Evolutionary optimization of prestressed steel frames

Эволюционная оптимизация предварительно напряженных стальных рам

Abstract. A method for the optimal synthesis of prestressed steel frame structures is developed. The search for a solution is being carried out by applying discrete sets of variable parameters, including cross-sections of structural rods and cross-sections of prestressing tie-rods. It is possible to vary the location of the prestressing system. To optimize the cost of the objects under consideration, an improved method of evolutionary simulation has been used, including a combined scheme for constraints set and an unusual scheme for the formation of populations. Strength and stiffness considerations for a number of objects within a population are not strict, and for variants with breached constraints, a penalty function has been applied. In forming the current population used to modify the individuals, multipoint genetic operators, random generation, and a strategy of elitism have been applied. To vary the location of the prestressing system and take into account the multivariance of loads, parallel evolving populations have been introduced, between which a limited exchange of individuals is allowed. Examples of optimization of prestressed frames with girders and trusses were considered.

Аннотация. Разрабатывается методика оптимального синтеза преднапряженных стальных рамных конструкций. Поиск решения осуществляется на дискретных множествах варьируемых параметров, включающих сечения стержней конструкции и сечения преднапрягающих затяжек. Предусматривается возможность варьирования расположения системы преднапряжения. Для оптимизации стоимости рассматриваемых объектов используется усовершенствованная схема эволюционного моделирования, включающая комбинированную схему учета ограничений и неординарную схему формирования популяций. Учет ограничений по прочности и жесткости для ряда объектов в популяции не является строгим, при этом для вариантов, имеющих нарушение ограничений, используется штрафная функция. При формировании текущей популяции для модификации объектов используются многоточечные генетические операторы, случайная генерация и стратегия элитизма. Для варьирования расположения системы преднапряжения и учета многовариантности нагрузений вводятся параллельно эволюционирующие популяции, между которыми допускается ограниченный обмен особями. Рассмотрены примеры оптимизации преднапряженных рам с балочным и ферменным ригелями.
1. Introduction

Reduction of materials used in steel structures is one of the topical problems arising during building and structure design. At present, algorithms of evolutionary simulation established on the basic principles of the evolution of species in an animated nature can be successfully used to optimize designs, by applying discrete sets of variants of structures and design parameters (dimensions, cross-sections sizes of rod, coordinates of nodal joints, grades of steels, etc.). Studies have proven that this approach to optimization for this class of problems is quite effective.

Reduced costs of frame structures can be attained via prestressing. The issue of optimization of prestressed rod, plate, and shell objects is crucial and has been a focus of close attention for many researchers. Numerous papers have been devoted to the optimization of prestressed reinforced concrete slabs [1–4], beams [5–10], trusses [11, 12], bridge structures [13–15], tensegrity systems [16], cable-stayed domes [17], membrane [18] structures and poles with prestressed anchor stays [19]. It is describes the two-stage general approach to optimization of reinforced concrete structures [20], where, at the first stage, linear programming is applied to select the prestressing force, and at the second stage - the specific dimensions of an element.

This method of cable-stayed trusses optimization based on evolutionary simulation has been developed [21] with variables divided into three groups: prestressing force, sizes of cross-sections, and location of structural elements. In [11], steel trusses were optimized according to a two-stage scheme, where, at the first stage, cross-sections of elements were selected, and, at the second stage, prestress was calculated via linear programming.

A number of important research areas are related to prestressed structure optimization issues. Primarily, assessment of the prestressed systems technical condition, which can be performed, for example, by an oscillatory method [22], as well as through experimental studies. For example, in [23], an experimental analysis of the behavior of steel trusses was made by taking into account different ways of linking ropes and prestressed elements. For such trusses with tubular cross-sections of elements, the effect of prestressing compressed and tensioned rods made of various steel grades was studied [24]. The stability of prestressed steel columns was investigated [25, 26].

This paper suggests a method for optimizing prestressed frame structures based on a multi-threaded iterative procedure. At the same time, based on the use of parallel populations, the issue of finding the minimum cost of construction is being solved. The distinctive feature of the suggested approach is that single computational process is applied to vary the standard sizes of the bar cross-section, location, and dimensions of the prestressing system, as well as the values of prestressing forces.

2. Methods

2.1. Formulation of the optimization problem

We consider the steel frame structure, which includes the prestressing system in the form of stops, tie-ropes, and elements fixing the tie-rope position. The design model is discretized according to the scheme of the finite element method, with rigid consoles (inserts) used to attach ropes to rods. System prestressing is taken into account by introducing multidirectional paired forces, the points of application of which coincide with the stops location, and the lines of such forces action coincide with the rope longitudinal axis. Technological characteristics, which depend on the conditions of initial loading and step-by-step prestressing, have not been taken into account. Frame bars are described by means of multilayer bar finite elements, in which each layer can be subject to tension-compression strains [27]. In general, Bernoulli’s hypothesis of flat sections is used for the layer assembly. We set the following tasks.

**Personal objective (I):** cost $C$ of the prestressed structure, for which the value of prestress force and location of the prestressing system does not change, is minimized.

$$C(\{X\}, P) \rightarrow \min, \{X\} = \{X_1, \ldots, X_n\},$$  \hspace{1cm} (1)

where $\{X\}$ – set of variable sizes of cross-section profiles in rods represented by integral geometric characteristics, $n$ – the number of variable parameters; $P$ – constant parameters of the prestressing system.

Variable

\[ X_i = \left\{ A_i; x_{i1}, y_{i1}, \ldots, x_{ik}, y_{ik} \right\}, \ i = 1..n, \]

where \( A_i \) – bar cross-section \( i \), \( x_{i1}, y_{i1}, \ldots, x_{ik}, y_{ik} \) – coordinates of points in bar cross-section \( i \), \( k \) – number of cross-section points in which stresses are calculated.

**Personal objective (II):** Search for optional design with minimum cost. Taking into account the change in the prestressing system location. The prestress force remains constant.

\[ C(X, P, \{ Y \}) \rightarrow \min, \]

\[ \{ Y \} = \{ Y_1, \ldots, Y_R \}, \]

\[ \{ Y_i \} = \{ B_i(x_{bi}, y_{bi}), E_i(x_{ei}, y_{ei}) \}, \ i = 1..R, \]

where \( \{ Y \} \) – set of the prestressing system locations, \( R \) – the number of such variant in the structure, \( B_i, E_i \) – points of prestressing forces application.

The considered private objectives allow us to formulate the **general objective (III):** minimization of the frame construction cost \( C \) by taking into account the variable level of prestress and variations in the prestressing system location. The species target function is minimized

\[ C(X, P, \{ Y \}, \{ N \}) \rightarrow \min, \]

\[ \{ N \} = \{ N_1, \ldots, N_D \}, \ i = 1..D, \]

where \( \{ N \} \) – set of values of prestressing forces, \( D \) – number of prestress levels of the structure which, in a particular case, may coincide with the number of tie-rope standard dimensions used.

For all the tasks set, constraints on the strength, stability, and stiffness of the frame structure should be taken into account, in accordance with current design standards. It is required from the prestressing system not to exceed the value of the tensile strength of the rope above the permissible design value. Conditions of the symmetry, unification, and design features of the object should also be taken into account.

### 2.2. Solving algorithm

The optimization algorithm is based on a modification of the combined evolution strategy detailed in [28]. We perform a number of steps.

1. **Formation of the structure finite element model.** In this case, for private objective achieving (I), this model shall be constructed traditionally. Stiffness of the prestressing ropes is set to be constant. Necessary design values (topology, material characteristics, loads, kinematic constraints, etc.) need to be entered.

In solving the objective tasks (II), a \( R \) of variants of finite element models with its own location of the prestressing system shall be formed. In this case, all variants are considered in a single multi-threaded iterative scheme and are united by a common system of constraints. For each \( R \) variant of the prestressing system location, an evolving population, for which a limited exchange of individuals from other populations is allowed, should be organized in the genetic algorithm in parallel.

For general formulation of the task, the process of the computational model formation is similar to the one described in the objective (II), but, in this event, in calculations, for each reinforcement system location variant, \( D \) different loads formed by prestressing forces shall be taken into consideration. Each value of the force can be associated with a certain rope cross-section. In this event, stiffness of the rope also changes, depending on the current value of the force.

At this stage, information on the operational loads applied to the structure, the sets of values of variable parameters that can be chosen, and other data necessary for the implementation of optimization and evaluation procedures for the object stress-strain state shall also be entered.
2. Random generation of the initial pool of prestressed system variants. Values required for problem solving in various constructive solutions of the object have been obtained from the corresponding sets of variable parameters. In this event, the possibility of grouping the elements by taking into account the symmetry of the system, constructive, and technological requirements shall be made use of. As a result, for each of the tasks posed, the first generation of individuals (set of the object variants) further considered in the genetic iterative procedure can be represented as (5):

\[
G_1^{(I)} = F_1: X_{\tilde{i}}, \ldots, X_{n\tilde{i}}; \quad G_1^{(II)} = F_1: X_{\tilde{i}}, \ldots, X_{n-1\tilde{i}}, Y_{nj};
\]

\[
F_{ng}: X_{\tilde{i}}, \ldots, X_{n\tilde{i}}; \quad F_{ng}: X_{\tilde{i}}, \ldots, X_{n-1\tilde{i}}, Y_{nj};
\]

\[
G_1^{(III)} = F_1: X_{\tilde{i}}, \ldots, X_{n-2\tilde{i}}, Y_{n-1j}, N_{nk};
\]

\[
F_{ng}: X_{\tilde{i}}, \ldots, X_{n-2\tilde{i}}, Y_{n-1j}, N_{nk};
\]

where \(G_1^{(I)}, G_1^{(II)}, G_1^{(III)}\) – initial populations for the formulation of tasks I, II, III, respectively;

\(\tilde{i}, \tilde{j}, \tilde{k}\) – whole numbers randomly selected at intervals \([1..np_i], [1..R], [1..D]\), \(np_i\) – number of values permitted for selected variable parameter \(i\).

3. Calculation of the stress-strain state and the fitness of the objective function for each structure variant. The decision shall be made on a system of linear algebraic equations for the discretized object finite-element method to determine displacements, and then, forces and stresses in structural elements. If the object meets the set constraints, calculations of its cost \(C\) shall be made. Based on the value \(C\), a decision shall be made as to the acceptance of the structure variant for further stages of the genetic iterative procedure. The cost shall include conditional cost values of materials and construction.

4. Creating the Elite Objects Database (EOB). EOB shall include structure variants with the best \(C\) value. Initially, objects from the initial pool generated at stage 2 shall be placed there, provided that all constraints have been met. This stage shall be implemented if the EOB has not been created, otherwise the next stage is to be started.

5. EOB correction. For any object to be entered into the database the following requirements shall be met:

- the cost of the object-applicant intended for inclusion in the database should be less than the maximum cost of the objects already existing in the EOB;
- The object-applicant should not be a copy of any object from the EOB.

6. Formation of the current pool (generation) of individuals (structure variants). New variants of the object are obtained through the implementation of genetic operators on the objects with the best cost values randomly selected from the EOB. Selection shall be made via the roulette wheel method, depending on the \(C\) value. The mechanism for the structure variant change is demonstrated by example of 20 objectives making up the pool in Table 1.

7. Check of compliance with the calculation termination criterion. If the contents of the EOB for 200-300 iterations does not change, then the solution obtained after checking passive constraints and verification calculations, taking into account physical, geometric, and constructive nonlinearities, for example by [29], can be deemed final. Otherwise, stages 3, 5-7 shall be repeated.
Table 1 Principles of population formation

<table>
<thead>
<tr>
<th>Individuals number</th>
<th>Name and description of the operator</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Single-point mutation</td>
</tr>
<tr>
<td>2</td>
<td>Ditto</td>
</tr>
<tr>
<td>3</td>
<td>- // -</td>
</tr>
<tr>
<td>4</td>
<td>Two-point mutation</td>
</tr>
<tr>
<td>5</td>
<td>Ditto</td>
</tr>
<tr>
<td>6</td>
<td>- // -</td>
</tr>
<tr>
<td>7</td>
<td>Regulated n-point mutation [30]</td>
</tr>
<tr>
<td>8</td>
<td>Ditto</td>
</tr>
<tr>
<td>9</td>
<td>- // -</td>
</tr>
<tr>
<td>10</td>
<td>Single point crossover</td>
</tr>
<tr>
<td>11</td>
<td>Ditto</td>
</tr>
<tr>
<td>12</td>
<td>Two-point crossover</td>
</tr>
<tr>
<td>13</td>
<td>Ditto</td>
</tr>
<tr>
<td>14</td>
<td>Regulated n-point crossover and inversion [31]</td>
</tr>
<tr>
<td>15</td>
<td>Ditto</td>
</tr>
<tr>
<td>16</td>
<td>Single-point inversion</td>
</tr>
<tr>
<td>17</td>
<td>Multipoint inversion</td>
</tr>
<tr>
<td>18</td>
<td>Random generation of the object. Values of variable parameters are randomly selected from a discrete set of corresponding acceptable values.</td>
</tr>
<tr>
<td>19</td>
<td>Ditto</td>
</tr>
<tr>
<td>20</td>
<td>- // -</td>
</tr>
</tbody>
</table>

3. Results and Discussion

3.1. Example 1. Optimal design of the steel frame with the search for the prestressing tie-rod position (personal objective (I))

The object in question is shown in Figure 1,a. It is assumed that the frame has become unfastened through the loss of stability from the plane by longitudinal girders and capping beams. The rods with the cross-section area shown in Figure 1,b are made of Fe 430 steel. Mechanical characteristics: modulus of elasticity $E = 2.06 \cdot 10^5$ MPa, yield strength $\sigma_y = 255$ MPa. In calculating structure variants during the course of the optimization process, stresses were limited by the value $\sigma = 235$ MPa. Restrictions on displacements were imposed: $\delta_x \leq 0.012$ m, $\delta_y \leq 0.072$ m. The tie-rod 3 is made of a spiral rope (Figure 1,c) of grade SS45 as per EN12385-10, with a modulus of elasticity $E_r = 1.7 \cdot 10^5$ MPa. The prestressing force was assumed to be 1000 kN, making up 84% of the design load. The structure model was discretized into finite elements measuring 1 m.
The frame was designed for the following loads: structure weight $q_1$, wind loads $w_1 = 1.1 \text{kN/m}$ $w_2 = 0.83 \text{kN/m}$, deck weight $q_2 = 12 \text{kN/m}$, snow load $s = 10.8 \text{kN/m}$, and payload $p = 7.2 \text{kN/m}$ acting on the entire span of the frame, snow $s$ and payload $p$ acting on half the span of the frame and having the same intensity as loads $s$ and $p$ (Figure 1,d-f).

The following loading combinations (6) were considered:

$$L_1 = q_1 + q_2 + p + 0.9(s + w_1 + w_2); \quad L_2 = q_1 + q_2 + s + 0.9(p + w_1 + w_2); \quad L_3 = q_1 + q_2 + 0.9(w_1 + w_2).$$

Sizes of profiles in the rod cross-sections and location of the prestressing tie-rods 3 were varied. Grade profiles were as per ASTM A6. The tie-rod can be located in positions a-a, b-b or c-c (Figure 1, a).

Four groups of structural elements were introduced: columns 1 and three equal parts of beam 2. For each group of elements, cross-sections were varied independently. A discrete set of parameters acceptable for selection during the course of the optimization process is presented in Table 2.

### Table 2. Variable cross-sections of rods

<table>
<thead>
<tr>
<th>N</th>
<th>Designation</th>
<th>Cross-sections in Fig. 1,b, $10^{-2} \text{ m}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>W1</td>
<td>$W 8 \times 8x31$</td>
<td>18.1 0.72 20.3 1.1</td>
</tr>
<tr>
<td>W2</td>
<td>$W 10 \times 10x49$</td>
<td>22.46 0.86 25.4 1.42</td>
</tr>
<tr>
<td>W3</td>
<td>$W 12 \times 12x65$</td>
<td>27.72 0.99 30.5 1.54</td>
</tr>
<tr>
<td>W4</td>
<td>$W 14 \times 10x68$</td>
<td>32.04 1.05 25.5 1.83</td>
</tr>
<tr>
<td>W5</td>
<td>$W 16 \times 10x77$</td>
<td>38.14 1.16 26.1 1.93</td>
</tr>
<tr>
<td>W6</td>
<td>$W 18 \times 11x86$</td>
<td>42.78 1.22 28.2 1.96</td>
</tr>
<tr>
<td>W7</td>
<td>$W 21 \times 8x93$</td>
<td>50.18 1.47 21.4 2.36</td>
</tr>
<tr>
<td>W8</td>
<td>$W 24 \times 9x94$</td>
<td>57.26 1.31 23 2.22</td>
</tr>
<tr>
<td>W9</td>
<td>$W 27 \times 10x102$</td>
<td>64.58 1.31 25.4 2.11</td>
</tr>
<tr>
<td>W10</td>
<td>$W 30 \times 10x108$</td>
<td>71.94 1.38 26.6 1.93</td>
</tr>
<tr>
<td>W11</td>
<td>$W 33 \times 11.5x118$</td>
<td>79.74 1.4 29.2 1.88</td>
</tr>
<tr>
<td>W12</td>
<td>$W 36 \times 12x135$</td>
<td>86.28 1.52 30.4 2.01</td>
</tr>
</tbody>
</table>

In varying sizes of profiles, the eccentricity of tie-rod $e_i$ relative to the longitudinal axis of the frame girder was defined as $e_{ri} = h_{wi}/2 + t_{ri}$, $i = [1;12]$. Vertical frame rods with length $e_{ri}$ connecting tie-rod 3 to beam 2, by their integral characteristics were considered to be close to absolutely rigid bodies. To assess prestress efficiency, optimal synthesis of the frame without prestressing was performed, with the results shown in Figure 2,a. Optimal solutions for the frame with prestressing are shown in Figure 2,b-d.

![Figure 2. Solution results: W6-W12 – cross-section numbers from Table 2](image)

The optimization process required the performance of no more than 100 iterations of the evolutionary algorithm, and 2000 variants of the frame were calculated. For the considered conditions for the solution, the location of b-b tie-rod 3 was the most expedient (Figure 1,a). Figure 2 illustrates that prestressing makes it possible to reduce the conventional cost of the considered frame structure by more than 20%.

### 3.2. Example 2. Search for the optimal frame solution by taking into account variations in tie-rod stiffness and tension force (personal objective (II))

The frame of example 1 is considered. The tie-rod is fixed to the structure in the b-b position, and its position does not change. Materials, loads, kinematic constraints, and the variable parameters of rods remain the same as in Example 1. In addition, the values of tie-rod section 3 and corresponding tension forces are independently varied. Permissible values for the tie-rods variable parameters are given in Table 3.

Some typical results of optimal frame design are shown in Fig. 3. Analysis of the results obtained indicate that the most efficient level of the prestressing value is $N = 1400$ kN. The optimization process required execution of no more than 120 iterations of the evolutionary algorithm, which allowed calculating 2400 variants of the frame.

**Table 3. Design characteristics of spiral ropes**

<table>
<thead>
<tr>
<th>$D$</th>
<th>$25$</th>
<th>$30$</th>
<th>$35$</th>
<th>$40$</th>
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<td>$124$</td>
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<td>$295$</td>
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<td>$P$</td>
<td>$370$</td>
<td>$524$</td>
<td>$719$</td>
<td>$931$</td>
<td>$1190$</td>
<td>$1460$</td>
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<td>$1400$</td>
<td>$1700$</td>
<td>$2000$</td>
<td>$2400$</td>
</tr>
</tbody>
</table>
3.3. Example 3. Optimization of the constructive solution for a single-story building frame by varying the location of the tie-rod and the level of its prestress (personal objective (III)).

To prevent the loss of stability from plane, the frame is fastened by spacers along the upper and lower belts of the roof truss and capping beams along the column length. Action of two combinations of loads $L_1$ and $L_2$, and $g$ of the frame weight, decking structure, and snow load $s$. The frame rods are made of A529 steel, Grade 55, yield strength $\sigma_y = 380 \text{ MPa}$. In calculating the structure variants, stresses were limited by $\sigma = 360 \text{ MPa}$. Dimensions of the profiles shown in Table 2 were varied for the columns and the top chord of truss, rectangular pipe profiles as per ASTM A500 were used for the remaining elements of the truss, dimensions which are given in Table 4.

### Table 4. Discrete set of rods cross-sections

<table>
<thead>
<tr>
<th>N</th>
<th>Designation</th>
<th>$d_1$</th>
<th>$d_2$</th>
<th>$t_1 = t_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>T1</td>
<td>4x3</td>
<td>6.7056</td>
<td>9.2456</td>
<td>0.457</td>
</tr>
<tr>
<td>T2</td>
<td>5x3</td>
<td>6.7056</td>
<td>11.785</td>
<td>0.457</td>
</tr>
<tr>
<td>T3</td>
<td>6x4</td>
<td>9.2456</td>
<td>14.3256</td>
<td>0.457</td>
</tr>
<tr>
<td>T4</td>
<td>7x5</td>
<td>11.1099</td>
<td>16.1896</td>
<td>0.795</td>
</tr>
<tr>
<td>T5</td>
<td>8x6</td>
<td>14.3256</td>
<td>19.4056</td>
<td>0.457</td>
</tr>
<tr>
<td>T6</td>
<td>10x6</td>
<td>14.3256</td>
<td>24.4856</td>
<td>0.457</td>
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<tr>
<td>T7</td>
<td>10x8</td>
<td>18.4150</td>
<td>23.4950</td>
<td>0.953</td>
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<tr>
<td>T8</td>
<td>12x8</td>
<td>19.0500</td>
<td>29.2100</td>
<td>0.635</td>
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<tr>
<td>T9</td>
<td>14x10</td>
<td>23.4950</td>
<td>33.6550</td>
<td>0.953</td>
</tr>
<tr>
<td>T10</td>
<td>16x12</td>
<td>28.5750</td>
<td>38.7350</td>
<td>0.953</td>
</tr>
</tbody>
</table>

Tie-rods were allowed to be made of grades SS 30, SS 40, SS 50 with the initial prestress forces equal to 500, 900, and 1400 kN, respectively. The tie-rod could be located in one of three possible positions (1, 2, 3) shown in Figure 4, a by the dashed line. Considering the object symmetry, the following group of variable parameters was used: group provided $g_1 - g_8$ – (Figure 4,a), within the limits of which the rod cross-section is assumed to be the same. Here, group $g_1$ – columns, $g_2$ contains all elements of the top chord of truss, $g_3 - g_6$ – are elements of the bottom chord, $g_7$ – all pillars, $g_8$ – all of the web members. A total of 10 parameters were varied independently. The optimization process was completed in no more than 200 iterations of genetic algorithm. The best solution found is shown in Figure 4,c.
3.4. Discussion

Problems of prestressed structures optimization were effectively solved based on various modifications of genetic algorithms, as those presented in [21, 22]. The step-by-step decision-making process was applied. The presented algorithm contains the unified iteration scheme for all design stages, to make possible obtaining the solution of very complex problems with a large number of variable parameters. Comparison of solutions obtained through the use of our proposed computational scheme with the results obtained by other authors [4], where is concluded that the amounts of the material and the cost of a steel plane truss can be reduced up to 19.9 %, guarantees very good results regarding objective function and convergence values. At the same time, one of the factors improving repeatability is the introduction of the penalty function described in [10].

4. Conclusions

1. The novel search method for a rational construction solution for steel frames with prestressed structural elements was developed on the basis of evolutionary simulation. The optimization algorithm enables consideration of different loading variants, independent variation of configurations and parameters of the projected structures and the prestressing system in one iterative process.

2. The effect of cost reduction compared with the same structure designed without prestress, considering the requirements of design standards, exceeded 18–20 %.

3. Considered examples demonstrated that the proposed algorithm used for specific structures allows determining the rational prestressing force value and the prestressing system location. This algorithm is recommended for use in computer-aided design engineering of buildings and structures made of steel frames.

5. Acknowledgement

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