

$$m = \sum_{i=0}^3 q_{m_i} \cdot (\sigma/R_b)^i,$$

where q_δ , q_α , q_m are parameters, which should be determined empirically for defined aggressive environment and initial mechanical characteristics of concrete.

Also it should be noted, that the function of corrosion damage depth $\delta(t, t_0)$ can take the trajectory of avalanche (1) or fading (2) development once more after an additional dynamic impact on the reinforced concrete element with depth of corrosion damage about δ_{cr} at time t_1 , as it is shown in Figure 1 (a).

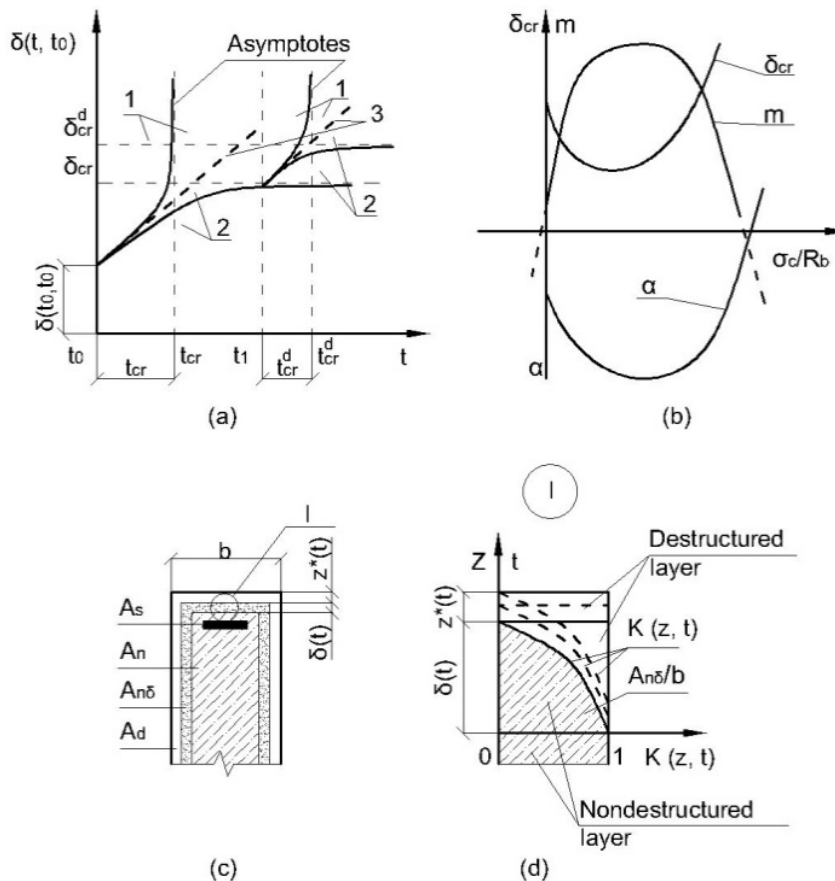


Figure 1. Scheme of the corrosion damage kinetics of compressed concrete
(a): scheme to determine parameters δ_{cr} , α , m ;
(b): cross-section of the corrosion-damaged reinforced concrete element;
(c): general view of the corrosion damage function $K(z, t)$;
(d): 1 corresponds to the area of avalanche corrosion penetration ($m < 0$);
2 corresponds to the area of colmatation ($m > 0$);
3 is the boundary line ($m = 0$)

3. Results and Discussion

In the present article we consider only colmatational process of corrosion damage development in accordance to which the stiffness of the cross section of corrosion damaged element can be determined from the following expression:

$$B_{red}(\delta(t)) = E_t \cdot I_{red}(\delta(t)) \quad (2)$$

where E_t is variable strain modulus of concrete, which depends on deformation, caused by loading and creep [21];

$I_{red}(\delta(t))$ is the reduced inertia moment of cross section, which can be calculated by the formula:

$$I_{red}(\delta(t)) = A_n r_{cn}^2 + \sum_{i=1}^l A_{n\delta,i} r_{cn\delta,i}^2 + I_n + \sum_{i=1}^l I_{n\delta,i} + \frac{\omega_s}{\psi_s} \cdot \alpha \cdot A_s \cdot r_{cs}^2 \quad (3)$$

where A_n is area of part of section without corrosion damage; $A_{n\delta,i}$ is area of i -th part of section exposed to corrosion, which is determined in accordance to function of corrosion damage $K(z, t)$ as it is shown in Figure 1 (c); A_s is area of reinforcement cross section; r_{cn} , $r_{cn\delta,i}$ are distance from center of mass of not damaged section to center of mass of whole section and distance from center of mass of i -th part of section damaged by corrosion to center of mass of whole section, which is determined in accordance to function of corrosion damage $K(z, t)$, respectively; r_{cs} is distance from center of mass of reinforcement cross section to center of mass of whole section; I_n is inertia moment of not damaged section with respect to own center of mass; $I_{n\delta,i}$ is inertia moment of i -th part of damaged section with respect to own center of mass; ω_s is coefficient of corrosion damage of reinforcement; ψ is coefficient of influence of tensiled concrete; $\alpha = E_s / E_t$ is coefficient of reduction of reinforcement to concrete.

Assuming that the corrosion damage of the section can be modeled by the function $K(z, t)$, as it is shown in Figure 1 (d), taking into account the scheme shown in figure 1 (c), we obtain:

$$\begin{aligned} I_{red}(t) = & (b - 2z^*(t)) \frac{4\delta(t)}{3} \left(\frac{h}{2} - z^*(t) - \frac{3}{5}\delta(t) \right)^2 + \frac{(b - 2z^*(t))}{6} \left(\frac{2\delta(t)}{3} \right)^3 + \\ & + [h - 2z^*(t) - 2\delta(t)]^3 \frac{\delta(t)}{9} + \frac{1}{12} [b - 2z^*(t) - 2\delta(t)] \cdot [h - 2z^*(t) - 2\delta(t)]^3 + \\ & + \frac{\omega_s}{\psi} \cdot \alpha \cdot A_s \cdot (h - 2a)^2 \end{aligned} \quad (4)$$

where h and b are the height and width of the intact section respectively; a is the distance from the center of gravity of the reinforcement to the face of the reinforced concrete element as it is shown in Figure 1 (c); $\delta(t)$ is the thickness of the layer, is partially damaged by corrosion which can be determined from the expression

$$\delta(t) = \delta(t, t_0) - z^*(t) \quad (5)$$

In which $\delta(t, t_0)$ can be obtained by the formula (1); $z^*(t) = t \frac{d}{dt} (z^*)$ is the approximate relationships for the depth of the entirely destructured concrete; approximate model of corrosion destruction ω_s of steel reinforcement can be calculated from the equations:

$$\begin{cases} \omega_s = 1 & \text{at } \delta(t, t_0) < a - d/2; \\ \omega_s = \frac{\pi}{4A_s} \left[d - \frac{t - t_s}{8760} \frac{d}{dt} (\omega_s) \right]^2 & \text{at } \delta(t, t_0) > a - d/2; \\ \omega_s = 0 & \text{at } z^*(t) \geq a - d/2. \end{cases} \quad (6)$$

Let us consider a fragment of a multi-storey frame (Figure 2) to develop an algorithm for analytical stability analysis of corrosion-damaged concrete structural system of the building. For this construction, we define the value of the parameter k_{cr} of special functions of the method of displacement, which characterize critical force and can be determined from the expression:

$$k_{cr}^2 = \frac{P_{cr} l^2}{B_{red}} \quad (7)$$

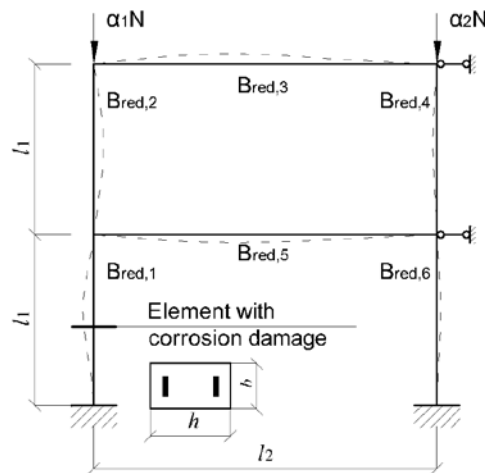


Figure 2. Computational model of the fragment of a multi-storey reinforced concrete frame

Let us assume that the elements of the structural system are compressed by longitudinal forces $N_i = \xi_i \cdot P_{cr}$, where ξ_i is the safety factor of stability of the i -th element of the system at the most unfavorable calculated combination of forces, P_{cr} – Euler’s critical force for the structural system.

Taking into account that the k_{cr} parameter depends only on the boundary conditions at the end sections of the frame structural element, we carried out an expression for the critical value of the bending stiffness of the corrosion-damaged element, at which it will lose the stability:

$$B_{red,cr} = \frac{\xi_i P_{cr} l^2}{k_{cr}^2} \quad (8)$$

Equating expressions (8) and (2) and taking into account expressions (1), (5), (6), we obtain an equation, in which all parameters can be expressed through a critical time t_{cr} , corresponding to the moment of loss of the stability due to reaching a critical level of corrosion damage:

$$\begin{aligned} \frac{\xi_i P_{cr} E_t l^2}{k_{cr}^2} = & (b - 2z^*(t_{cr})) \frac{4\delta(t_{cr})}{3} \left(\frac{h}{2} - z^*(t_{cr}) - \frac{3}{5}\delta(t_{cr}) \right)^2 + \frac{(b - 2z^*(t_{cr})) \left(\frac{2\delta(t_{cr})}{3} \right)^3}{6} + \\ & + [h - 2z^*(t_{cr}) - 2\delta(t_{cr})]^3 \frac{\delta(t_{cr})}{9} + \frac{1}{12} [b - 2z^*(t_{cr}) - 2\delta(t_{cr})] \cdot [h - 2z^*(t_{cr}) - 2\delta(t_{cr})]^3 + \\ & + \frac{\omega_s}{\psi} \cdot \alpha \cdot A_s \cdot (h - 2a)^2 \end{aligned} \quad (9)$$

Let us analyze the stability of a fragment of a reinforced concrete frame (figure 2), one of the bearing elements of which (the lower left rack) is subject to the action of an aggressive environment. Initial parameters of the frame: $l_1 = 3$ m, $l_2 = 6$ m, initial section width $b = 0.2$ m, initial section height $h = 0.35$ m, the distance from the most compressed face to the center of gravity of the reinforcement $a = 40$ mm as it is shown in Figure 1 (a). Concrete B25 class, reinforcement $A_s = A'_s = 4$ rod A400 class with a diameter of 16 mm. The level of aggressiveness of the environment is medium ($z'(t) = 1$ mm/year). Values of parameters δ_{cr} , α , m are taken by paper V.M. Bondarenko [15]. At the moment $t_0 = 0$ the safety factor of the stability of the corrosion-damaged rack $\xi_i = 0.8$, and the value of the critical force $P_{cr} = 2860$ kN, the ratio of the load parameters $\alpha_2 / \alpha_1 = 4$.

Solving equation (9) in accordance with expressions (1), (5), (6), we obtain the graph $B_{red} - t$, which is shown in Figure 3 (b). The intersection point of this graph with the line of the ultimate value of stiffness $B_{red,cr}$ corresponds to the value of the critical time of the aggressive environment impact on the loaded structure t_{cr} , after which the frame will lose the stability.

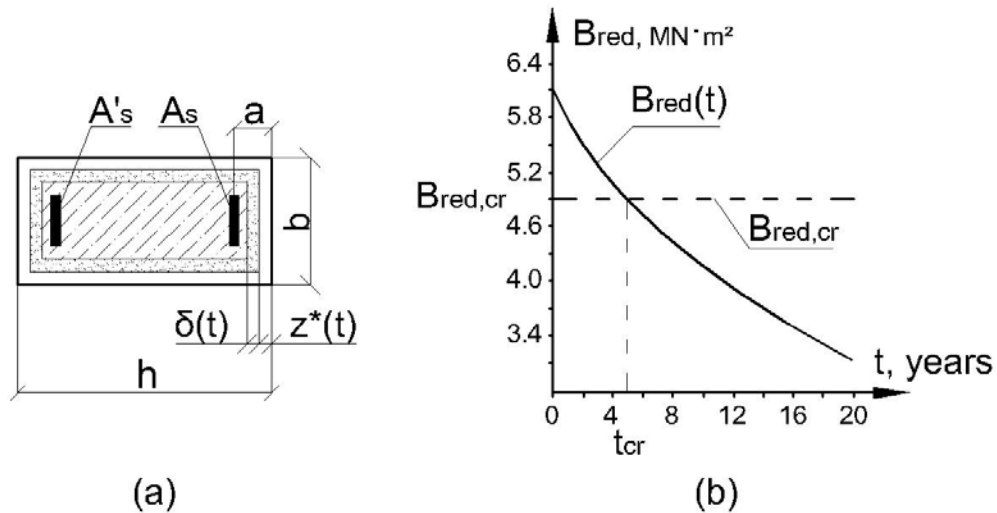


Figure 3. Scheme of the rack cross section (a) and graph $B_{red} - t$ (b) for corrosion-damaged structural element

The ultimate value of the cross-section stiffness of the damaged structural element $B_{red,cr}$ or critical time t_{cr} , after which the structural system will lose stability, can be used as criteria to assess the survivability of reinforced concrete structural system at exceeding of the limit states, caused by a sudden loss of stability of the bearing element at acting of the loading and corrosion.

4. Conclusions

1. Analytical relationships for the critical stiffness value of the cross-section $B_{red,cr}$ of the corrosion-damaged element of the reinforced concrete frame are obtained.
2. "Exposure time of survivability" (by the terminology of V.M. Bondarenko, V.I. Kolchunov [29]) t_{cr} is determined on the basis of the stability criteria of a structural system, subjected acting of load and aggressive environment impact.
3. It is proposed criteria to assess the survivability of reinforced concrete structural system at exceeding of the limit states, caused by a sudden loss of stability of the bearing element at acting of the loading and corrosion.

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