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Method for calculating strongly damped systems with non-proportional damping

Метод расчета сильно демпфированных систем с непропорциональным демпфированием

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Abstract. The calculation of systems under seismic excitations is performed both dynamically by time integration and quasistatically under inertial seismic loads using linear response-spectra method (RSM). Dynamic timing calculation can be performed either using direct integration of the initial system of motion equations, or by using the spectral decomposition of motion equations by shape modes. RSM is completely based on spectral decomposition. However, the spectral decomposition was worked out only for systems with proportional damping, when the eigenvectors of the undamped and damped systems coincide. With regard to RSM, even for proportional damping, the existing Guide Lines do not allow to take into account the actual damping in the system. There are proposals for the explicit calculation of damping within the framework of the RSM for proportional damping in literature. Their results can be used both for constructing RSM and for integrating motion equations with arbitrary damping using the spectral decomposition of the motion equations. But so far the mentioned mathematical results have not been connected with calculating structures. The authors propose a variant of the RSM for calculating highly damped systems under earthquake impact. To this aim, complex eigenvectors and eigenvalues of the motion equation system were obtained, and this system was reduced to a tridiagonal form. As a result, the assumed equation system of the order equal to N was decomposed into N pairs of independent real equations. The base oscillation accelerogram and its derivative present the input in the right part of the motion equations. In this way two matrices of seismic forces are generally obtained. To sum up these forces, the shape mode correlation coefficients were analyzed. An example of mass damper calculation is given in the paper. For 4-6-story buildings constructed in ordinary conditions the proposed variant of the RSM leads to the same data that existing Guide Lines. But the proposed variant of RSM makes it possible to calculate systems with heterogeneous damping including seismic isolated systems, mass dampers and systems with soil-structure interaction which is impossible to do on the base of the existing Guide Lines.

Аннотация. Расчет систем на сейсмическую нагрузку выполняется динамически, путем интегрирования по времени, и квазистатически, по инерционным сейсмическим нагрузкам с использованием линейно-спектрального метода (ЛСМ). Динамический расчет может быть выполнен либо путем прямого интегрирования исходной системы уравнений движения, либо с использованием спектрального разложения уравнений движения по формам. ЛСМ полностью основан на спектральном разложении. Однако спектральное разложение было разработано только для систем с пропорциональным затуханием, когда собственные векторы незатухающих и затухающих систем совпадают. Что касается ЛСМ, даже для пропорционального демпфирования существующая методика в нормативной документации не позволяет учитывать фактическое демпфирование в системе. В литературе существуют предложения по явному учету затухания в рамках ЛСМ для пропорционального затухания. Данные результаты могут быть использованы в расчетах как по ЛСМ, так и путем интегрирования уравнений движения с произвольным затуханием с использованием спектрального разложения уравнений движения. Но до сих пор упомянутые математические результаты не были использованы при расчете конструкций зданий и сооружений. Авторы предлагают вариант ЛСМ для расчета сильно демпфированных систем при расчете на землетрясения. С этой целью были получены комплексные собственные векторы и собственные значения системы уравнений движения, и эта система была сведена к трехдиагональной форме. В результате принятая система уравнений порядка N была разложена на N пар независимых вещественных уравнений. Правая часть уравнения представляет собой ускорение основания и его производную. Таким образом, получается две матрицы сейсмических сил. Чтобы суммировать эти силы, были проанализированы коэффициенты корреляции форм. В статье приведен пример расчета здания с динамическим гасителем. Для 4-6-этажных зданий, построенных в обычных условиях, предлагаемый вариант ЛСМ приводит к тем же данным, что и нормативный вариант ЛСМ. Но предлагаемый вариант ЛСМ позволяет рассчитать системы с неоднородным затуханием, включая системы с сейсмоизоляцией, с динамическими гасителями и системы с учетом основания, что невозможно сделать на основе существующей нормативной документации.

1. Introduction

The object of the study is the method of calculating damping for analyzing seismic stability of strongly damped systems.

In the past 20 years, various kinds of dampers have been increasingly used to decrease dynamic loads on building structures in earthquake engineering in particular [1–7]. However, in the theory and practice of seismic load calculations, the problem of allowing for damping causes certain difficulties. At present, the basic method for evaluating the seismic stability of structures is the method of decomposition by vibration modes. In this case, in the process of calculating it is necessary to build a damping matrix, write down the system of seismic oscillation equations, expand the system according to vibration modes, estimate loads for each mode and sum up these loads taking into account the random nature of the impact. When calculating the structure by the linear spectral method (LSM), the response spectra are used to estimate the seismic loads, and in the case of dynamic calculation a package of accelerograms with further averaging of the result or some deliberately dangerous accelerogram [8, 9] is used. At present the construction of linear system damping matrix is not a difficult problem. This task is included in program complexes, for example, MicroFE [10], Midas [11], etc. Some aspects of this problem were considered by the authors in [1]. As a result, the system of seismic oscillation equations has the following form

$$M\ddot{q} + B_{eq}\dot{q} + Rq = -M\ddot{Y}_0, \quad (1)$$

where M is inertia matrix; B_{eq} is the matrix of equivalent viscous damping;

R is the stiffness matrix; q is the column of generalized displacements;

$\ddot{Y}_0$ is the column of kinematic inputs

To solve equation (1), the decomposition of motion by the oscillation modes of the undamped system is used in research on the subject [1, 11]. Such decomposition is admissible at damping of less than 15 % of the critical value, or in the case when the matrices $M^{-1}R$ and $M^{-1}B$ have the same eigenvector matrices [12]. Damping, which allows the decomposition of the motion equations by the oscillation modes of the undamped system, is called proportional.

Calculating structures with a non-proportional damping have caused certain difficulties so far. Meanwhile, this kind of calculations is necessary for seismic-protection systems, tuned mass dampers, structures consisting of different materials, soil-structure interaction analysis etc. Especially relevant are such calculations for selecting parameters of special seismic protection at the early stages of its designing, when no design accelerograms for the building site are available yet.

The aim of investigation is bringing out features of calculating systems with a non-proportional damping using the motion decomposition by oscillation modes. To do this, it is necessary to solve the following problems

1. Building the spectral distribution of strongly damped systems and developing the response spectra method (RSM) for their calculating.
2. Presenting some example of proposed method

2. Methods

System (1) is reduced to a system of equations of the first order.

$$A \cdot \dot{\Theta} = Q\,, \tag{2}$$

where

$$A = \begin{Vmatrix} -M^{-1}B_{eq} & -M^{-1}R \\ E & 0 \end{Vmatrix}; \qquad \Theta = \begin{pmatrix} q \\ \dot{q} \end{pmatrix}; \quad Q = \begin{pmatrix} \ddot{Y}_0 \\ 0 \end{pmatrix}$$

System (2) is characterized by a set of complex eigenvalues presented in the following form

$$\Lambda = \left[-\frac{\gamma_1 k_1}{2}, -\frac{\gamma_2 k_2}{2}, \dots -\frac{\gamma_n k_n}{2}, -\frac{\gamma_1 k_1}{2}, -\frac{\gamma_2 k_2}{2}, \dots -\frac{\gamma_n k_n}{2} \right] + i \left[\omega_1, \omega_2, \cdots \omega_n, -\omega_1, -\omega_2, \cdots - \omega_n \right], \tag{3}$$

where n is the number of the system degree of freedom

$$\lambda_{i,i+1} = -\frac{\gamma_i k_i}{2} \pm i\omega_i\,,\ \omega_i = k_i \sqrt{1 - \frac{\gamma_i^2}{4}} \tag{4}$$

$\lambda, k, \omega, \gamma$ are an eigenvalue, eigenfrequency, the frequency of damped oscillations, and the modal inelastic resistance coefficient of the system respectively.

Peculiarities of constructing the damping matrix and determining the eigenvalues and vectors of the matrix A were considered in [12]

For strongly damped systems, appearance of real eigenvalues is possible. This aspect of the problem was considered in [13].

According to [14], there exists a real transformation of the variables of the equation system (2)

$$T = \begin{pmatrix} V_1 & W_1 & T_1 \\ V_2 & W_2 & T_2 \end{pmatrix}, \tag{5}$$

by replacing the original variables $\{q, \dot{q}\}$ with new ones $\{\Xi, \mathrm{H}, \mathrm{I}\}$

$$\begin{cases} \dot{q} = V_1\Xi + W_1\mathrm{H} + T_1\mathrm{I} \\ q = V_2\Xi + W_2\mathrm{H} + T_2\mathrm{I} \end{cases}, \tag{6}$$

and reducing the matrix A to a tridiagonal form.

In transformation (5), the matrices $\{V_1, V_2\}$ and $\{W_1, W_2\}$ are respectively real and imaginary parts of the eigenvector matrix of the matrix A.

In new variables, the system of equations (2) takes the form:

$$\begin{cases} \dot{\Xi} = -\frac{1}{2}\Gamma\Lambda\Xi + \Lambda_* \mathrm{H} - P_1 \ddot{Y}_0 \\ \dot{\mathrm{H}} = -\Lambda_* \Xi - \frac{1}{2}\Gamma\Lambda \mathrm{H} - Q_1 \ddot{Y}_0 \\ \dot{\mathrm{I}} = NI - U_1 \ddot{Y}_0 \end{cases} \tag{7}$$

In the system of equations (7)

$\Gamma=\lceil\gamma_1,\ \gamma_2,\ \ldots\ \gamma_{nc}\rfloor$ is the diagonal matrix of inelastic resistance modal coefficients; $\Lambda=\lceil k_1,\ k_2,\ \ldots\ k_{nc}\rfloor$ is the diagonal matrix of natural frequencies of system oscillations; $\Lambda=\lceil k_1,\ k_2,\ \ldots\ k_{nc}\rfloor$ is the diagonal matrix of frequencies of the damped system; $N=\lceil \nu_1,\ \nu_2,\ \ldots\ \nu_{nr}\rfloor$ is the diagonal matrix of the real eigenvalues of the matrix A, where nc is the number of complex eigenvalues pairs, nr is the number of real eigenvalues ($2\cdot nc+nr=n$).

$P_{1,2}$, $Q_{1,2}$, $U_{1,2}$ are blocks of the matrix inverse to transformation matrix (5)

$$\mathrm{T}^{-1}=\begin{bmatrix} P_1 & P_2 \\ Q_1 & Q_2 \\ U_1 & U_2 \end{bmatrix} \tag{8}$$

The decoupling of oscillation equations similar to (7) is known in the vibration theory [13, 15–17].

The last matrix equation of system (7) is a system of independent first-order equations of the form

$$\dot{\upsilon}_j-\nu_j\upsilon=-d_j^{(U)}\ddot{y}_0 \tag{9}$$

Here the variable υ_j is an element of the vector of variables I.

The kinematic input vector is written in the form $\ddot{Y}_0=V_p\ddot{y}_0$, where $\ddot{y}_0$ is the accelerogram of base oscillations, V_p is the vector of excitation projections on the directions of generalized coordinates; $d_j^{(U)}$ is an element of the vector U_1V_p.

The first two equations of system (7) are written down in pairs of independent second-order equations

$$\begin{aligned}\ddot{\xi}_j+\gamma_j k_j\dot{\xi}_j+k_j^2\xi_j&=-d_j^{(1)}\ddot{y}_0+d_j^{(2)}\dddot{y}_0\\ \ddot{\eta}_j+\gamma_j k_j\dot{\eta}_j+k_j^2\eta_j&=-d_j^{(3)}\ddot{y}_0+d_j^{(4)}\dddot{y}_0\end{aligned} \tag{10}$$

The coefficients $d_j^{(1)}$, $d_j^{(2)}$, $d_j^{(3)}$, $d_j^{(4)}$ are the elements of the vectors $(\Lambda{*}Q_1+0.5\Gamma\Lambda P_1)V_p$; P_1V_p; $(\Lambda{*}P_1+0{,}5\Gamma\Lambda Q_1)V_p$ и Q_1V_p respectively.

If damping in the system is proportional, i.e. condition (11) takes place,

$$B_{eq}=RX\Phi X^{-1}, \tag{11}$$

then the coefficients $d_j^{(2)}$, $d_j^{(3)}$, $d_j^{(4)}$ become 0.

In condition (1), Φ is an arbitrary real diagonal matrix.

For arbitrary damping, movement in the direction of i-th generalized coordinate is presented as the sum

$$y_i=\sum_j \chi_{ij}^{(1)}\xi_j^{(1)}+\chi_{ij}^{(2)}\xi_j^{(2)}+\chi_{ij}^{(3)}\eta_j^{(3)}+\chi_{ij}^{(4)}\eta_j^{(4)}+\sum_s \chi_{is}^{(5)}\upsilon_s \tag{12}$$

$\chi_{ij}^{(k)}$ are coefficients similar to the mode coefficient η_{ij} introduced in the applied earthquake engineering theory and used in the existing Guidelines [14], they are defined as follows: $\chi_{ij}^{(1)}=v_{ij}d_j^{(1)}$, $\chi_{ij}^{(2)}=v_{ij}d_j^{(2)}$, $\chi_{ij}^{(3)}=w_{ij}d_j^{(3)}$, $\chi_{ij}^{(4)}=w_{ij}d_j^{(4)}$, $\chi_{ij}^{(5)}=u_{ij}d_j^{(U)}$;

$\xi_j^{(1)}$ and $\eta_j^{(3)}$ are determined from the solution of the following equations:

$$\ddot{\xi}_j^{(1)} + \gamma_j k_j \dot{\xi}_j^{(1)} + k_j^2 \xi_j^{(1)} = \ddot{y}_0 \text{ and } \ddot{\eta}_j^{(3)} + \gamma_j k_j \dot{\eta}_j^{(3)} + k_j^2 \eta_j^{(3)} = \ddot{y}_0; \tag{13}$$

$\xi_j^{(2)}$ and $\eta_j^{(4)}$ are determined from the solution of the following equations:

$$\dddot{\xi}_j^{(1)} + \gamma_j k_j \dot{\xi}_j^{(1)} + k_j^2 \xi_j^{(1)} = \dddot{y}_0 \text{ and } \dddot{\eta}_j^{(3)} + \gamma_j k_j \dot{\eta}_j^{(3)} + k_j^2 \eta_j^{(3)} = \dddot{y}_0; \tag{14}$$

The function υ_j is obtained from the solution of equation (9).

We note that the appearance of the derivative of the load (the third derivative of the base displacements) in the right-hand side of presentation (10) was mentioned in paper on damping forced oscillations [1].

The obtained presentation of solution (11) makes it possible both numerical integration of the equations of motion with retention of the required number of vibration modes and the application of the response spectra method (RSM) with some refinement.

When the RSM is used, seismic forces in the absence of real eigenvalues should be summed up according to the formula

$$s_i = \left[\sum_{k=1}^{nf} \sum_{j=1}^{nf} \sum_{m=1}^{2} \sum_{n=1}^{2} s_{ij}^{(m)} s_{ik}^{(n)} \varepsilon_{jk}^{(m,n)} \right]^{1/2} \tag{15}$$

where 'nf' is the number of the considered oscillation modes;

m = 1, 2 is the index of the displacement component ξ or η;

N = 1, 2 the index of the component of the displacement caused by $\ddot{y}_0$ or $\dddot{y}_0$;

$\varepsilon_{jk}^{(m,n)}$ is the corresponding correlation coefficient.

If we take into account that the correlation coefficient of functions ξ_j or η_j is equal to 1, and the correlation coefficient between the function and its derivative is equal to 0, then expression (15) is simplified and with the account of damped modes of oscillations takes the form

$$s_i = \left[\sum_j \sum_k s_{ij} s_{ik} \varepsilon_{jk} + \sum_j \sum_k s'_{ij} s'_{ik} \varepsilon''_{jk} + \sum_{j_{j\neq}} \sum_{k} s_{ij} s'_{ik} \varepsilon'_{jk} + \sum_s \sum_r s^*_{is} s^*_{ir} \varepsilon^*_{rs} + \right]^{1/2} \tag{16}$$

In formula (16), the first term is conventional seismic force, determined on the basis of equations (13) and having a standard form for a system with concentrated masses m_i: [2]

$$s_{ij} = A \cdot g \cdot m_i \cdot \beta(T_j) \cdot K_\psi(\gamma_j) \cdot \left(\chi_{ij}^{(1)} + \chi_{ij}^{(2)} \right) \tag{17}$$

The second term in formula (17) is completely analogous to the first, but is obtained on the basis of equations (14), in which the derivative of the input accelerogram is used as the input excitation. In this case, the seismic forces formula is the following

$$s'_{ij} = A \cdot g \cdot m_i \cdot \beta(T_j) \cdot K_\psi(\gamma_j) \cdot \frac{\left(\chi_{ij}^{(3)} + \chi_{ij}^{(4)} \right)}{k_j} \tag{18}$$

The third term appears due to a possible correlation between seismic forces S_{ij} and S'_{ij} for i≠j.

Finally, the fourth term is determined on the basis of equation (9). For seismic forces, it is also possible to obtain an analogue of the dynamic factor by presenting seismic load in the form

$$s_{ij}^{*}=A\cdot g\cdot m_{i}\cdot\beta\left(\nu_{j}\right)\cdot\chi_{ij}^{(5)} \quad (19)$$

If the modes are not correlated, then from (16) we obtain a formula similar to that used in the current Russian standards of earthquake-proof construction.

$$s_{i}=\left[\sum s_{ij}^{2}+\sum s_{ij}'^{2}+\sum\left(s_{ij}^{*}\right)^{2}\right]^{1/2} \quad (20)$$

An important problem of estimating seismic loads on a damped system is calculating correlation coefficients, which are determined by formulas

$$\varepsilon_{ij}=\int_{0}^{\infty}\frac{S(\omega)e^{i\omega t}d\omega}{Z_{i}Z_{j}};\quad \varepsilon_{ij}'=\int_{0}^{\infty}\frac{\omega S(\omega)e^{i\omega t}d\omega}{Z_{i}Z_{j}};\quad \varepsilon_{ij}''=\int_{0}^{\infty}\frac{\omega^{2}S(\omega)e^{i\omega t}d\omega}{Z_{i}Z_{j}}; \quad (21)$$

Here $S(\omega)$ is the input spectral density, $Z_j=(\omega^2-k_j^2)^2-\gamma_j^2\,k_j^2\,\omega^2$.

For $\varepsilon_{ij}*$ the correlation coefficient has the following form

$$\varepsilon_{ij}^{*}=\int_{0}^{\infty}\frac{S(\omega)e^{i\omega t}d\omega}{\left(\omega-\nu_{i}\right)\left(\omega-\nu_{j}\right)} \quad (22)$$

Many studies have been devoted to the problem of calculating integrals (21–22). Due to the fact that Zj functions have a pronounced peak at $\omega = kj$, the form of the function $S(\omega)$ does not play an important role. The results obtained by formulas of A.A. Petrov [18], A. Ter Kuryugian [19, 20] and other authors diverge by 3–7%. The authors of this paper prefer to use the formula of A.A. Petrov. The development of formulas of A. Petrov and A. Ter Kurrigyan is presented in [21].

3. Results and Discussion

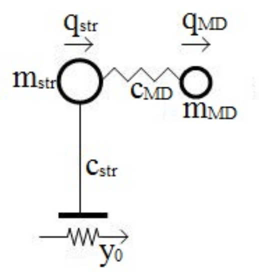

Figure 1. The design diagram for analyzing systems with the MD

The above proposed method allows one to consider linear systems with inhomogeneous and non-proportional damping. The most important area of application of the proposed technique is calculating the tuned mass damper (MD) and the soil- structure-interaction. For example, let us consider calculating the MD, shown in Figure 1. The MD is considered for tuning equal to 0.98 and the relative mass equal to 0.1 and damping equal to 5% of the critical value. The MD tuning $\kappa=\frac{k_{MD}}{k_{str}}$, where $k_{MD}=\sqrt{\frac{C_{MD}}{m_{MD}}}$, $k_{str}=\sqrt{\frac{C_{str}}{m_{str}}}$; the relative mass $\nu=\frac{m_{MD}}{m_{str}}=0.1$.

In Figure 2 the graphs show the dependence of the MD efficiency (relative decrease of the shearing force on the structure base) on damping in the MD spring. The first graph (curve 1) shows the conventional solution with the motion decomposition by the oscillation modes of the undamped system obtained earlier

[22], the second graph (curve 2) shows the exact solution with harmonic excitation and the third graph (curve 3) - the proposed solution.

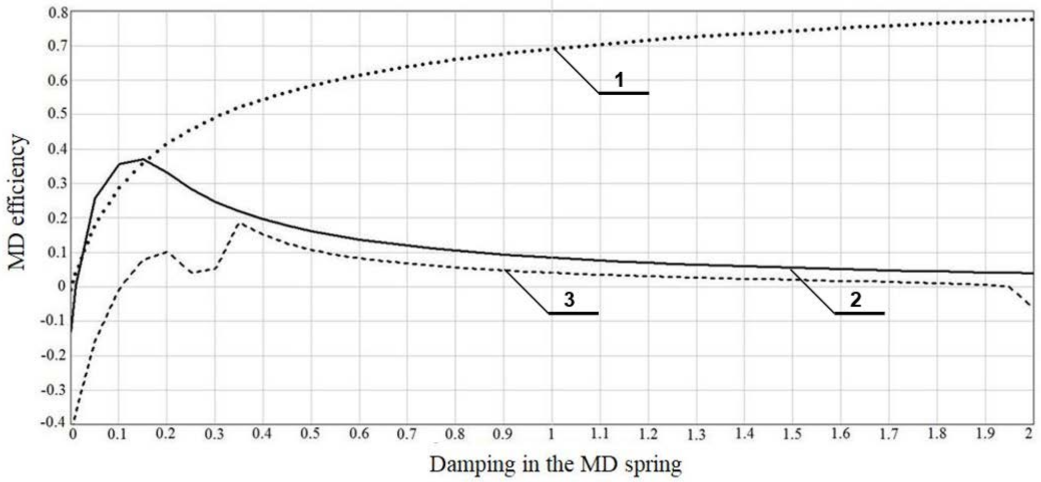

Fig.2. Dependence of the MD efficiency on damping in the MD spring;
Curve 1 – conventional solution with the motion decomposition by the oscillation modes of the undamped system;
Curve 2 – solution with harmonic excitation;
Curve 3 – the proposed solution

In the proposed solution, a normative spectral curve is used to set the input excitation. It can be seen from Figure 2 that the motion equation decomposition by to the oscillation modes of the undamped system is acceptable for solving the problem under consideration with damping of less than 0.2 of the critical value, but for greater damping such decomposition gives a qualitatively incorrect result that does not provide the safety margin. The exact solution with a harmonic excitation qualitatively coincides with the proposed one, and the existing difference between the solutions is due to the difference in the spectra of the given inputs.

4. Conclusions

1. The method for calculating strongly damped systems using their spectral distribution and estimating seismic loads on the basis of the RSM is proposed.

2. The proposed method of calculating systems with a non-proportional damping allows one to carry out linear calculations of a wide range of systems, including seismoprotection systems, the MD, structures h a soil massif, structures consisting of different materials, etc. The use of conventional methods for culating this sort of structures can result in significant errors, distorting calculation results by 2 or more times. The peculiarities of the proposed method are calculating complex eigenvalues and complex eigenvectors of the system under consideration, introducing an amendment to seismic forces for each oscillation mode and taking into account the mode correlation

3. Using this method, one can set the design input excitation by the normative response spectrum. This makes it possible to select the parameters of special seismic protection at the initial stages of designing, when no design accelerograms for the building site are available.

4. Proposed results are illustrated as applied to the calculating the tuned mass damper

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