are given of some papers that use the induction method to generalize particular solutions in the derivation of formulas suitable for a sufficiently wide class of problems. The number of schemes of such regular-type trusses with a periodic structure is relatively small. R.G. Hutchinson and N.A. Fleck [8, 9] announced the "hunt" for schemes of statically determinate periodic trusses. For some types of spatial regular trusses an attempt [10] was made to sort the periodic structures. Formulas for the dependence of deflection and effort on the number of panels in the rods of various planar [11–16] and spatial [17–20] statically determinate trusses are known. In [21], an algorithm is given for analyzing the limiting state of spatial rod frames using kinematic relations. Almost all formulas are designed to evaluate statically determinate structures with a periodic structure of trusses, to which the induction method with one parameter is applicable, for example, the number of panels in the span. Analytic solutions with two independent natural parameters of the structure are much more interesting. In this paper, we propose a two-parameter analytic calculation of the core frame with an arbitrary number of panels in the crossbar and the number of panels in the vertical support parts of the frame. The problems of the topology of the trusses and their optimization are considered in [22–24].

2. Methods

2.1. The calculation of the stresses in the rods

A frame with a cross-shaped grate and four supports, one of which is fixed, contains $2n_1$ panels of length a in the crossbar and panels in side parts of height h (Figure 1).

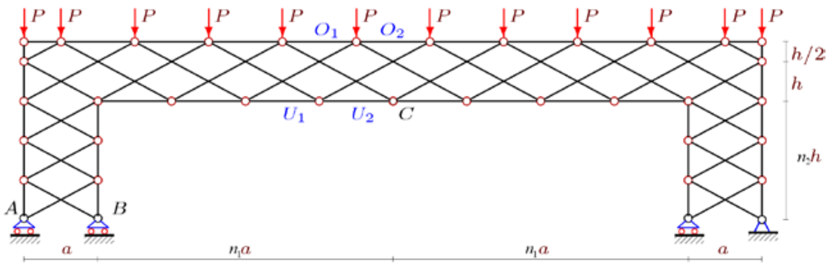

Figure 1. The scheme of the truss, $n_1 = 4,\ n_2 = 3$

Calculation of the forces in the rods of individual trusses with specific numbers of panels n_1 and n_2 is done by cutting out the nodes. A distinctive feature of the considered truss is its external static indeterminacy. This means that the forces in the supports can only be found from the general system of equilibrium equations for all nodes, together with the forces in the rods. The periodic structure of the truss allows you to write equations of node equilibrium in cycles using the Maple programming language. To do this (also using the loop operator), the node coordinates and the connection structure of the rods are defined by analogy with the task of graphs.

The elements of the matrix **G** of the system of equations of equilibrium of nodes contain the direction cosines of the forces. Here is a fragment of the program for specifying a matrix in the Maple language:

```
> M5:=M+5:
> for i to M5 do
> Lxy[1]:=x[N[i][2]]-x[N[i][1]]:
> Lxy[2]:=y[N[i][2]]-y[N[i][1]]:
> L[i]:=sqrt(Lxy[1]^2+Lxy[2]^2);
> for j to 2 do
> t:=2*N[i][2]-2+j:
> if t<= M5 then G[t,i]:=-Lxy[j]/L[i]:fi;
> t:=2*N[i][1]-2+j:
> if t<= M5 then G[t,i]:= Lxy[j]/L[i]:fi;
```

```
> od;
> od:
```

Here, `M=8(n1+n2)+13` – the number of rods, `Lxy` – the projection of the oriented rods on the coordinate axis, `x,y` – the coordinates of the hinges. The vector `N[i][1]` indicates the hinge number in the conditional beginning of the rod `i`, in `N[i][2]` – the end. In the vector of the right-hand part `B` of the system of equations `G.S=B` where `S` is the force vector in the rods, information about the load is recorded. For the case of uniform load in the upper-belt nodes, we have the following entry in the program

```
for i from 3*n2+2*n1+4 to 3*n2+4*n1+7 do B[2*i]:=1: end:
```

To solve a system of linear algebraic equations in symbolic form, the inverse matrix method is used in a very simple form: `1/G`. This way the solution is faster than with the help of the special operator `LinearSolve`.

The first solutions for a number of trusses with different numbers n_1 and n_2 have shown that for n_1 = 2, 5, 8, 11, ... and for any n_2 the determinant of the system of equilibrium equations vanishes. This indicates a kinematic variability of the structure with such a number of panels. In general, inadmissible numbers of panels with step 3 can be represented as the following formula $n_1 = 3k - 1,\ k = 1, 2, \ldots$. For some other flat trusses, this effect was previously seen in [11, 15, 16, 20]. To confirm the found feature, you can specify a scheme of possible speeds of the hinges (Figure 2).

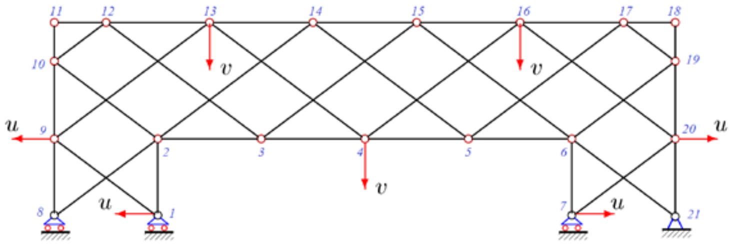

Figure 2. Scheme of possible speeds of an instantly variable truss

The joints 2, 3, 5, 6, 8, 10-12, 14-15, 17-19 remain stationary, rods 89 and 9-10 perform a rotation about the instantaneous joint 9, 12-13 and 13-14 are rotated about the joint 13, the rods 9-10 and 4-13 move instantaneously. The same movements make symmetrical rods on the right side of the truss. Obviously the ratio of speeds. To notice this feature is not easy. In calculations this is manifested in the degeneration of the system of equilibrium equations. In this case, numerical methods can skip this feature hidden behind the errors of the count. And when modeling a truss with finite elements with rigid nodes instead of hinged, this feature is not visible at all. A review of similar cases, where the values of panel numbers in regular trusses were also found, and the schemes of possible speeds of nodes of variable trusses are given in [6].

Degeneration of the system of node equilibrium (or kinematic changeability of the truss) at can also be noticed without cutting out all the nodes. It suffices to consider the nodes of the chain 1-9-13-4-16-20-7, recording only one equation for each node. If we write the equations of equilibrium of nodes 13, 14 and 16 on the vertical axis, and for the remaining nodes on the horizontal axis, then the inconsistency of the resulting system of equations becomes immediately clear. For example, if the entire truss is free of loads and only the support node 7 is loaded with horizontal force, then the forces in the rods 1-9, 9-13, 13-4, 4-16, 16-20 and 20-7 must be zero, that contradicts the equation of equilibrium of the loaded node 7 in the projection to the horizontal, from which it follows that the force in rod 20-7 is not zero.

2.2. Deflection

We use the formula of Maxwell-Mohr for the computation of deflection:

$$\Delta = \sum_{j=1}^{M} \frac{S_j s_j l_j}{EF}, \tag{1}$$

where E and F – the modulus of elasticity and the cross-sectional area of the rods, l_j and S_j – length and stress in *j*-th core under the action of a given load, s_j – forces under a single vertical force applied to the central node *C*. In order to construct a sequence of solutions, the generalization of which gives the required dependence of the deflection on the number of panels, while avoiding degeneracy, we give the following relationship $n_1 = (6k - 1 + (-1)^k)/4$. The obtained sequence, when changing $k = 1, 2, 3...$ passes the unacceptable values and gives for the deflection of the structure under the action of the load distributed over the upper belt, a solution of the form

$$\Delta = P(C_{1,n_2,k}a^3 + C_{2,n_2,k}c^3 + C_{3,n_2,k}h^3)/(2h^2EF). \quad (2)$$

The coefficients in this expression depend only on k and n_2 form sequences for which common terms can be found. For the coefficient at a^3 the operator rgf_findrecur returns an equation of the ninth order

$$C_{1,n_2,k} = C_{1,n_2,k-1} + 4C_{1,n_2,k-2} - 4C_{1,n_2,k-3} - 6C_{1,n_2,k-4} + 6C_{1,n_2,k-5} + 4C_{1,n_2,k-6} - 4C_{1,n_2,k-7} - C_{1,n_2,k-8} + C_{1,n_2,k-9}.$$

The initial conditions obtained from the solutions of the problems of deflection of trusses with $k = 1,...,9$ and arbitrary numbers n_2 have the form

$$C_{1,n_2,1} = 8, C_{1,n_2,2} = 72, C_{1,n_2,3} = 268, ..., C_{1,n_2,9} = 15016.$$

The operator *rsolve* gives the following solution of the recurrence equation

$$C_{1,n_2,k} = (30k^4 + 4((-1)^k + 15)k^3 + (6(-1)^k + 34)k^2 + 20(1 - (-1)^k)k + 13(-1)^k - 13)/16. \quad (3)$$

Similarly, we obtain other coefficients:

$$C_{2,n_2,k} = 3(2k^2 + 2(1 - (-1)^k)k - (-1)^k + 1)/8, \quad (4)$$

$$C_{3,n_2,k} = (2(n_2 + 2 - 2(n_2 + 1)(-1)^k)k - (11n_2 + 8)(-1)^k + 7n_2 + 8)/2. \quad (5)$$

The coefficients of a^3 and c^3 do not depend on the number of panels n_2. And to obtain a coefficient $C_{3,n_2,k}$, which also depends on the number n_2 of panels along the vertical, it was necessary to perform induction on this parameter. Solving the problem sequentially for $n_2 = 1, 2, ..., 6$, and performing each time by induction on n_1, we obtain a series of solutions

$$C_{3,1,k} = (6 - 8(-1)^k)k - 19(-1)^k + 15)/2,$$
$$C_{3,2,k} = (8 - 12(-1)^k)k - 30(-1)^k + 22)/2,$$
$$C_{3,3,k} = (10 - 16(-1)^k)k - 41(-1)^k + 29)/2,$$
$$C_{3,4,k} = (12 - 20(-1)^k)k - 52(-1)^k + 36)/2,$$
$$....$$

The coefficients in these formulas form the obvious arithmetic sequences in steps of 2, 4, 11 and 7. This allows us to obtain the general formula (5) for $C_{3,n_2,k}$. Thus, formula (2) with coefficients (3–5) gives the required dependence of the deflection of the structure on its dimensions, the magnitude of the load and the number of panels.

A similar solution is obtained for the case of loading the truss by the lower belt of the crossbar (Figure 3). For this, the program needs to write the vector of loads in the following form

```
for i from n2+2 to n2+2*n1 do B[2*i]:=1: end:
```

The form of the solution (2) does not change. By induction for the coefficients, we obtain expressions

$$C_{1,n_2,k} = (30k^4 + 4((-1)^k + 15)k^3 - 2(5(-1)^k + 1)k^2 - 9(-1)^k + 9)/16,$$
$$C_{2,n_2,k} = 3(4k^2 + 4(1-(-1)^k)k - 2(-1)^k + 2)/16,$$
$$C_{3,n_2,k} = (2(n_2 + 2 - 4(n_2+1)(-1)^k)k - 3n_2(-1)^k + n_2)/2.$$

The recurrence equations for the coefficients remain the same as for loading the nodes of the upper belt. Only the initial data differ.

The method of double induction by *k* and n_2 is used for obtaining $C_{3,n_2,k}$.

The coefficients in the deflection formula (2) have a somewhat simpler form, in the case of the action on the structure of one force at the point *C*:

$$C_{1,n_2,k} = (4k^3 + 6(1-(-1)^k)k^2 + 2(4-3(-1)^k)k + (-1)^k - 1)/4,$$
$$C_{2,n_2,k} = 3(2k + 1 - (-1)^k)/4,$$
$$C_{3,n_2,k} = 3n_2 + 2 - 2(n_2+1)(-1)^k.$$

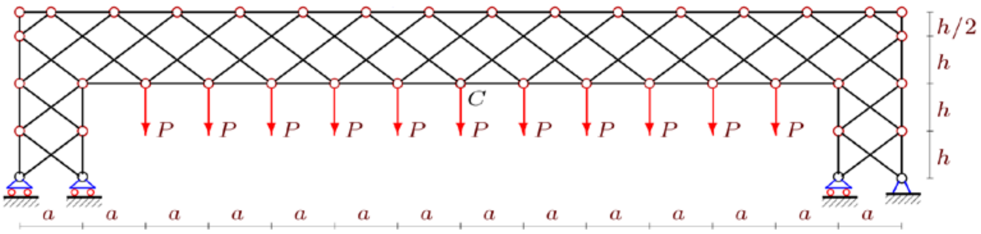

Figure 3. The scheme of the truss, $n_1 = 6,\ n_2 = 2$

The formulas obtained require verification. This can be done in the derivation with an inverse induction sequence. First, we perform an induction on the parameter n_2, then on the parameter n_1. In some cases, changing the sequence of inductions significantly speeds up the solution. The final check can be a count on the found formulas in the numerical mode of the program, the speed of which is much larger than the symbolic transformations. For example, with a dozen panels along the vertical of the truss and 40-50 in the crossbar, the time for symbolic obtaining of the formula is unrealistically large and is calculated in days. The numerical solution for the same program is executed in fractions of seconds. This fact is one of the reasons for applying the induction method. Otherwise, you could get an analytical solution each time for a truss with a specific number of panels and a given load. A similar feature is possessed by other systems of symbolic mathematics (Mathematica, Maxima, etc.).

2.3. Reaction of supports and forces in critical rods

Simultaneously with the derivation of formulas for deflection by induction, formulas were obtained for the reactions of supports and forces in certain rods (Figure 1). On the basis of these formulas it is possible in the future to evaluate the stability and strength of the rods. Consider the rods from the middle of the span. When loading the upper belt, we have the following expressions

$$Y_A = P(k((-1)^k + 3) + 5 + (-1)^k)/2,$$
$$Y_B = -P(2(-1)^k k + (-1)^k + 3)/4,$$
$$O_1 = -Pa(6k^2 + 2((-1)^k + 3)k + (-1)^k - 1)/(8h),$$
$$O_2 = -U_1 = -Pa(6k^2 + 2(5(-1)^k + 3)k + 5(-1)^k + 3)/(8h),$$
$$U_2 = Pa(6k^2 + 2((-1)^k + 3)k + (-1)^k + 7)/(8h).$$

The case of loading the lower belt:

$$Y_A = Pk((-1)^k + 3)/2,$$
$$Y_B = -P(2(-1)^k k - (-1)^k + 3)/4,$$
$$O_1 = -Pa(6k^2 + 2((-1)^k + 3)k + (-1)^k - 1)/(8h),$$
$$O_2 = -Pa(6k^2 + 2(5(-1)^k + 3)k + 5(-1)^k + 3)/(8h),$$
$$U_1 = Pa(6k^2 + 2(5(-1)^k + 3)k - 3(-1)^k - 9)/(8h),$$
$$U_2 = Pa(6k^2 + 2((-1)^k + 3)k + (-1)^k - 5)/(8h).$$

The case of force loading in the middle of the span:

$$Y_A = P(1-(-1)^k)/2,\ Y_B = P(-1)^k/2,$$
$$O_1 = O_2 = -Pa(2k + 1 - (-1)^k)/(4h),$$
$$U_1 = U_2 = Pa(2k - 1 - (-1)^k)/(4h).$$

3. Result and Discussion

In the formulas obtained, some features of the solution are hidden, which the graphical representation of them for specific cases will help to see.

In order to clarify the qualitative picture of the deformation of the construction that the solution obtained, we construct the curves of the dependence (2) of the deflection on the number of panels in the crossbar in the case of loading the lower belt of the truss at $n_2 = 1$. We assume that the span length is constant and does not depend on the number of panels $L = 40m$, hence $a = L/(2n_1)$. Let us also fix the total load on the structure $P_{sum} = (2n_1 - 1)P$. We introduce the dimensionless deflection $\Delta' = \Delta EF/(P_{sum}L)$. With an increase in the number of panels, the deflection in this formulation of the problem decreases first, then increases (Figure 4). In this case, the curve has very significant jumps, but the minimum is weakly expressed.

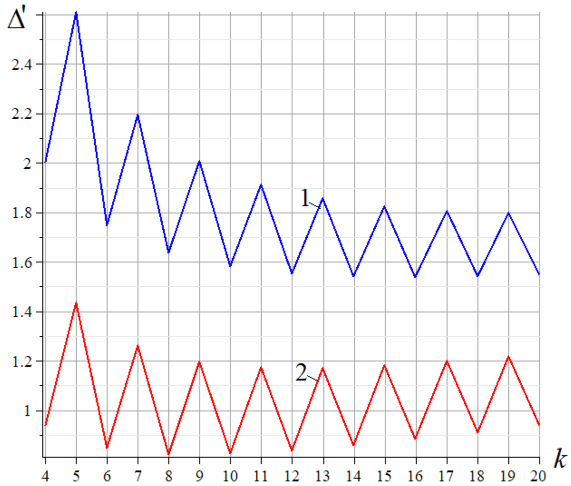

Figure 4. The dependence of the deflection of the panels. 1 – $h = 4m$, 2 – $h = 6m$

Despite this, this fact can be used to optimize stiffness. Analytical expressions for the minimum value cannot be obtained.

The dependence of the deflection on the number of panels n_2 along the vertical reveals a more pronounced minimum (Figure 5, $k = 5$, $L = 40m$). In addition, the curves have an intersection point, after which the effect of the height of the truss on the deflection is reversed. Here the height of the truss *H* is adopted, which is independent of the number of panels in the vertical direction: $h = H/n_2$. For small values of n_2, the deflection increases with the greater the height of the truss. After the intersection point (in Figure 5 this is n_2 = 7), the dependence is inverse.

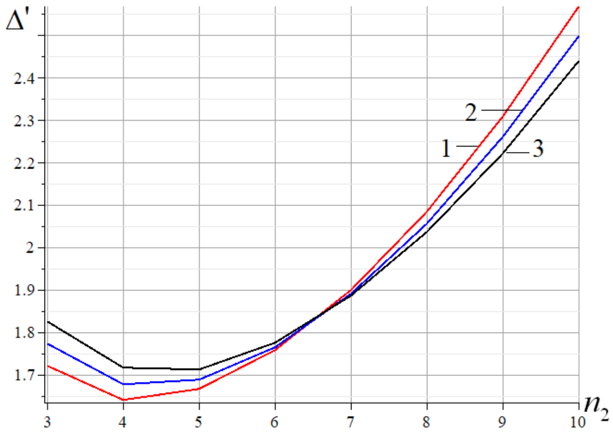

Figure 5. The dependence of the deflection of the number n_2. 1 – $H = 50m$; 2 – $H = 52m$; 3 – $H = 54m$

The obtained dependences of the forces in the rods on the number of panels also have some peculiarities. Help them to help with graphics. Consider the case of loading the lower belt at $n_2 = 1$, $h = 4m$, $L = 40\,m$ (Figure 6). The force values depend essentially on the parity of the number of panels n_1 and do not depend on the number of panels in height. The same effect can be observed under other loads. In this case, the maximally stretched rod can be not a rod U_2 in the middle of the span, but a rod U_1 adjacent to it. The same alternation is observed for compressed elements O_1 and O_2. With increasing k, the amplitude of the oscillations of the graph decreases.

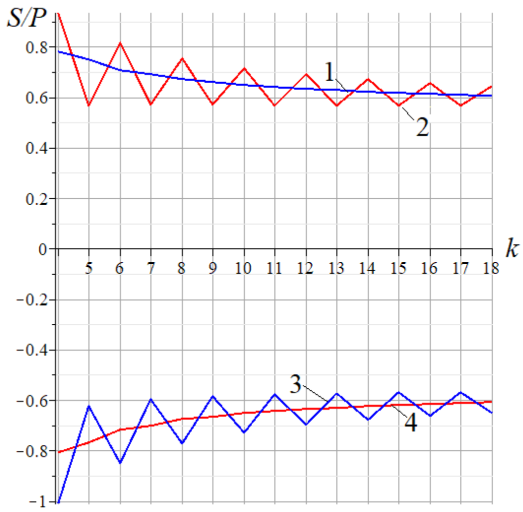

Figure 6. Dependence of the forces in the rods on the number of panels. 1—U_1; 2—U_2; 3 – O_1; 4 – O_2

4. Conclusion

1. The analytical dependences of the deflection of a planar truss structure on its dimensions, load and number of panels have been found and analyzed. To generalize particular solutions, we carry out induction on two parameters.

2. It is revealed that the proposed scheme of the truss has some hidden feature, which can be considered a defect. Externally, a fully functional design with certain values of the panel numbers turns out to be kinematically variable.

3. Another feature of the proposed scheme is its external static uncertainty. This creates difficulties in determining the reaction of supports, but allows modeling of structures with many supports with a statically determinate scheme. Of course, in design practice, such schemes can be used as basic ones for more complex statically indeterminate structures with additional constraints, rigid knots and rods of different cross-sections and elastic properties. However, the resulting formulas, due to their simplicity, accuracy and ease of use, can always be useful both for estimating the structural deformity and for estimating the accuracy and reliability of numerical solutions.

4. The solutions essentially depend on the parity of the number of panels in the crossbar. This means that changing the number of panels in a project by just one can significantly reduce or increase the rigidity of the structure or the force in the rod. Judging from Figure 6, the force in the middle rod of the lower belt with a small number of panels varies in this setting to 30 %. Interesting is also the effect of changing the alternation of curves depending on the height of the truss when the number of panels is changed vertically.

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