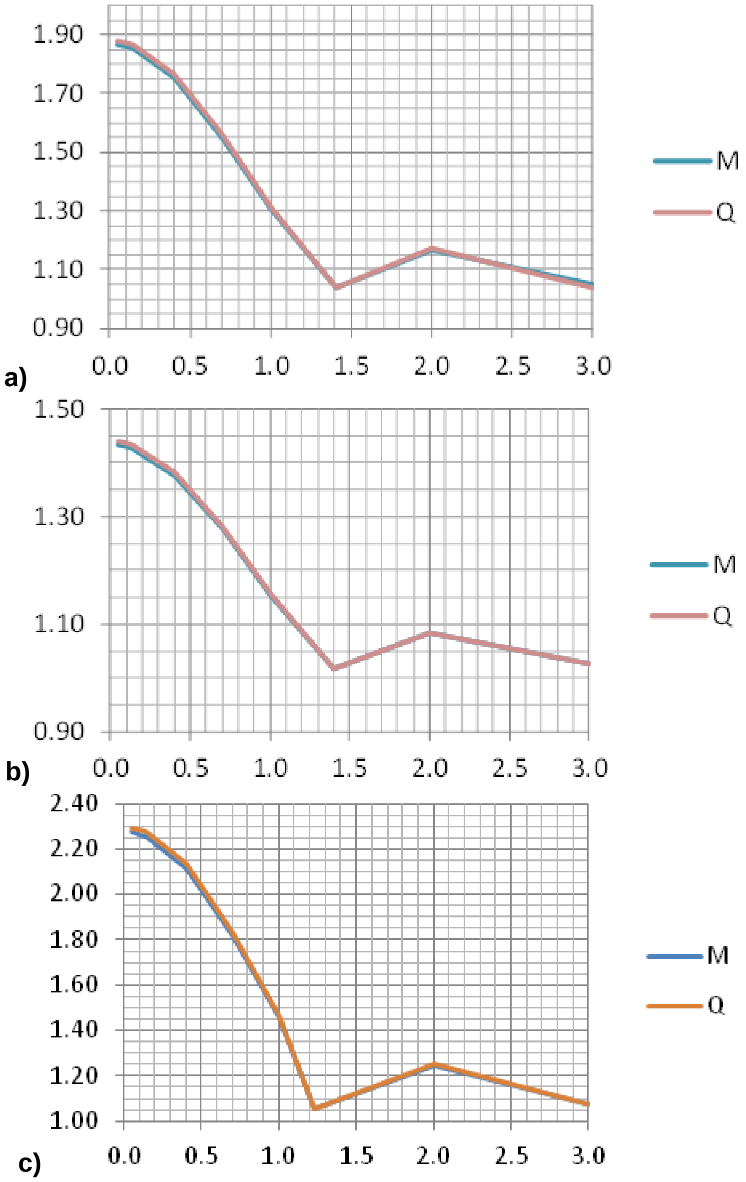

Figure 11. Relation of the dynamic factors to time of support reaction reduction
a – type 1; b – type 2; c – type 3
M - dynamic factor for the moment
Q - dynamic factor for the shear force

Figure 12 shows the ration of support reaction of the end supports of the doublespan beam in case of the load in the span by the concentrated loads (Fig. 5) to the removal of the support during the 0.14 sec.

Table 2 gives forces and dynamic factors of the doublespan beam loaded in the span depending on the time of support reaction reduction at the damage of midsupport. The forces of the static load at the concentrated loads in the span are maximum bending moment: *M*=39.15*3=117.45 kN·m, maximum shear force: *Q*=39.15 kN. At the uniformly distributed load, the static forces are *M*=6.525*12^2/8=117.45 kN·m, *Q*=6.525*12/2=39.15 kN.

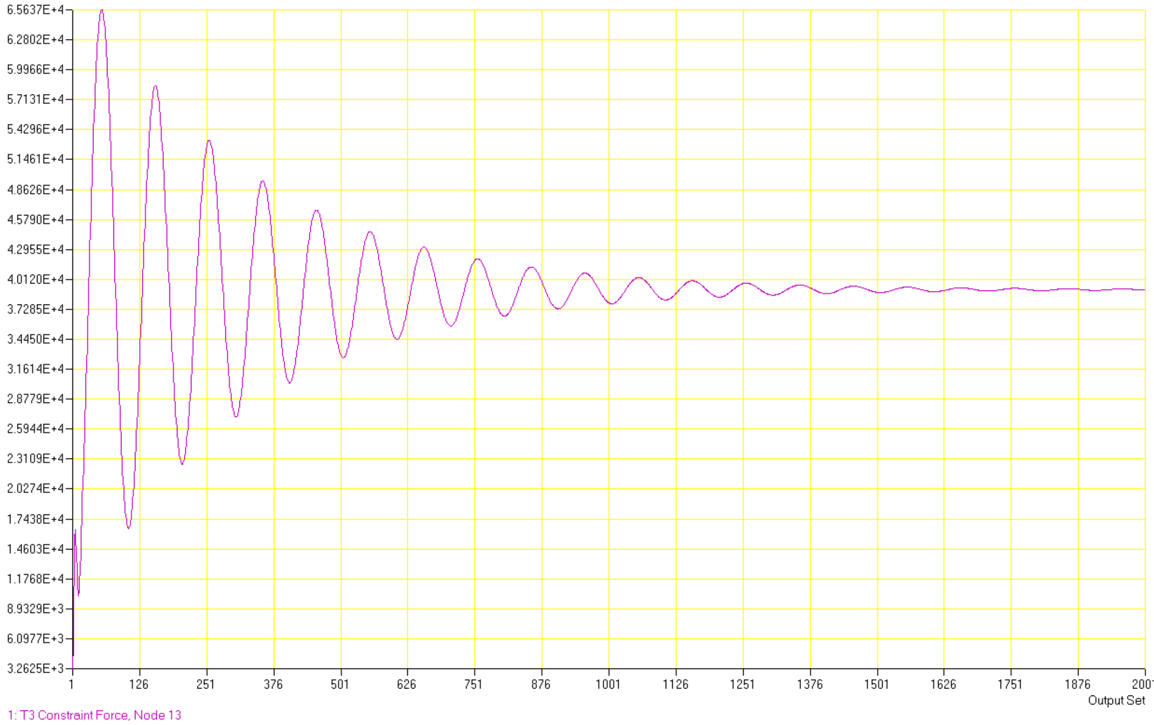

Figure 12. Change of end support reaction at the concentrated loads in the span and at the removal of the midsupport within 0.14 sec

Table 2. Forces and dynamic factors at the double span beam loaded in the span when the middle support is damaged

Parameter	Forces and dynamic factors at the time of reaction reduction								
	0.05	0.10	0.14	0.40	0.70	1.00	1.260(1.232)	2.00	3.00
Concentrated loads in the span									
M, kN m	217.97	217.38	216.07	200.93	170.39	137.05	123.39	132.90	128.57
kd	1.86	1.85	1.84	1.71	1.45	1.17	1.05	1.13	1.09
Q, kN	72.57	72.30	71.94	66.91	56.84	45.57	41.13	44.28	42.85
kd	1.85	1.85	1.84	1.71	1.45	1.16	1.05	1.13	1.09
Uniformly distributed load									
M, kN m	221.81	219.99	219.80	204.40	173.69	139.26	122.87	133.86	128.62
kd	1.89	1.87	1.87	1.74	1.48	1.19	1.05	1.14	1.10
Q, kN	66.14	65.92	65.64	61.58	53.63	44.70	40.42	43.28	41.91
kd	1.69	1.68	1.68	1.57	1.37	1.14	1.03	1.11	1.07

If the damage time of midsupport is decreasing, the dynamic factor will rise. When the time for support reaction reduction equals to the period of the first frequency of self-induced vibration, the dynamic factor is close to 1. The maximum dynamic factor at the load in the span received numerically can differ from the theoretical value which is 1.9723 no more than by 6 % (theoretical value is bigger than the numerical).

Figure 13 shows the ratio of dynamic factor to the time of support reaction reduction at the load in the span.

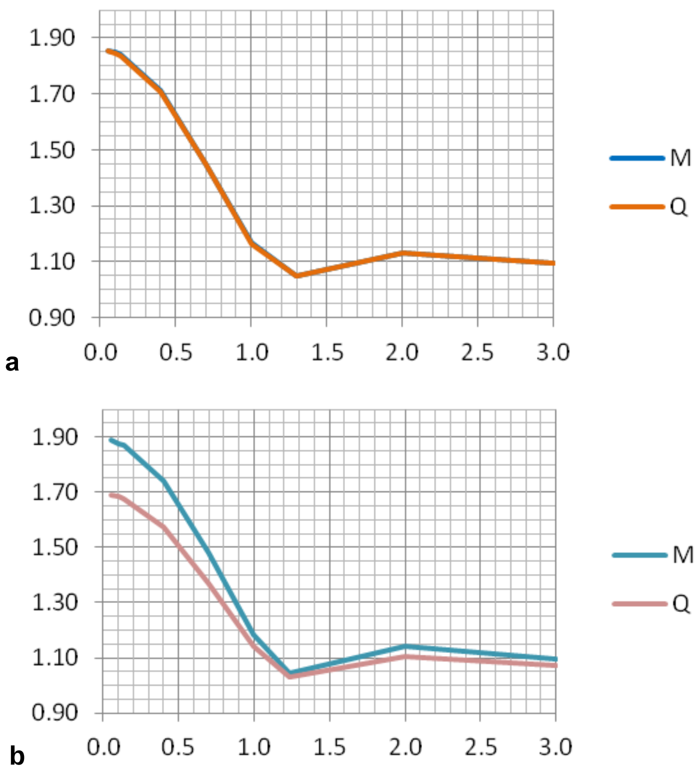

Figure 13. Ratio of dynamic factor at the load in the span to the time of support reaction reduction
a – concentrated load; b – distributed force
M – dynamic factor for the moment
Q – dynamic factor for the shear force

In the threespan continuous beam (Fig. 7), the maximum bending moment appears in the part of the beam where the support was removed and comprises 141.92 kN·m according to the data of dynamic calculation, and 159.05 kN·m – according to the data of static calculations with the dynamic factor. The maximum support reaction appears in the undamaged midsupport and comprises 113.5 kN (dynamic calculations) and 139.5 kN (static calculations).

In the fourspan continuous beam (Fig. 8), the maximum bending moment appears in the area of removed support and comprises 90.32 kN·m according to the data of dynamic calculation and 110.11 kN·m – according to the data of static calculations with the dynamic factor 2. The maximum support reaction appears in the closest to the middle of the beam undamaged supports and comprises 92.54 kN (dynamic calculations) and 118.7 kN (static calculations).

The comparison of the results of the dynamic and static calculations demonstrated that the bending moment and support reactions received by the dynamic calculations are smaller than the values received by the static calculations by 11–22 %.

Table 3 provides the forces and dynamic factors in the continuous main beam and in the secondary beams (Fig. 9) depending on the time of the support reaction reduction with the damaged midspan. The forces of the static load without the dynamic factor in the damaged construction are: bending moment in the main beam in the area of the removed support is M=26.93 kN·m, the support reaction of the span which is closest to the damaged one is R=34.68 kN, in the secondary beam with the hinged fastening, the bending moment in the beam span is M_1=4.90 kN·m, the shear force is Q_1=3.26 kN, for the continuous secondary beam in the place of its attachment to the main beam M_1=3.26 kN·m, shear force is Q_1=3.26 kN.

Table 3. Forces and dynamic factors in the continuous main beam and in the secondary beams when the middle support is damaged

Parameter	Forces and dynamic factors at the time of reaction reduction								
	0.01	0.05	0.10	0.14	0.40	0.70	1.00	1.416(1.318)	2.00
Simply supported secondary beam									
M, kN m	46.80	45.68	42.31	38.43	33.78	32.11	30.23	28.07	28.44
kd	1.74	1.70	1.57	1.43	1.25	1.19	1.12	1.04	1.06
R, kN	46.18	45.60	43.69	41.59	38.19	37.13	36.14	35.54	35.19
kd	1.33	1.31	1.26	1.20	1.10	1.07	1.04	1.02	1.01
M1, kN m	6.61	6.58	6.52	6.46	6.23	5.74	5.37	4.98	5.06
kd	1.35	1.34	1.33	1.32	1.27	1.17	1.10	1.02	1.03
Q, kN	4.18	4.17	4.12	4.08	3.82	3.65	3.38	3.17	3.23
kd	1.28	1.28	1.26	1.25	1.17	1.12	1.04	0.97	0.99
Continuous secondary beam									
M, kN m	56.20	55.72	54.33	52.62	40.38	31.00	30.94	30.46	29.11
kd	2.09	2.07	2.02	1.95	1.50	1.15	1.15	1.13	1.08
R, kN	50.58	50.31	49.50	48.56	41.87	36.98	36.87	36.60	35.82
kd	1.46	1.45	1.43	1.40	1.21	1.07	1.06	1.06	1.03
M1, kN m	5.62	5.52	5.43	5.26	4.04	3.10	3.09	3.05	3.58
kd	1.72	1.69	1.67	1.61	1.24	0.95	0.95	0.94	1.10
Q, kN	6.87	6.82	6.65	6.46	4.98	3.75	3.76	3.70	3.25
kd	2.11	2.09	2.04	1.98	1.53	1.15	1.15	1.13	1.00

The final force appears in the continuous beam and secondary beams at the minimum time of support destruction. At the same moment, the dynamic factor for construction with continuous beams is bigger, than for constructions with the simply supported secondary beams.

The calculation of the damaged construction with the loaded secondary beams in the area of load damage with the dynamic factor – 2. While other secondary beams are loaded with the initial load. Basing on the static calculations, the bending moment in the beam area located over removed support was 57.12 kN·m, the reaction of support closest to the damaged one – 59.02 kN. For the secondary beams, the bending moment in the hinged secondary beams is 9.79 kN·m, in the continuous secondary beams - 6.53 kN·m. The shear force in the secondary beams is 6.53 kN.

The moment determined by the static calculation with the increased load on the beams in the area of support destruction exceeds the moment obtained by the dynamic calculation in the main beam by 2–18 %, in the secondary beam by 14–34 %, the "static" support reaction of the main beam is more "dynamic" by 14–22 %, and the "dynamic" shear force in the secondary beam takes 64–105 % of the "static" one. Thus, for a spatial construction in the form of a continuous beam to which the secondary beams are adjoined, the static calculation can be used with a load doubled in the area of damage. Such a calculation is carried out with a reserve of load-carrying capacity, with the exception of the shear force in continuous secondary beams, which in case of a static calculation with a dynamic factor of 2 is less than dynamic by no more than 5 %.

A suggestion regarding the continuous beams was not confirmed on the fact that the biggest dynamic factors should appear at the coincidence of time for construction damage with the period of the first form of vibrations of such construction. Maximum dynamic factors of such beams are determined at the time length of damage which is lower than 0.1 sec.

4. Conclusions

Depending on the initial state of the continuous doublespan beam, the theoretical value of the dynamic factor varies in the range from 1 to 3. The possibility of numerical calculation of damaged constructions in a dynamic setting using the Nastran computer complex has been developed, and a numerical calculation method for the action of concentrated and distributed load has been worked out. The discrepancy between the theoretical dynamic factors and the factors determined by the numerical calculation is 4–12 %. It is revealed that in the considered constructions, the shorter the time of the midsupport destruction, the greater the dynamic factor. At the time of damage of the midsupport, equal to the period of the first form of vibrations, there are no significant values of the dynamic factors; the dynamic factors, in this case, are from 1.02 to 1.06.

For simple beam constructions (in the form of individual beams, girders, consisting of main and secondary beams) with the load capacity reserve, it is allowed to perform a static calculation in the design using loads with corresponding dynamic factors.

Taking into account the carried out studies on constructions in the form of simply supported and continuous beams, including those with secondary beams adjoining to them, in the absence of initial gaps or bends; in static calculations, it is recommended to use loads with a dynamic factor equal to 2 in the spans adjacent to the damaged support.

References

1. *Report of the Inquiry into the Collapse of Flats at Ronan Point*. Caning Town. MSO, 1968
2. Marchand K.A., Alfawakhive F. *Blast and Progressive Collapse.* AISC, 2005.
3. Strugatskiy Yu.M., Shapiro G.I. Bezopasnost moskovskikh zhilykh zdaniy massovykh seriy pri chrezvychaynykh situatsiyakh [Security of Moscow residential buildings of mass series in emergency situations]. *Industrial and Civil Engineering*. 1998. № 8. Pp. 37–41. (rus)
4. Strugatskiy Yu.M. Obespecheniye prochnosti panelnykh zdaniy pri lokalnykh razrusheniyakh ikh nesushchikh konstruktsiy [Ensuring the strength of panel buildings with local destruction of their load-bearing structures]. *Sb.: Issledovaniya nesushchikh betonnykh i zhelezobetonnykh konstruktsiy sbornykh mnogoetazhnykh zdaniy.* Moscow: MNIITEP, 1980. Pp. 3–19. (rus)
5. Zenin S.A., Sharipov R.Sh., Kudinov O.V., Shapiro G.I., Gasanov A.A. Methods of calculating of large-panel buildings: how to prevent progressing collapse. *Academia. Arkhitektura i stroitel'stvo.* 2016. No. 4. Pp. 109–113. (rus)
6. Yeremeyev P.G. Predotvrashcheniye lavinoobraznogo (progressiruyushchego) obrusheniya nesushchikh konstruktsiy unikalnykh bolsheproletnykh sooruzheniy pri avariynykh vozdeystviyakh [Prevention of avalanche-like (progressive) collapse of load-bearing structures of unique long-span structures during emergency operations]. *Structural Mechanics and Analysis of Constructions.* 2006. No. 2. Pp. 65–72. (rus)
7. Danilov A.I. Kontseptsiya upravleniya protsessom razrusheniya stroitelnogo obyekta [The concept of controlling the process of destruction of a building object] // *Industrial and Civil Engineering.* 2014. No. 8. Pp. 74–77. (rus)
8. Nazarov Yu.P., Gorodetskiy A.S., Simbirkin V.N. K probleme obespecheniya zhivuchesti stroitelnykh konstruktsiy pri avariynykh vozdeystviyakh [To the problem of ensuring the survivability of building structures under emergency influences]. *Structural Mechanics and Analysis of Constructions.* 2009. No. 4. Pp. 5–9. (rus)
9. Xavier F.B., Macorini L., Izzuddin B.A.. Contribution of masonry cladding for robustness enhancement of multi-storey buildings under sudden column loss. *Proceedings of the 16th International Brick and Block Masonry Conference*. Padova, Italy, 2016.
10. Nethercot D.A. Design of building structures to improve their resistance to progressive collaps. *The Twelfth East Asia-Pacific Conference on Structural Engineering and*

Литература

1. Report of the Inquiry into the Collapse of Flats at Ronan Point, Caning Town. – MSO, 1968 (ЦИНИС, перевод 18736).
2. Marchand K.A., Alfawakhive F. Blast and Progressive Collapse. AISC, 2005.
3. Стругацкий Ю.М., Шапиро Г.И. Безопасность московских жилых зданий массовых серий при чрезвычайных ситуациях // Промышленное и гражданское строительство. 1998. № 8. С. 37–41.
4. Стругацкий Ю.М. Обеспечение прочности панельных зданий при локальных разрушениях их несущих конструкций // Сб.: Исследования несущих бетонных и железобетонных конструкций сборных многоэтажных зданий. М.: МНИИТЭП, 1980. С. 3–19.
5. Зенин С.А., Шарипов Р.Ш., Кудинов О.В., Шапиро Г.И., Гасанов А.А. Расчёты крупнопанельных зданий на устойчивость против прогрессирующего обрушения методами предельного равновесия и конечного элемента // Academia. Архитектура и строительство. 2016. № 4. С. 109–113.
6. Еремеев П.Г. Предотвращение лавинообразного (прогрессирующего) обрушения несущих конструкций уникальных большепролетных сооружений при аварийных воздействиях // Строительная механика и расчет сооружений. 2006. № 2. С. 65–72.
7. Данилов А.И. Концепция управления процессом разрушения строительного объекта // Промышленное и гражданское строительство. 2014. № 8. С. 74–77.
8. Назаров Ю.П., Городецкий А.С., Симбиркин В.Н. К проблеме обеспечения живучести строительных конструкций при аварийных воздействиях// Строительная механика и расчет сооружений. 2009. № 4. С. 5–9.
9. Xavier F.B., Macorini L., Izzuddin B.A. Contribution of masonry cladding for robustness enhancement of multi-storey buildings under sudden column loss // Proceedings of the 16th International Brick and Block Masonry Conference. Padova, Italy, 2016.
10. Nethercot D.A. Design of building structures to improve their resistance to progressive collaps // The Twelfth East Asia-Pacific Conference on Structural Engineering and Construction. Procedia Engineering. 2011. № 14. Pp. 1–13.
11. Yu H., Izzuddin B.A., Zha X.-X. Progressive collapse of steel-framed buildings: influence of modelling approach // Advanced Steel Construction. 2010. Vol. 6. № 4. Pp. 932–948.
12. Stylianidis P.M., Nethercot D.A., Izzuddin B.A.,

Construction. Procedia Engineering. 2011. No. 14. Pp. 1–13.
11. Yu H., Izzuddin B.A., Zha X.-X. Progressive collapse of steel-framed buildings: influence of modelling approach. *Advanced Steel Construction*. 2010. Vol. 6. No. 4. Pp. 932–948.
12. Stylianidis P.M., Nethercot D.A., Izzuddin B.A., Elghazouli A.Y. Robustness assessment of frame structures using simplified beam and grillage models. *Engineering Structures*. 2016. Vol. 115. Pp. 78–95.
13. Ellingwood B.R., Smilowitz R., Dusenberry D.O., Duthinh D., Lew H.S., Carino N.J. *Best Practices for Reducing the Potential for Progressive Collapse in Buildings.* National Institute of Standards and Technology, NISTIR 7396, USA. 2007.
14. Marchand K.A., Stevens D.J. Progressive Collapse Criteria and Design Approaches Improvement. *Journal of Performance of Constructed Facilities*. 2015. No. 29(5). B4015004.
15. Stylianidis P.M., Nethercot D.A. Modelling of connection behaviour for progressive collapse analysis. *Journal of Constructional Steel Research.* 2015. Vol. 113. Pp. 169–184.
16. Astaneh-Asl A. Progressive Collapse of steel truss brides, the case of I-35W collapse. *Proceedings, 7th International Conference on Steel Bridges.* Guimaraes, Portugal, 2008. Pp. 1–10.
17. Kandil K.S., Ellobody E.A.E.F., Eldehemy H. Progressive Collapse of Steel Frames. *World Journal of Engineering and Technology*. 2013. No. 1. Pp. 39–48.
18. Belostotskiy A.M. et al. Raschety zdaniy na ustoychivost protiv progressiruyushchego obrusheniya s uchetom fizicheskoy i geometricheskoy nelineynostey [Calculations of buildings for stability against progressive caving in view of physical and geometric nonlinearities]. Teoriya i praktika rascheta zdaniy, sooruzheniy i elementov konstruktsiy. Analiticheskiye i chislennyye metody: Proceedings of international scientific and practical conference. Moscow: MGSU, 2008. Pp. 183–193. (rus)
19. Kaewkulchai G., Williamson E.B. Beam element formulation and solution procedure for dynamic progressive collapse analysis. Computers & Structures.vol.82, issues 7-8, varch 2004, pp. 639-651
20. Canisius T.D. Robustness of structural systems – a new focus for the joint committee on structural safety (JCSS). *Applications of Statistics and Probability in Civil Engineering.* London. 2007. 8 p.
21. Styliandis P.M., Nethercot D.A., Izzuddin B.A., Elghazouli A.Y. Modelling of beam response for progressive collapse analysis. *Structures.* 2015. Vol. 3. Pp. 137–152.
22. Belyayev N.M. *Soprotivleniye materialov* [Strength of materials]. Moscow, 1965. 856 p. (rus)
23. Kudishin Yu., Drobot D. Zhivuchest konstruktsiy v avariynykh situatsiyakh [Stability of structures in emergency situations]. *Metallicheskiye zdaniya*. 2008. No. 4(8). Pp. 20-22. 2008. No. 5(9). Pp. 21–23. (rus)
24. Drobot D. Yu. Otsenka zhivuchesti Krytogo Konkobezhnogo tsentra v Krylatskom [Assessment of survivability of the Indoor Skating Center in Krylatskoye]. *Vestnik MGSU.* 2009. No. 2. Pp. 116–119. (rus)
25. Kudishin Yu.I. Kontseptualnyye problemy zhivuchesti stroitelnykh konstruktsiy [Conceptual problems of survivability of building structures]. *Vestnik MGSU.* 2009. No. 2(spec.). Pp. 28–36. (rus)
26. Kudishin Yu.I., Drobot D.Yu. K voprosu o zhivuchesti stroitelnykh konstruktsiy [To the question of the survivability of building structures]. *Structural Mechanics and Analysis of Constructions.* 2008. No. 2(217). Pp. 36–43. (rus)
27. Kudishin Yu.I., Drobot D.Yu. Zhivuchest stroitelnykh

Elghazouli A.Y. Robustness assessment of frame structures using simplified beam and grillage models // Engineering Structures. 2016. Vol. 115. Pp. 78–95.
13. Ellingwood B.R., Smilowitz R., Dusenberry D.O., Duthinh D., Lew H.S., Carino N.J. Best Practices for Reducing the Potential for Progressive Collapse in Buildings. National Institute of Standards and Technology, NISTIR 7396, USA. 2007.
14. Marchand K.A., Stevens D.J. Progressive collapse criteria and design approaches improvement // Journal of Performance of Constructed Facilities. 2015. № 29(5). B4015004.
15. Stylianidis P.M., Nethercot D.A. Modelling of connection behaviour for progressive collapse analysis // Journal of Constructional Steel Research. 2015. Vol. 113. Pp. 169–184.
16. Astaneh-Asl A. Progressive Collapse of steel truss brides, the case of I-35W collapse // Proceedings, 7th International Conference on Steel Bridges. Guimaraes, Portugal, 2008. Pp. 1–10.
17. Kandil K.S., Ellobody E.A.E.F., Eldehemy H. Progressive Collapse of steel frames // World Journal of Engineering and Technology. 2013. № 1. Pp. 39–48.
18. Белостоцкий А.М. и др. Расчеты зданий на устойчивость против прогрессирующего обрушения с учетом физической и геометрической нелинейностей // Теория и практика расчета зданий, сооружений и элементов конструкций. Аналитические и численные методы: Сб. трудов международной научно-практической конференции. М.: МГСУ, 2008. С. 183–193.
19. Kaewkulchai G., Williamson E.B. Beam element formulation and solution procedure for dynamic progressive collapse analysis // Computers & Structures. 2004. Vol. 82. № 7-8. Pp. 639–651.
20. Canisius T.D. Robustness of structural systems – a new focus for the joint committee on structural safety (JCSS) // Applications of Statistics and Probability in Civil Engineering. London. 2007. 8 p.
21. Styliandis P.M., Nethercot D.A., Izzuddin B.A., Elghazouli A.Y. Modelling of beam response for progressive collapse analysis // Structures. 2015. Vol. 3. Pp. 137–152.
22. Беляев Н.М. Сопротивление материалов. М., 1965. 856 с.
23. Кудишин Ю., Дробот Д. Живучесть конструкций в аварийных ситуациях // Металлические здания. 2008. № 4(8). С. 20–22. 2008. № 5(9). С. 21–23.
24. Дробот Д.Ю. Оценка живучести Крытого Конькобежного центра в Крылатском // Вестник МГСУ. 2009. № 2. С. 116–119.
25. Кудишин Ю.И. Концептуальные проблемы живучести строительных конструкций // Вестник МГСУ. 2009. № 2(спец.). С. 28–36.
26. Кудишин Ю.И., Дробот Д.Ю. К вопросу о живучести строительных конструкций // Строительная механика и расчет сооружений. 2008. № 2(217). С. 36–43.
27. Кудишин Ю.И., Дробот Д.Ю. Живучесть строительных конструкций – важный фактор снижения потерь в условиях аварийных ситуаций // Металлические конструкции. 2009. № 1. С. 61–72.
28. Sanches R.A.K., Coda H.B. Flexible multibody dynamics finite element formulation applied to structural progressive collapse analysis // Latin American Journal of Solids and Structures. 2017. № 14. Pp. 52–57.

konstruktsiy –vazhnyy faktor snizheniya poter v usloviyakh avariynykh situatsiy [Stability of building structures is an important factor in reducing losses in emergency situations]. *Metallicheskiye konstruktsii.* 2009. No. 1. Pp. 61–72.

28. Sanches R.A.K., Coda H.B. Flexible multibody dynamics finite element formulation applied to structural progressive collapse analysis. *Latin American Journal of Solids and Structures.* 2017. No. 14. Pp. 52–57.

Alexandr Tusnin,
+7(916)115-14-21; valeksol@mail.ru

Александр Романович Туснин,
+7(916)115-14-21; эл. почта: valeksol@mail.ru

Tusnin A. Dynamic factors in case of damaging continuous beam supports. *Magazine of Civil Engineering*. 2018. No. 2. Pp. 47–64. doi: 10.18720/MCE.78.4.