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## Reliability estimation of industrial building structures

## Оценка надежности конструкций промышленных зданий

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С.Ф. Пичугин,***Полтавский национальный технический университет имени Юрия Кондратюка, г. Полтава, Украина***Key words:** construction; building; structure; reliability; failure; probabilistic method; load; redundant system**Ключевые слова:** сооружение; здание; конструкция; надежность; отказ; вероятностный метод; нагрузка; статически неопределимая система

**Abstract.** The article is devoted to solving the problem of the reliability of industrial building structures: crane and roof beams, columns, redundant building systems. For this aim, the probabilistic method of structural reliability estimation is developed after the criterion of bearing strength, in which the main component of structure reliability is faultlessness. The method takes account of the random loads and material strength, loads joint action, the specific character of work and failure elements, nodes and the whole structure as well. For the ground of method, the large amount of statistic results on crane load is examined for the bridge cranes of the different types. A large amount of wind and snow meteorological data is collected for the territory of Ukraine. Stationary probabilistic model of crane load and quasi-stationary model of the snow and mean wind load are substantiated. The most widely spread probabilistic presentations of random loads are observed. They are as follows: stationary random process and its absolute maxima, random sequence of independent and correlated loads, discrete presentation and extreme model. It was deduced that the redundant structure reliability estimation is a very complicated problem as depends upon the system complexity. The method of states, a probabilistic method of ultimate equilibrium and logic and probabilistic method are developed for solving this problem. On the base of the determined method, the numerical reliability computations of a wide range of industrial building structures are realized. It is shown that the structures have quite different levels of reliability. In particular, the light roof structures are not reliable enough being under the great influence of snow load. At the same time, the Design Code allows over-estimation of reliability for industrial columns. The estimation of industrial redundant structures with a different degree of redundancy is obtained on the base of developed approach. It gave the possibility to evaluate the high safety level of redundant structures in comparison with separate members and statically determined structures. With regard to mentioned results, it is recommended to correct some load factors, a combination factor and a factor for model uncertainties of the Design Codes of structures and loads.

**Аннотация.** В статье рассмотрены вопросы оценки надежности конструкций промышленных зданий: подкрановых и стропильных балок, колонн, статически неопределимых конструктивных систем. Для этого разработан вероятностный метод оценки надежности конструкций по критерию несущей способности, основой которого является безотказность конструкции. Метод учитывает случайные нагрузки и прочность материалов, совместное действие нагрузок, специфический характер работы и отказов элементов, узлов и сооружений в целом. Для обоснования метода был сформирован обширный объем статистических данных по нагрузкам мостовых кранов различных типов. Большое количество метеорологических данных по ветру и снегу было собрано для территории Украины. Обоснованы стационарная вероятностная модель для крановой нагрузки и квазистационарная модель для снеговой и ветровой нагрузок. Рассмотрены наиболее распространенные вероятностные представления случайных нагрузок: стационарный случайный процесс и его абсолютные максимумы, случайная последовательность независимых и коррелированных нагрузок, дискретное представление и экстремальная модель. Было подтверждено, что оценка надежности статически неопределимых систем – достаточно сложная проблема, поскольку она зависит от сложности таких систем. Для решения данной проблемы разработаны метод состояний, вероятностный метод предельного равновесия, логико-вероятностный метод. На основе разработанного метода выполнены численные расчеты надежности широкого круга строительных конструкций. Показано, что различные конструкции

имеют разный уровень надежности. В частности, недостаточно надежными могут быть легкие стропильные конструкции при действии значительных снеговых нагрузок. В то же время нормы проектирования закладывают в колонны промышленных зданий излишний запас надежности. На основе разработанного подхода выполнены расчеты надежности статически неопределимых систем разной степени сложности, которые дали возможность оценить повышенный уровень надежности таких систем по сравнению с отдельными элементами и статически неопределимыми системами. В результате были обоснованы коэффициенты норм проектирования, такие как коэффициенты надежности по нагрузке, коэффициент сочетания нагрузок, коэффициент условий работы.

## 1. Introduction

The building structure reliability is the problem of a high priority today. It can be treated as a scientific trend, which does not need both material and financial expenses.

The eminent scientists created the classic works in building reliability [1–3], numerous foreign and native scientists were involved in the study of this problem [4–12]. The general probabilistic models of wind, snow and crane loads can be considered as a statistic basis of reliability estimation of industrial building structures. Loads of bridge cranes were considered in [13], the description of the wind load was summarized in [14]. Active researches of the wind load proceed the last years [15–17]. The analysis of snow load was resulted in [18], the influence of global temperature rise on the snow load is analyzed in work [19]. In different countries, systematic researches of climatic influences are conducted with the purpose of Codes correcting [20]. The perspective model is a correlated casual sequence of loads (based on the method of the generalized covariance, developed by A.P. Kudzis [21]). The linearizing of the functions of random parameters worked out in work [22]. However, these approaches lack both systematic analysis and model comparison, which can cause different results in structure reliability design.

The probabilistic computation of structures has not been developed with regard to joint loads application, their real distributions, and frequency characteristics. At the same time, active studies of probabilistic models for concrete elements proceed the last years [23], in particular, the initial reliability of reinforce-concrete elements was defined [24]. However, the estimation of reliability is yet absent for many buildings and structures. The problem of reliability continues to be probed [25–30] and expects further investigations and solutions. The exploitation experience demonstrated the existence of quite different reliability of various structures. The reliability of existing structures has to be examined together with the reliability of elements (sections) as well as nodes [31–33]. The assessment of a redundant structure safety is rather a difficult task. In spite of many studies that have been done the problem of reliability analysis of these structures has not been solved yet.

As follows from the preceding, the evolution of reliability computation theory and design Codes of building structures are still of interest because of the complexity of a problem on the one hand and on the other of the ignoring the random loads.

The article is devoted to solving the problem related to the assessment of the reliability of industrial building structures: crane and roof beams, columns, redundant building systems. This task deals with the developing of the general method of reliability estimation of industrial building structures after the criterion of bearing strength, in which the main component of structure reliability is faultlessness. At such interpretation the estimation of structure reliability includes the decision of the following questions: probabilistic description of loads; mechanical descriptions of materials, joints and other random parameters; reliability estimation of elements of building structures; calculation of reliability of the structural systems taking into account the possible character of their failure; quantitative estimation of reliability of buildings and structures of the different setting.

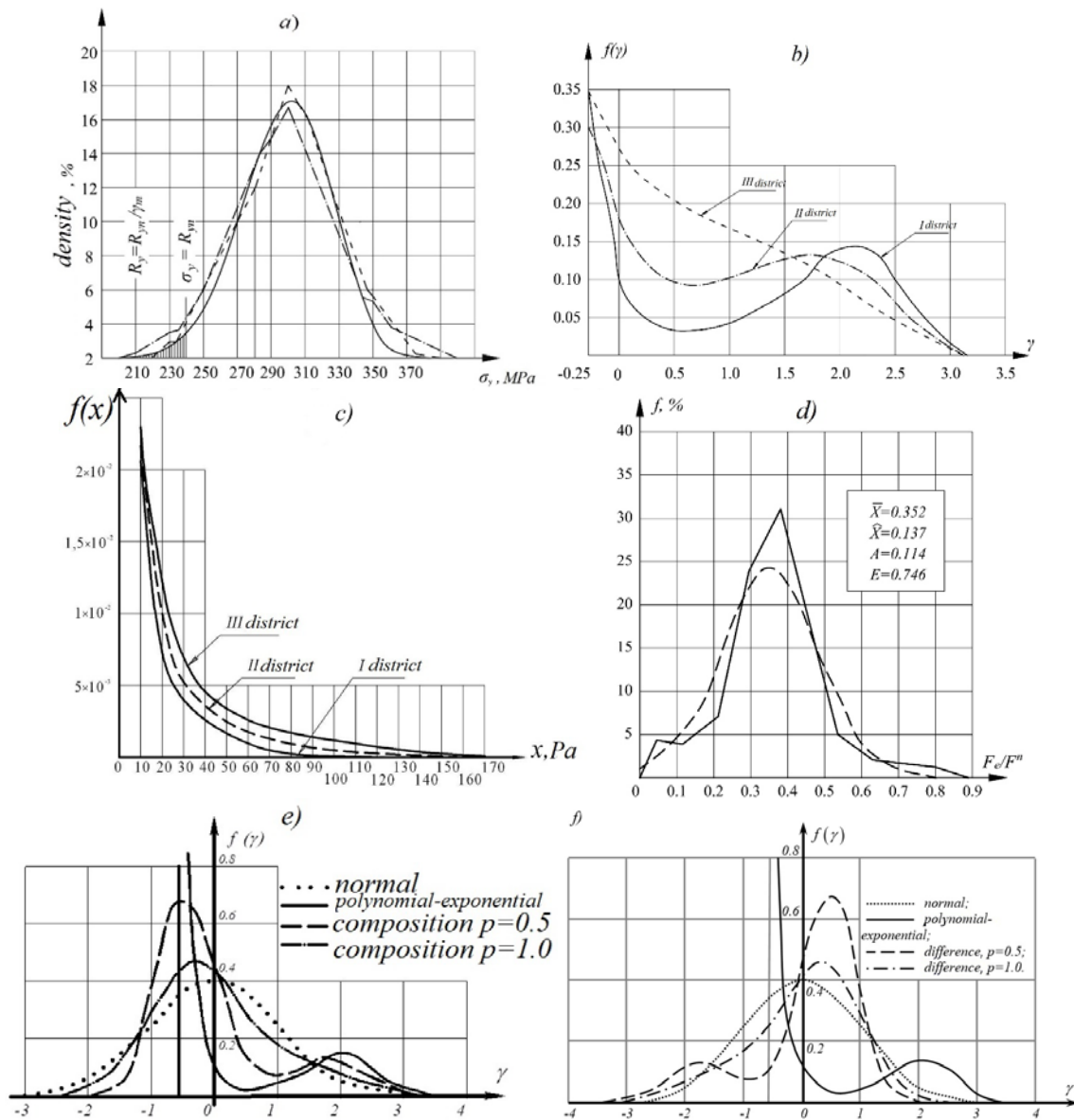
## 2. Methods

It was put and decided the complicated and difficult task of formation of the generalized probabilistic models of crane and atmospheric loads. The decision was based on sufficient statistical material, took into account the specific features of the different loads, type of their distributions and frequency structure.

The stochastic model of the crane loads was built as a result of experimentally statistical researches. The vertical and horizontal loads were investigated for bridge cranes with rigid and flexible hanged cargo of different carrying capacity and regimes which were exploited in the conditions of metallurgical and machine-building shops from 10 to 30 years of service. The generalized numerical and frequency descriptions of the crane loads were grounded and the possibility of application for them of normal law was substantiated (Figure 1, d). The special attention in model experiments was spared lateral forces of bridge cranes.

The systematic information about the wind velocity measurements done with ten minutes average at 70 Ukrainian meteorological stations were used as an initial data. The mean value of wind load is of quasi-stationary origin with the constant frequent parameters and normalized distribution. For the description of ordinate density of wind mean value Veibull's law was used, for wind dynamic value the normal distribution was based. Having integrated some initial data all necessary mean parameters of probabilistic wind models were determined for territory of Ukraine.

The results of regular snow measurements for 15–40 years at 62 Ukrainian meteorological stations have been taken as the reference statistic material for the snow probabilistic model. It is substantiated that the ground snow load is of a quasi-stationary origin. Its mathematical expectation and standard have a seasonal trend. At the same time snow frequent characteristics and normalised ordinate distribution remain constant during the season. For its description it was first applied polinomo-exponential distributing (Figure 1, b). In future in PoltNTU there were executed the researches of deposits of snow on coverages with the overfalls of heights, the estimation of influence of roofs heating descriptions on the value of the snow load, the calculation of the snow load on the cold roofs of buildings with the positive internal temperatures of air.



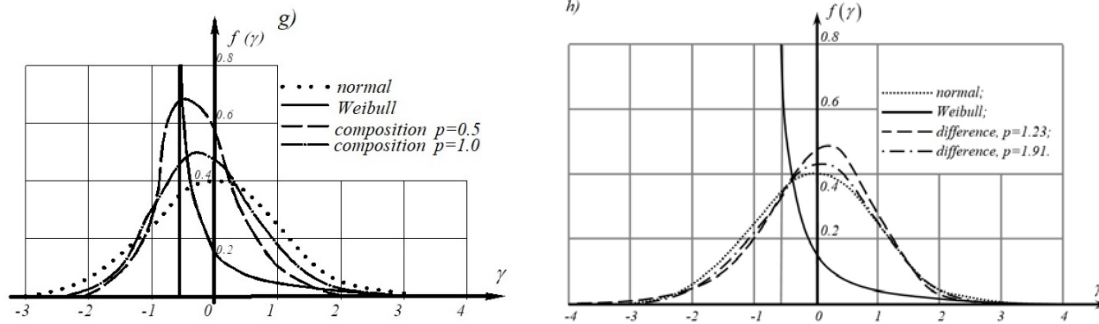


Figure 1. Distributions of random arguments:

a – durability of steel; b – snow load; c – wind load; d – crane load; e – composition of polinomo-exponential and normal distributions; f – difference of polinomo-exponential and normal distributions; g – composition of Veibull's and normal distributions; h – difference of Veibull's and normal distributions

The systematic analysis of random loads was realized for the six most commonly used probabilistic models. The main one is presented in the form of a stationary (crane load) or quasi-stationary (wind and snow loads) random processes (Figure 2, a, pos.1); their parameters are an effective frequency  $\omega$  and the coefficient of trend  $K_{tr}$  which accounts the atmospheric load season change.

The absolute maxima of random process are one of simpler model; they are determined by the tail part of the distribution of outliers and are higher then characteristical maximum level  $\gamma_0$ . The letter is a solution of the following equation  $N_+(\gamma_0; 0 \leq \tau \leq t) = 1$ , where  $N_+(\cdot)$  – the number of outliers of random process.

The model in the form of a random sequence of independent random loads with the intensity  $\lambda$  is widely spread ( $\lambda$  – the number of loads in per-unit time  $t_\lambda$ ). For discrete presentation of loads (Figure 2,a, pos. 2) the frequent parameter of which is the mean duration of overloading  $\bar{\Delta}$  connected with the intensity by the ratio  $\bar{\Delta} = t_\lambda / \lambda$ .

The analysis of the problem has demonstrated that the load values sampling can be classified as the exponential type. That's why their maximum values can be presented correctly by the extreme double exponential Gumbel distributions of a normalized type:

$$y = \alpha_n (\gamma - u_n), \quad (1)$$

where  $u_n$  – characteristic extremum;  $\alpha_n$  – extreme intensity;  $y = -\ln[-\ln F(t)]$  – Gumbel distribution argument.

The normalised load level was taken into account  $\gamma = (x - \bar{x})/\hat{x}$ , where  $x$  – the load ordinate,  $\bar{x}$  – mathematical expectation,  $\hat{x}$  – standard deviation.

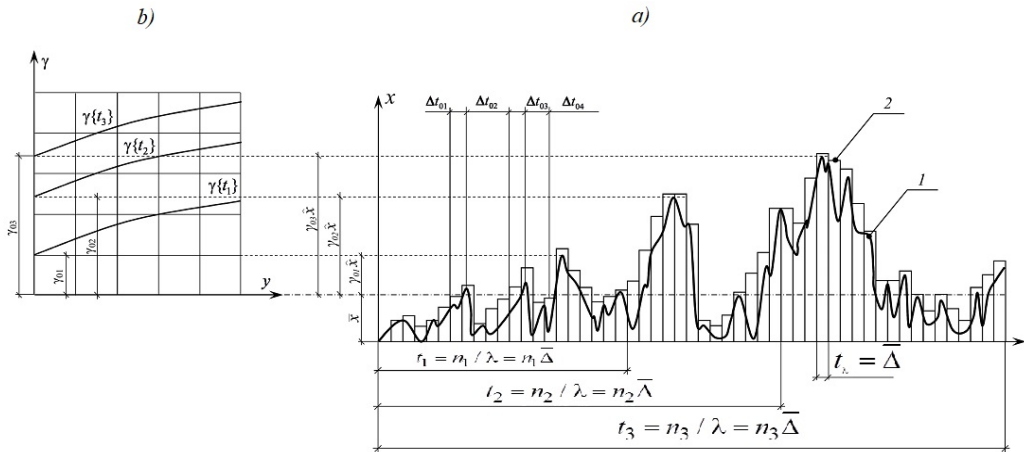
The choice of models of loads depends on a specific of solving probabilistic problems: the more complicated ones are solved in the manner of random processes which, however, are difficult for description and take much more computation time. More accessible and simple models, mentioned earlier, are based on the random values and corresponding frequent characteristics and provide not less exact solution if they are proper grounded. In particular, all the examined models are close to its sense of the following evaluation:

$$Q(t) = \frac{\omega f(\gamma) t}{\sqrt{2\pi}} = \frac{f(\gamma)}{f(\gamma_0)} = \lambda t [1 - F(\gamma)] = \frac{t[1 - F(\gamma)]}{\bar{\Delta}} = [r(1 - \rho) + \rho][1 - F(\gamma)], \quad (2)$$

where  $Q(t)$  – probability of exceeding  $\gamma$  – level during  $t$ ;  $F(t)$  and  $f(\gamma)$  – accordingly integral and differential functions of load distribution;  $r$  – amount of calculation cuts of casual sequence of loads;  $\rho$  – generalized coefficient of correlation of sequence.

Load probabilistic comparison can be well performed at the extreme scale which is illustrated in Figure 2, b. On the axis of ordinate of a scale the standardized load is laid off, on the axis of abscissa the

Gumbel's distribution argument  $y$  is laid of which is connected with the load return period  $T$ . Gumbel's distribution (1) is described on the scale in the form of straight lines. The models of a random process, a random sequence of independent loads and discrete presentation are introduced as different curves. The main advantage of this scale is its visual effect of the tail parts of the load distributions which have rather small distinctions in the usual form of presentation. It enables to present the visual comparison and correspondence of parameters of different load models.



**Figure 2. Presentation of random load:**  
**a – realization of random process: 1 – model of continuous process; 2 – discrete model;**  
**b – general form of load presentation**

A model of random processes, without regard to its relative complication, is for today most studied, the model of absolute maximums of random processes also enabled to get interesting scientific results. It should be noted that other perspective models, the especially random sequences of independent and correlated overloads, and also discrete presentation of loads, are less studied and recommended for the use in subsequent researches of loads.

The worked out random parameters give possibility to develop the reliability estimation of building structures.

### 3. Results and Discussion

#### 3.1. Reliability estimation of building structures in the technique of random values

The failure of an element takes place when a stochastic stress under the joint applied loads  $S(t)$  ( $t$ -time) exceeds the resistance of an element  $R(t)$ . The failure of the element is defined by the equation

$$\tilde{Y}(t) = \tilde{R}(t) - \tilde{S}(t) < 0, \tag{3}$$

where  $Y(t)$  – margin function of load carrying capacity.

The probabilistic technique of random values without the account of time factor is applied in this section. The technique is grounded at the action of loads with little change in time (dead and some technological loads) or those which have non-permanent character. Then the margin function will be written down as

$$\tilde{Y} = \tilde{R} - \tilde{S} < 0. \tag{4}$$

Mathematical expectation and standard mean deviation of margin function are determined as for a linear function:

$$\bar{Y} = \bar{R} - \bar{S}; \hat{Y} = \sqrt{\hat{R}^2 + \hat{S}^2}. \tag{5}$$

The safety characteristic is of great importance and is derived from equation

$$\beta = \bar{Y} / \hat{Y} = (\bar{R} - \bar{S}) / (\hat{R}^2 + \hat{S}^2)^{1/2}, \tag{6}$$

where  $\bar{Y}, \bar{R}, \bar{S}$  – the corresponding mathematical expectations,  $\hat{Y}, \hat{R}, \hat{S}$  – the corresponding standard mean deviations.

This characteristic determines probability of structural failure

$$Q(Y \leq 0) = F_Y(0) = F_Y(\bar{Y} - \beta \hat{Y}), \quad (7)$$

where  $F_Y(\cdot)$  – an integral function of margin function distributing.

In the case of normal distribution  $f(Y)$  the safety characteristic is very convenient:

$$Q(Y < 0) = 0.5 - \Phi(\beta), \quad P(Y \geq 0) = 0.5 + \Phi(\beta). \quad (8)$$

Here  $\Phi(\beta)$  – the well-known function of Laplace contained in widespread statistical tables.

The element resistance and margin function of the load carrying capacity (3) are generalized by the function of a few arguments:

$$Y = f(X_1, X_2, \dots, X_n).$$

In general case this function can be nonlinear, and for simplification of calculations, its linearizing is often executed with replacement of initial expression the linear function:

$$Y = f(\bar{X}_1, \bar{X}_2, \dots, \bar{X}_n) + \sum_{i=1}^n \frac{\partial Y}{\partial X_i}(\bar{X}_1, \bar{X}_2, \dots, \bar{X}_n)(X_i - \bar{X}_i) = \bar{Y} + \sum_{i=1}^n D_i(X_i - \bar{X}_i). \quad (9)$$

In the calculations of reliability the linearizing is executed near the mathematical expectation of function, here the angular coefficient of entered line is determined by the partial differentiation of initial function on the proper argument. Error in determination of  $\bar{Y}$  as a result of linearizing for (8) it is possible to estimate the following expression:

$$\Delta \bar{Y} = \frac{1}{2} \sum_{i=1}^n \frac{\partial^2 Y}{\partial X_i^2}(\bar{X}_1, \bar{X}_2, \dots, \bar{X}_n) \hat{X}_i.$$

Here  $\hat{X}_i$  – dispersion of  $i$ - argument.

At insignificance of this error a formula for determination of mathematical expectation (9) is simplified and taken to the substitution of mathematical expectations of arguments in an initial function:

$$\bar{Y}(X_1, X_2, \dots, X_n) = f(\bar{X}_1, \bar{X}_2, \dots, \bar{X}_n). \quad (10)$$

Standard deviation of the margin function is determined, in the case of independence of arguments, from equation:

$$\hat{Y} = \sqrt{\hat{Y}} = \sqrt{\sum_{i=1}^n \left[ \frac{\partial Y}{\partial X_i}(\bar{X}_1, \bar{X}_2, \dots, \bar{X}_n) \right]^2 \hat{X}_i^2} = \sqrt{\sum_{i=1}^n D_i^2 \hat{X}_i^2}. \quad (11)$$

Error of this expression as a result of linearizing is estimated as follows

$$\Delta \hat{Y} = \Delta(\hat{Y}^2) = \frac{1}{2} \sum_{i=1}^n \left[ \frac{\partial^2 Y}{\partial X_i^2}(\bar{X}_1, \bar{X}_2, \dots, \bar{X}_n) \right]^2 \hat{X}_i^2 + \sum_{i>j} \left[ \frac{\partial^2 Y}{\partial X_i \partial X_j}(\bar{X}_1, \bar{X}_2, \dots, \bar{X}_n) \right]^2 \hat{X}_i \hat{X}_j. \quad (12)$$

### 3.2. Practical reliability estimation of structure elements

In quality an example we will consider the reliability calculation of reinforced concrete beams with carbon-plastic external strengthening. For the receipt of reliability estimations we will use the worked out reception with the substitution of probabilistic parameters in the deterministic decisions of durability of reinforced concrete beams. Taken into account thus, that most random arguments of the margin function of load carrying capacity of reinforced concrete beams can be reasonably described by a normal law, in particular, durability of concrete, armature, carbon-plastic, and also row of loads (dead, technological, crane etc.).

Random value of maximum bend moment which is perceived by a beam with a double armature:

$$\tilde{M}_{ult} = f(\tilde{\sigma}_b, \tilde{\sigma}_s, \tilde{\sigma}_{sc}) = \tilde{\sigma}_s A_s (h_0 - 0.5\tilde{x}) + \tilde{\sigma}_{sc} A'_s (0.5\tilde{x} - a'), \quad (13)$$

where:  $\tilde{\sigma}_b$  – random value of concrete resistance to the compression for the limit states of the first group;  $\tilde{\sigma}_s$  – random value of the armature durability to tension;  $A_s$  – section area of the stretched armature;  $\tilde{\sigma}_{sc}$  – random value of concrete resistance to tension;  $A'_s$  – section area of the compressed armature;  $a'$  – distance from resultant effort in the compressed armature to the compressed verge of element;  $h_0$  – calculation height of section;  $x$  – height of the compressed zone of concrete, equal

$$\tilde{x} = (\tilde{\sigma}_s A_s - \tilde{\sigma}_{sc} A'_s) / \tilde{\sigma}_b b,$$

where:  $b$  – width of section.

We put expression for  $x$  in a formula (13):

$$\tilde{M}_{ult} = \tilde{\sigma}_s A_s h_0 - \tilde{\sigma}_{sc} A'_s a' - \frac{0.5}{\tilde{\sigma}_b b} (\tilde{\sigma}_s A_s - \tilde{\sigma}_{sc} A'_s)^2. \quad (14)$$

We have the mathematical expectation of limit moment putting the expected values of casual arguments in this common expression.

We will define coefficients for the calculation of standard of limit moment:

$$D_{sc} = \frac{\partial M_{ult}}{\partial \sigma_{sc}} = \frac{A'_s}{\sigma_b b} [-\sigma_b b a' + (\sigma_s A_s - \sigma_{sc} A'_s)]; \quad (15)$$

$$D_s = \frac{\partial M_{ult}}{\partial \sigma_s} = \frac{A_s}{\sigma_b b} [\sigma_b h_0 b - (\sigma_s A_s - \sigma_{sc} A'_s)]; \quad (16)$$

$$D_b = \frac{\partial M_{ult}}{\partial \sigma_b} = \frac{0.5}{\sigma_b^2 b} (\sigma_s A_s - \sigma_{sc} A'_s)^2. \quad (17)$$

The numerical values of coefficients we get, putting the expected values of random arguments in these expressions.

The standard of limit bend moment is determined as

$$\hat{M}_{ult} = \sqrt{(D_b \hat{\sigma}_b)^2 + (D_s \hat{\sigma}_s)^2 + (D_{sc} \hat{\sigma}_{sc})^2}. \quad (18)$$

For the reliability estimation of beams we determine a safety characteristic, having in this case next kind:

$$\beta = (\bar{M}_{ult} - M_{cal}) / \hat{M}_{ult}, \quad (19)$$

where:  $M_{cal}$  – calculation value of external bend moment in a beam.

Farther we will consider a beam strengthened by carbon-plastic (FAP). Random value of the limit bend moment, perceived by an increased beam:

$$\tilde{M}_{ult} = f(\tilde{\sigma}_b, \tilde{\sigma}_s, \tilde{\sigma}_{sc}, \tilde{\sigma}_{fu}) = \tilde{\sigma}_{fu} A_f (h - 0.5\tilde{x}) + \tilde{\sigma}_s A_s (h_0 - 0.5\tilde{x}) + \tilde{\sigma}_{sc} A'_s (0.5\tilde{x} - a'), \quad (20)$$

where:  $\tilde{\sigma}_{fu}$  – random value of durability on tension of FAP;  $A_f$  – area of section of carbon-plastic armature;  $h$  – height of section;  $x$  – height of the compressed zone of concrete, equal

$$\tilde{x} = (\tilde{\sigma}_{fu} A_f + \tilde{\sigma}_s A_s - \tilde{\sigma}_{sc} A'_s) / \tilde{\sigma}_b b.$$

We put expression for  $x$  in a formula (20):

$$\tilde{M}_{ult} = \tilde{\sigma}_{fu} A_f h + \tilde{\sigma}_s A_s h_0 - \tilde{\sigma}_{sc} A'_s a' - \frac{0.5}{\tilde{\sigma}_b b} (\tilde{\sigma}_{fu} A_f + \tilde{\sigma}_s A_s - \tilde{\sigma}_{sc} A'_s)^2. \quad (21)$$

Mathematical expectation of limit moment we get, putting the expected values of random arguments in this expression.

We will define coefficients for the calculation of standard of limit moment:

$$D_{fu} = \frac{\partial M_{ult}}{\partial \sigma_{fu}} = \frac{A_f}{\sigma_b b} \left[ \sigma_b h b - (\sigma_{fu} A_f + \sigma_s A_s - \tilde{\sigma}_{sc} A'_{sc}) \right]; \quad (22)$$

$$D_{sc} = \frac{\partial M_{ult}}{\partial \sigma_{sc}} = \frac{A'_s}{\sigma_b b} \left[ -\sigma_b b a' + (\sigma_{fu} A_f + \sigma_s A_s - \sigma_{sc} A'_s) \right]; \quad (23)$$

$$D_s = \frac{\partial M_{ult}}{\partial \sigma_s} = \frac{A_s}{\sigma_b b} \left[ \sigma_b h_0 b - (\sigma_{fu} A_f + \sigma_s A_s - \sigma_{sc} A'_s) \right]; \quad (24)$$

$$D_b = \frac{\partial M_{ult}}{\partial \sigma_b} = \frac{0.5}{\sigma_b^2 b} \left[ (\sigma_{fu} A_f + \sigma_s A_s - \sigma_{sc} A'_s) \right]^2. \quad (25)$$

The numerical values of coefficients we get, putting the expected values of random arguments in these expressions.

The standard of limit bend moment is determined as

$$\hat{M}_{ult} = \sqrt{(D_{fu} \hat{\sigma}_{fu})^2 + (D_b \hat{\sigma}_b)^2 + (D_s \hat{\sigma}_s)^2 + (D_{sc} \hat{\sigma}_{sc})^2}. \quad (26)$$

For the reliability estimation of beams, we determine a safety characteristic utilizing formulae (8), (19) and the function of Laplace.

Application of the got results is illustrated on a numerical example with a reinforce-concrete beam. A beam has section sizes  $b = 300$  mm,  $h = 700$  mm, external calculation bend moment is  $M_{cal} = 500$  kNm. After the above-mentioned formulae, numerical descriptions of maximum moment of beam are  $\bar{M}_{ult} = 762.7$  kNm,  $\hat{M}_{ult} = 29.92$  kNm. The safety characteristic in relation to the calculation value of external bend moment in a beam makes  $\beta = (762.7 - 500) / 29.92 = 8.78$ , that answers the low probability of beam failure  $Q(\beta) = 4.0 \cdot 10^{-19}$ . Farther on the terms of example, a beam is loaded with the increase of calculation moment to  $M'_{cal} = 750$  kNm. The safety characteristic here is diminished substantially  $\beta = (762.7 - 750) / 29.92 = 0.42$ , the probability of failure  $Q(\beta) = 0.337$  becomes too high, that testifies to the accident rate of beam and necessity of its strengthening.

The strengthening of the beam is one layer of carbon-plastic fibre with width 250 mm, thickness 1.4 mm, area section of strengthening  $A_f = 350$  mm<sup>2</sup> = 3.5 cm<sup>2</sup>. Calculated durability of carbon-plastic fibre is  $R_{fu} = 1071$  MPa. Numerical descriptions of maximum moment of beam are increased –  $\bar{M}_{ult} = 991.0$  kNm,  $\hat{M}_{ult} = 35.5$  kNm. The safety characteristic in relation to the calculation value of heightened external bend moment in a beam makes  $\beta = (991.0 - 750) / 35 / 5 = 6 / 79$ , the probability of failure of the increased beam  $Q(\beta) = 1.3 \cdot 10^{-11}$  is substantially below than failure probability of beam without strengthening. The got estimation of failure probability testifies to sufficient reliability of beam and efficiency of its strengthening of carbon-plastic fibre.

The analysis of results of reliability calculation of the increased beam confirmed the validity of application for this purpose the described higher linearized probabilistic model, as an insignificant error is here assumed: - 0.12 % for the mathematical expectation of maximum moment and 0.43 % for its standard.

For elements which perceive a multicomponent dead load a stochastic model as a sum of random values allows to enter the reducing coefficient of combination  $\psi = 0.90 \dots 0.95$ . It allowed also estimating the reliability of linear part of main pipeline and faultlessness of bearings and non-load-bearing structures from steel of the thin-walled types.

The compositions of sums and differences of loads and strength should be done for several loads and for design of margin of carrying capacity (4). Convolution formulae are used for this purpose.

$$Y = X_1 + X_2, \quad f(Y) = \int_0^Y f_1(X_1) f_2(Y - X_1) dX_1;$$



$$Y = X_1 - X_2, \quad f(Y) = \int_{-\infty}^{\infty} f_1(X_1)f_2(X_1 - Y)dX_1. \quad (27)$$

They cannot be determined in a closed form for the mentioned distributions and therefore numerical integration is adopted. The convolutions are transformed (presented in an unified comparable form) for the design simplification and their parameters are combined. From 13 received analytical expressions of convolutions, only the formula for difference of normal and arbitrary  $f_2(z)$  density distributions is suggested

$$f(\gamma) = \frac{D}{\sqrt{2\pi}} \int_{z_1}^{z_2} \exp(-0.5E^2) f_2(z) dz, \quad (28)$$

where:  $\gamma = (Y - \bar{Y})/\hat{Y}$  – standardised variable;  $E = D \cdot \gamma + z/p$ ;  $D = \sqrt{1 + p^2}$ ;  $p = \hat{x}_n / \hat{x}_a$ ;  $v_n = \hat{x}_n / \bar{x}_n$ ;  $\bar{x}_n$ ;  $\hat{x}_n$  – parameters of the normal distribution,  $\hat{x}_a$  – standard deviation of arbitrary distribution,  $z_1 = \gamma\sqrt{1 + p^2} - p/v_n$  – lower limit of integration,  $z_2$  – upper limit of integration which is determined with the account of necessary calculation accuracy.

The examples of joint distributions based on the proposed formulae are presented on the Figure 1, d–g. They can be non-symmetrical and differ considerably from the normal distribution in solutions of structural reliability problems. Without the account of this factor the estimation of structure reliability can have noticeable inaccuracy.

### 3.3. Reliability estimation of structure elements with the account of time factor

The loads are presented in the form of stationary and quasistationary random processes. Hence the  $\tilde{Y}(t)$ -function is a random process as well. Therefore the element failure is a projection of random process of the load carrying capacity margin (3) into the negative region. In this case the probability of failure  $Q(t)$  is estimated as follows

$$Q(t) = \omega_q f_Y(\beta) t / (\beta_\omega \sqrt{2\pi}), \quad (29)$$

where:  $\omega_q$  and  $f_Y(\beta)$  – the effective frequency and density of the distribution of the random process  $\tilde{Y}(t)$  ordinate;  $\beta_\omega$  – the coefficient of structural complexity of the random process  $\tilde{Y}(t)$  which takes into account the spectrum of frequencies of the real loads;  $t$  – the work time of element (different from his term of service).

The solving of the problem (29) demands the presentation of the frequency characteristics of summary random processes of loads and  $\tilde{Y}(t)$ . Frequency description of load connection, presented in the form of quasistationary random processes is determined as

$$\omega_{12} = \frac{1}{\sqrt{1 + K^2}} \left[ (\omega_1 \cdot K_{tr1})^2 + (\omega_2 \cdot K \cdot K_{tr2})^2 \right]^{1/2}, \quad (30)$$

where:  $\omega_1, \omega_2, \omega_{12}$  – effective frequencies of separate random processes and their sum;  $K = \hat{x}_2 / \hat{x}_1$  – a relation of standards of loads which are composed;  $K_{tr1}, K_{tr2}$  – trend coefficients that take into account the slow seasonal changes of the atmospheric loads.

The frequency analysis of the composition of the random loads are constantly investigated by scientists [34, 35].

It is necessary to underline that the investigated random loads are substantially differed on frequency, it is evidently shown their realization (Figure 3, a,b) and also substantially different spectral densities (Figure 3, d). Effective frequencies of random processes of various loads are differed considerably: for crane loads  $\omega_c = 1700 - 5160$  1/day, for wind ones  $\omega_w = 5.4 - 6.6$  1/day, for snow

loads  $\omega_s = 0.073 - 0/141$  1/day. Therefore the joint action of several loads becomes multifrequent (Figure 3, c).

This peculiarity can be defined in the formula (29) by the coefficient of structural complexity of the random process  $\beta_\omega$  which equals the ratio of mean frequency on maximum  $\omega_m$  to the effective frequency on zero  $\omega_q$ :

$$\beta_\omega = \frac{\omega_m}{\omega_q} = \frac{[K^{IV}(0) \cdot K(0)]^{1/2}}{[-\ddot{K}(0)]} = \frac{\beta_i \cdot [(1+k^2 \cdot \Theta^4)(1+k^2)]^{1/2}}{(1+k^2 \cdot \Theta^2)}, \quad (31)$$

where:  $K(0)$ ,  $\ddot{K}(0)$ ,  $K^{IV}(0)$  – random process correlation function and its derivatives with zero argument;  $\beta_i$  – structure complexity coefficient of separate loads;  $k = \hat{x}_2 / \hat{x}_1$ ;  $\Theta = \omega_2 / \omega_1$  – standard deviations and effective frequency proportion of summed up random loads.

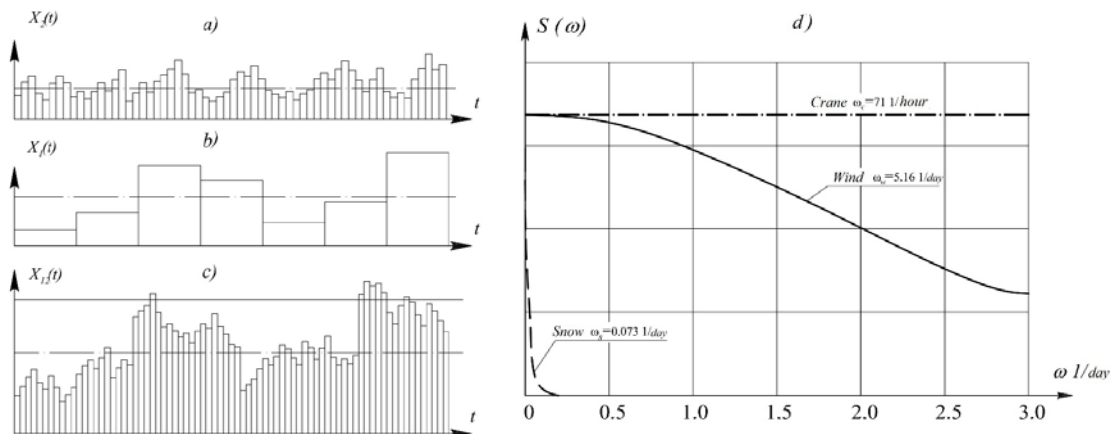
In accordance with equation (31) the frequency composition analysis can be realised if the load random process is differentiated sufficiently many times. For the atmospheric and crane loads the coefficient of structure complexity differs negligibly from  $\beta_i = 3$ . As a result of substantial difference of frequencies of loads, this coefficient of their composition can be considerable.

If the effective frequency of one of the loads prevails and  $\omega$  is large, then formula (31) is simplified as follows:

$$\beta_\omega = \beta_i(1+k^2)^{1/2} / k, \quad (32)$$

Crane beams as well as crane trestle structures can be treated like elements under only crane load. We will examine general stress of these elements without local effects and fatigue. In this case the probability of structure failure  $Q(t)$  depends on the ratio of a load stress of one bridge crane  $X_{M1}$  to the load stress of two bridge cranes  $X_{M2}$ , then on the ratio of a load carrying capacity of cranes to their mass and it also depends on crane work condition. The analysis showed that elements have a deficient reliability in the case of the one crane loads dominance. In the rest cases the elements reliability under crane loads is sufficient. In some cases the design crane load on structures can be decreased: if the approach of a loaded crane trolley to a crane track is limited or if the structure service term  $T$  is also limited.

The reliability of structures (beams, trusses) under snow load was estimated by a developed method. The calculation demonstrated the lack of reliability of these structures. That justifies the idea of understating of existing snow loads in the norms of SNiP, which operated on territory of Ukraine to 2006 year. Besides this fact validates the causes of steel truss failures. It applies to steel structures with lightweight roofs in the southern districts of Ukraine when there is much snow in winter. The increasing of snow design load for 1.5–2 times for our region can solve the reliability problem of steel structures under snow load. It was executed in national Code in Ukraine DBN V.1.2:2-2006 "Loads and influences". The reliability of steel roofs under snow load is constantly probed by the specialists of different countries [36].



**Figure 3. To the estimation of frequency of load composition**  
**a – high-frequency component; b – low-frequency component; c – total random process;**  
**d – comparison of the load spectral densities**

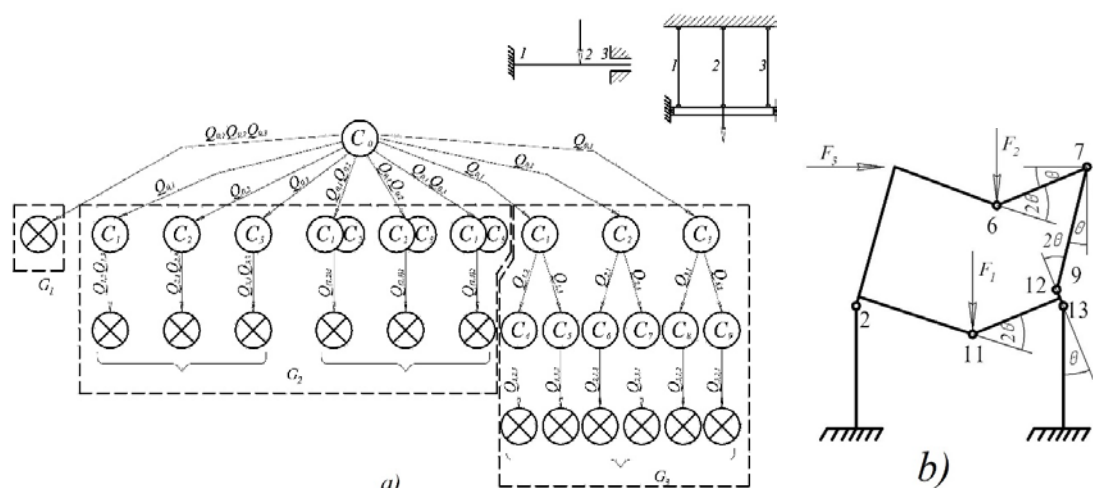
Structure elements under wind load designed in accordance with the existing code (glass elements, wind protection screens etc.) are of sufficient reliability. So if the service term of these elements will be limited it is possible to introduce the temporary coefficient  $\gamma_T \leq 1$  for the design wind load. It is recommended to introduce the increasing coefficients of combination  $\psi = 0.7 - 0.9$  into the structure design under snow, wind and crane loads.

The special attention was spared the evaluation of reliability of steel beam-column structures. Time factor, existing loads, random steel strength were taken into account during computation process of these elements. Existing steel columns of industrial buildings in a broad range of parameters were examined. The general conclusion is as follows: the reliability of steel columns of industrial buildings is quite sufficient. In quality tasks for subsequent researches it is possible to name the search of direct probabilistic decisions of tasks of compressed elements and subsequent development of probabilistic version of method of finite elements.

### 3.4. Reliability analysis of structure nodes and redundant systems

It was founded out that reliability of typical structure nodes could be compared with the schemes of successive connections of correlated elements. Thus, the node reliability depends upon the number of engaged independent load carrying elements.

Some beams and simple frames, as well as multi-storey and multi-span structures of industrial and residential buildings present the redundant structures. Redundant structure failures occur after some member failures in the form of transmission to different workable states. Thus, the redundant structure failure estimation is a very complicated problem as depends upon the system complexity. Illustration of it is a multibranch graph of the states of twice statically indefinite system (Figure 4, a). The complete estimation of the failure probability of elastic-plastic redundant systems with random durability and loading was got as a result of application of probabilistic method of limit equilibrium. A method allows, walking around the transient states of system, directly to examine the reliability of the last (true) mechanism of failure of the system (Figure 4, b). Corresponding formulae are derived; the algorithms and computing programs are developed. The estimation of a wide range of industrial redundant structures with different degree of redundancy was obtained on the base of this approach. It gave possibility to evaluate the safety level of redundant structures in comparison with separate members and statically determined structures. This level can be taken into account introducing the additional coefficient of work condition  $\gamma_c = 1.1 - 1.4$ .



**Figure 4. To the reliability estimation of redundant structures: a – graph of the states of twice statically indefinite system; b – true mechanism of frame failure**

Among questions, connected with the estimation of reliability of redundant systems, which remained unsolved, it follows to name formalization and translation on personal computer of method of the states and logic-probabilistic method, and also classification of redundant system elements, after the criterion of reliability. This direction becomes especially actual in connection with development of scenarios of failures, questions of vitality and progressive failure of buildings [37–39].

## 4. Conclusions

1. The reliability analysis of wide range of structures designed in accordance with existing Design Codes is examined in the article. Considered structures of industrial buildings are as follows: crane and roof beams, columns, redundant building systems.

2. This analysis exposes that the structures have quite different levels of reliability. It is shown that the light roof structures are not reliable enough being under the great influence of snow load. At the same time, the Design Code allows over-estimation of reliability for steel columns and redundant structures.

3. For the achievement of the put aim, the practical method of reliability analysis of structural elements is worked out. This method takes into account real load distributions and frequency characteristics.

4. The general probabilistic models of wind, snow and crane loads were designed and taken as a statistic basis of reliability estimation. These models are as follows: stationary and quasi-stationary random processes, its absolute maxima, random sequences of independent and correlated loads, discrete presentation and extreme model.

5. The simplest method of structure reliability estimation is realized in the technique of random values. Reliability estimation of reinforced concrete beams with carbon-plastic external strengthening is presented as an example.

6. Substantial results are got in the probabilistic techniques of random processes. It is important to notice that correlation functions and effective frequencies of random processes of various loads differ considerably. Therefore the joint action of several loads becomes multi-frequent. This peculiarity can be defined by the structure complexity coefficient  $\beta_\omega$  which equals the ratio of mean frequency on maximum to the effective frequency on zero.

7. There is confirmed that redundant structure failure estimation is a very complicated problem as depends upon the system complexity. The method of states, a probabilistic method of ultimate equilibrium and logic, as well as a probabilistic method is developed for solving this problem.

8. With regard to mentioned above, it is recommended to correct some load factors, a combination factor and a factor for model uncertainties of the Design Codes of steel structures and loads.

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