



DOI: 10.18720/MCE.85.5

The periodic temperature oscillations in a cylindrical profile with a large thickness

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Keywords: hollow cylinder; thermal conductivity equation; Hankel function; finite-difference scheme; temperature wave; cylindrical symmetry; damping coefficient; civil engineering; building; construction industry

Abstract. A hollow cylinder with thick walls is one of the most complex objects to calculate the unsteady temperature field, so this field is the least studied. However, such objects are found in many modern constructions of systems of generation and distribution of heat. In the proposed work it deals with the study of propagation of temperature waves in the wall of the hollow cylinder with harmonic temperature change of external environment arising from its diurnal fluctuations. The approximate analytical solution is presented by separation of variables in the complex domain with the use of cylindrical functions. The algorithm of calculation of temperature fields numerically is shown using an explicit finite-difference scheme of high accuracy in conditions of cylindrical symmetry with boundary conditions of the first kind. The results of calculations according to the considered algorithm, depending on the time since the start of heat exposure and their comparison with the analytic solution are given for its implementation. Calculated radial profiles of the temperature in the cylindrical wall within the temperature waves and the analytical approximation relations for the description of its damping coefficient are presented. The results are compared with the existing analytical solution in rectangular coordinates and it is marked that they have some differences but the common results are found regardless of the material and geometry of the cylinder, as well as of temperatures of inner and outer environment. Presented dependences are invited to apply for the analytical evaluation of the temperature amplitude on the inner surface of the heated cylindrical structures that will allow the use of engineering methods to verify compliance with industrial safety requirements.

1. Introduction

In this article, an unsteady temperature profile of a hollow thick wall cylinder under periodic thermal influence is studied.

The problem of unsteady thermal conductivity in bodies of various geometric forms is studied for a long time. Moreover, currently, due to achievements of computational equipment, numerical methods of solving the problem are more attractive. Nevertheless, in most cases, they are either related to a single dimension case or consideration is done in rectangular coordinates. This rather corresponds to the most really existing problems in both unsteady and steady modes [1–2]. Under the condition of cylindrical symmetry, heating and cooling of solid hollow cylinders are the most developed issues [1].

Over the last years a number of researches, where such issues are studied in both analytic and numerical ways, is emerged. However, the results obtained by the authors are either very difficult for use in engineering practice [3, 4, 9, 14, 17] or too rough, contrary [11]. The other solution variants are related to specific types of constructions used in limited areas and functioning in super critical modes [8], in nuclear power industry [13] or composite material production [5–7, 10], as well as in case of phase transitions [12], [15, 16, 18] or fuel combustion [19], or for underground pipes [20].

Samarin, O.D. The periodic temperature oscillations in a cylindrical profile with a large thickness. Magazine of Civil Engineering. 2019. 85(1). Pp. 51–58. DOI: 10.18720/MCE.85.5.

Самарин О.Д. Периодические температурные колебания в цилиндрическом слое при большой толщине стенки // Инженерно-строительный журнал. 2019. № 1(85). С. 51–58. DOI: 10.18720/MCE.85.5



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In the same time, the calculation of an unsteady temperature profile of thick walled hollow cylinders, including a periodic mode, is interesting. For instance, this can be useful for solving the problem of condensate formation either on the internal surface of fume stacks or on the external surface of thermal pipe insulation with daily fluctuation of ambient air temperature and heat carrier temperature. This is especially interesting for open-laid pipelines of heating systems with a significant moisture content of the outside air, which happens at a temperature close to zero and above, when it may be necessary to assess the conditions of freezing and thawing of atmospheric moisture and snow. In addition, such a calculation is needed to confirm the absence of condensation on the inner surface of the air ducts of exhaust ventilation systems with their outer gasket, which in some cases is allowed in case of structural necessity with the condition of their thermal insulation. The same applies to the outer surface of the air intake ducts, when they are laid as an exception inside the building, also in a heat-insulated form. Finally, the considered calculation may be required to assess the temperature fluctuations on the outer surface of cylindrical furnaces with changes in their operation mode in order to solve the question of the admissibility of this temperature for sanitary and hygienic requirements. The author of the published paper [21] has obtained a simple solution for thick walled cylinder cooling. However, the solution is related only to aperiodic mode conditions, whereas an analytic solution, but for a small diameter linear source is given in the paper [22]. Therefore, the relevance of the proposed research is in the necessity to search the precise and physically well-reasoned (in the same time acceptable for engineering usage) dependences of temperature alteration in periodically heated/ cooled cylindrical structures. Obtained results may be acceptable for a large number of energetic facilities of such structure.

The calculation of a temperature profile at harmonic fluctuations of ambient temperature near the external surface of a cylinder is the research purpose. Research tasks are as follows:

- building up an analytical solution for an equation of thermal conductivity in the cylinder wall for a regular mode;
- development of an algorithm, which implements finite-difference approximation of the equation;
- obtaining analytic dependences for temperature across cylinder cross section and temperature wave amplitude by results of correlating the theoretical results with software generation data.

2. Methods

Figure 1 shows a diagram of studied cylinder and some conventional symbols used further. The main of them are r_0 and r_1 , i.e. the external and internal radii of a cylinder, m , respectively.

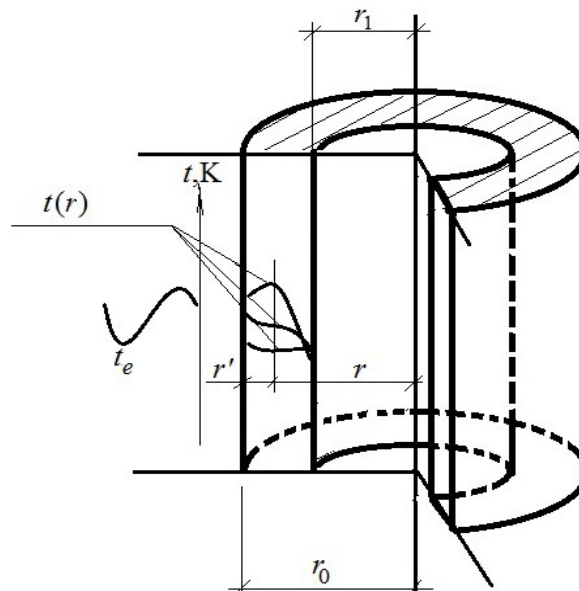


Figure 1. A diagram of a hollow cylinder and unsteady temperature profile in its wall.

Then a differential equation of unsteady heat conductivity for the cylindrical symmetry case may be written in the following way [1–2]:

$$\frac{\partial t}{\partial \tau} = a \left[\frac{\partial t}{r \partial r} + \frac{\partial^2 t}{\partial r^2} \right], \quad (1)$$

where $a = \frac{\lambda}{c\rho}$ – coefficient of cylinder body material heat conductivity, m²/s;

λ is its thermal conductivity, W/(m·K);

c and ρ are specific heat capacity, J/(kg·K), and density, kg/m³, respectively. The solution in this case will show a current temperature profile in the temperature wave distribution zone.

The boundary condition on the cylinder external surface for a periodic mode in the complex view of the harmonic oscillation simplest case may be written as follows:

$$t = A_{te} \exp(-i\omega\tau), \quad (2)$$

where A_{te} is oscillation amplitude of ambient air temperature t_e , K;

$\omega = 2\pi/T$ is wave circular frequency of t_e , s⁻¹. Here T is oscillation period, s. As the initial condition we may adopt $t(r) = 0$, considering an average period value $t_{e.av} = 0$, and, if it is not, we shall consider excessive temperature $\theta = t(r) - t_{e.av}$. Therefore, we adopt conditions of 1-st type, and in case of known heat exchange intensity on the surfaces, in first approximation it is possible to introduce additional layers with thickness equal to $\Delta r_{cond} = \lambda/\alpha$, where α – full heat exchange coefficient, W/(m²·K), at a corresponding side.

Then we may search for solution of equation (1) by the method of dividing the variables as the production of cofactors depending respectively only on τ and r :

$$t = \exp(-i\omega\tau)\varphi(r'), \quad (3)$$

where $r' = r_0 - r$ is radial coordinate, with respect to the fact that a temperature wave distributes from outsides (see Figure 1). After substituting in (1) we obtained a regular differential equation for φ :

$$r' \frac{\partial^2 \varphi}{\partial r'^2} + \frac{\partial \varphi}{\partial r'} + \frac{i\omega r'}{a} \varphi = 0. \quad (4)$$

Such equation is Bessel's equation in the complex area. The equation may be solved by Hankel cylindrical function of zero order first type $H_0^{(1)}$ [1] under applied boundary and initial conditions. The final shape of the function t may be found by substitution into (3):

$$t = A_{te} \exp(-i\omega\tau) H_0^{(1)} \left(r' \sqrt{\frac{i\omega}{a}} \right), \quad (5)$$

Through selecting a real component and using the known asymptotic approximation of the function $H_0^{(1)}$ at sufficiently large values of the argument, which may be used, since we are interested in the established oscillation process, we finally find:

$$\text{Re}(t) = A_{te} \cdot 2 \sqrt{\frac{1}{\pi r'} \sqrt{\frac{a}{\omega}}} \cos \left(\omega\tau - r' \sqrt{\frac{\omega}{2a}} + \frac{\pi}{n} \right) \exp \left(-r' \sqrt{\frac{\omega}{2a}} \right). \quad (6)$$

Therefore, unlike temperature waves in a flat wall, for a cylindrical wall it is more likely to have rapid decrease of amplitude as depth increases: not only due to the exponential factor, but also in a proportional way $1/\sqrt{r'}$. In this case, there is an additional shift of a phase in relation to oscillations t_e expressed by cosine argument summand equal to π/n . The theoretical value of the parameter n makes 1/8, however, taking into account the approximated character of the dependency (6), it is reasonable to clarify it through comparing with numeric calculation data based on the finite-differential approximation of the equation (1).

Since we need the obtained result precision, whereas used memory volume and quantity of made operations are not significant due to high computational resources of modern computers, a finite-differential diagram will be the most favorable due to its programming simplicity. Then a temperature value in i -n grid node at $j+1$ -n time moment may be calculated according to the following expression:

$$t_{i,j+1} = Fo_{\Delta} \left(\frac{2i-1}{2i-2} t_{i+1,j} + \left(\frac{1}{Fo_{\Delta}} - 2 \right) t_{i,j} + \frac{2i-3}{2i-2} t_{i-1,j} \right). \quad (7)$$

Here $t_{i,j}$, $t_{i-1,j}$ and $t_{i+1,j}$ are temperature values at j -n time moment in i -n node and two adjoining nodes on the right and left (node numbering from the cylinder axis to the external surface side); $FO_{\Delta} = \frac{a\Delta\tau}{(\Delta r)^2}$ is dimensionless local Fourier criterion, where $\Delta\tau$, s and Δr , m is respectively, time and coordinate steps representing parameters of finite-differential diagram. As it is known [1–2], the adopted diagram converges at $FO_{\Delta} \leq 1/2$. In this case, the value $FO_{\Delta} = 1/6$, providing high precision of 4-th order approximation by a space coordinate and 2-nd order approximation by a time coordinate, was chosen.

3. Results and discussion

Calculation results according to the formula (6) and author provided software in *Fortran* language according to the diagram (7) for $A_{te} = 1$ K, $r_0 = 1$ m, $a = 5.1 \cdot 10^{-7}$ m²/s and $\omega = 2\pi/86400 = 7.27 \cdot 10^{-5}$ s⁻¹, i.e. with daily oscillation period are given in Figure 2. On the internal radius $r_1 = 0.5$ m, temperature was kept constant.

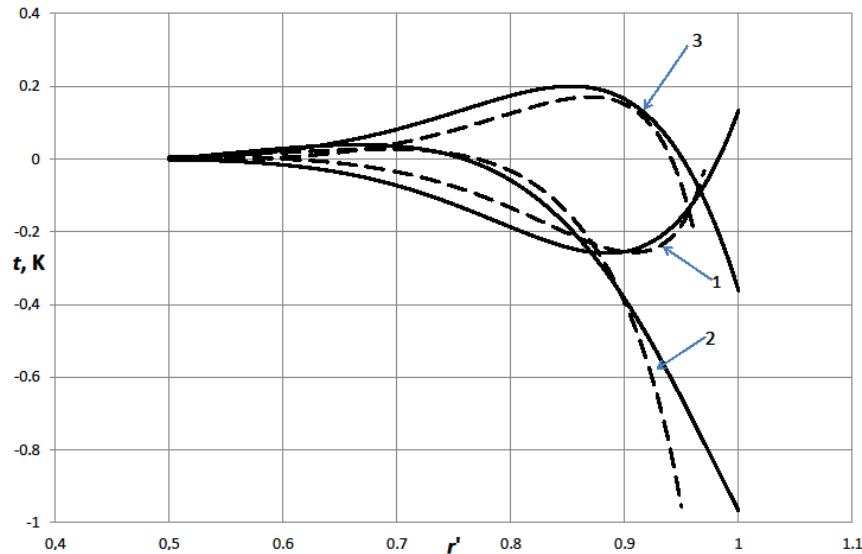


Figure 2. Temperature alteration along the cylinder radius (continuous lines – numeric calculation, dotted lines – according to the formula (6)); 1 – $\omega\tau = 23.7$; 2 – $\omega\tau = 47.4$; 3 – $\omega\tau = 71.1$.

In this case, for maximal matching it is necessary to adopt the value $n = 32$, i.e. real additional phase shift at cylindrical symmetry is very insignificant. It is clear, that in general, correctness of the relation (6) is confirmed, except $r' < 0.05$. It is true that the function approximation $H_0^{(1)}$ used in the conclusion (6) poorly works in this area, however for the considered problem it is meaningless, since in most cases a value of the relation $\Delta r_{cond}/r_0$, showing relative radius of the additional external cylindrical layer with considered external heat exchange is more than 0.05 or, at least, equal to 0.05.

Since in practical problems, temperature fluctuation amplitude is interesting, it is possible to introduce its attenuation coefficient $v_r = A_{te}/A_t$, which may be associated with thermal inertia of a cylindrical layer D_r in the same way as in the single dimension problem. From (6) the following expression may be obtained for it:

$$v_r = \frac{\sqrt{\pi D_r} \exp(D_r/\sqrt{2})}{2}, \text{ where } D_r = r' \sqrt{\frac{\omega}{a}}. \quad (8)$$

It is not hard to see that the obtained dependency differs from the single dimension case [2] only by the additional factor $\frac{\sqrt{\pi D_r}}{2}$. It is clear that at $D_r > 4/\pi \approx 1.26$ this factor is more than 1, therefore damping in a cylindrical layer is really more significant. In the considered example for the entire cylinder $D_r = (1 - 0.5) \sqrt{\frac{7.27 \cdot 10^{-5}}{5.1 \cdot 10^{-7}}} = 5.965$, then $v_r = 147$, or 2.16 times more than for a flat wall with the same heat inertia. For illustrative purpose, the ratio between damping coefficients for a flat and cylindrical layers shown in Figure 3.

Since the ratio (8) is written in a dimensionless form, it is common, and having no dependency on actual air temperature values, cylinder radius, cylinder wall material, and the ratio r_1/r_0 , at least at $r_1/r_0 > 0.5$, for which numeric calculations are done.

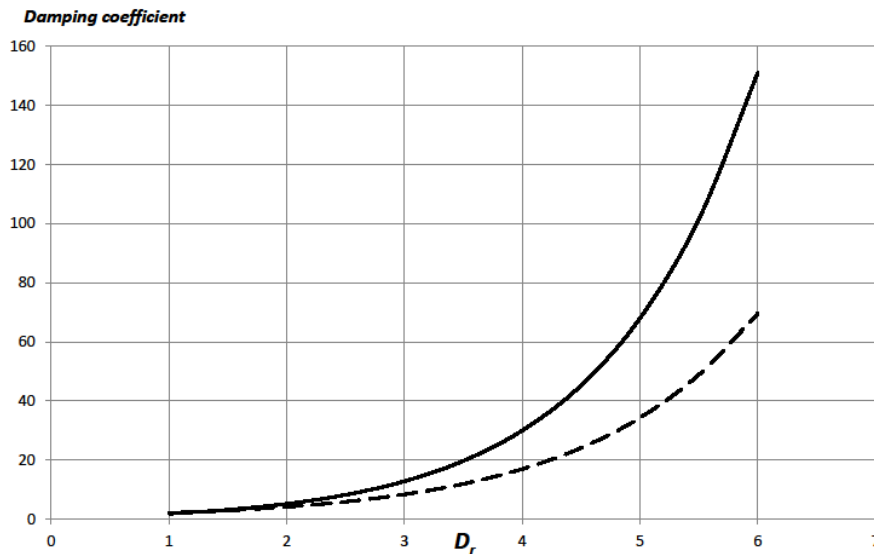


Figure 3. The dependence of the heat wave damping coefficient on heat inertia for a flat wall (dotted line) and cylindrical profile.

The results are principally matched with the data given in [2] for a flat wall with precision down to a factor $\frac{\sqrt{\pi D_r}}{2}$. Besides, a general form of calculated temperature profiles corresponds to results of some other authors, for instance, [15], whereas their analytical description discovers similarities in theoretical solutions of other sources, in particular, [13], [14].

To illustrate the practical use of the ratio (8), we calculate the temperature fluctuations in the insulation layer of an open air duct with the parameters $r_1 = 0.25$ m, $r_0 = 0.35$ m, i.e. with the insulation thickness of 100 mm, with the amplitude of the outdoor air temperature fluctuations $A_{te} = 2.7$ °C for the average conditions of the heating season. In this case the condition $r_1/r_0 > 0.5$ is satisfied. For thermal insulation of mineral wool $\lambda = 0.044$ W/(m·K), $c = 840$ J/(kg·K), $\rho = 50$ kg/m³, so $a = 1.05 \cdot 10^{-6}$ m²/s. Since we are primarily interested in the temperature on the inner surface of the air duct $t(r_1)$ from the point of view of assessing the possibility of condensation of water vapor, then $r' = r_0 - r_1 = 0.1$ m; and when the daily period of oscillations according to the formula (8) $D_r = 0.83$, where $v_r = 1.45$, and therefore the oscillation amplitude of the fluctuations of $t(r_1)$ is equal to $A_{te}/v_r = 2.7/1.45 = 1.86$ °C. This means that when checking the adequacy of the adopted thickness of the insulation will need to take into account that the minimum during the day the value of $t(r_1)$ will be below the average of 1.86 degrees.

4. Conclusion

– It is noted that temperature wave distribution is controlled by the same laws as in the single dimension case, but with a slightly different attenuation coefficient and phase shift.

– It is proved that unlike temperature waves in a flat wall, for a cylindrical wall it is more likely to have rapid decrease of amplitude as depth increases: not only due to the exponential factor, but also in reverse proportion to square root of a radial coordinate.

– It is shown that a real additional phase shift of temperature fluctuation at cylindrical symmetry in comparison with the single dimension case is very insignificant and equals to 1/32 of a period.

– It is shown that the ratio of the internal and external radii of a hollow cylinder does not explicitly influences on the temperature profile character in the temperature wave zone, at least, at $r_0/R > 0.5$.

– It is shown that temperature field distribution in the zone of temperature wave penetration into the cylinder, according to results of analytic and numeric solutions, coincides within engineering calculation limits, which means that the obtained dependences are correct.

–It is proposed to apply the ratios obtained in the research for analytic evaluation of temperature fluctuation amplitude on cylindrical surfaces of heated and cooled structures. It is necessary, first of all, to solve the problem of condensate formation on the internal surface of fume stacks in case of boiler unit load

fluctuation or on the external surface of heat line insulation as well as on the surfaces of the open-laid heat-insulated air ducts in case of daily ambient temperature alteration and on the outer surface of cylindrical furnaces in variable operating modes. It will allow to use not only software methods, but also engineering methods to check meeting the industrial safety requirements.

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DOI: 10.18720/MCE.85.5

Периодические температурные колебания в цилиндрическом слое при большой толщине стенки

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Ключевые слова: полый цилиндр; уравнение теплопроводности; функция Ханкеля; конечно-разностная схема; цилиндрическая симметрия; коэффициент затухания; гражданское строительство; здание; строительная индустрия

Аннотация. Полый цилиндр с толстыми стенками является одним из наиболее сложных объектов для расчета нестационарного температурного поля, поэтому такое поле является наименее изученным. Вместе с тем подобные объекты встречаются во многих современных конструкциях систем генерации и распределения теплоты. В предлагаемой работе рассматривается исследование распространения температурной волны в стенке полого цилиндра при гармоническом изменении температуры наружной среды, возникающем при ее суточных колебаниях. Представлено приближенное аналитическое решение задачи методом разделения переменных в комплексной области с использованием цилиндрических функций. Показан алгоритм расчета температурного поля численным методом с помощью явной конечно-разностной схемы повышенной точности в условиях цилиндрической симметрии при граничных условиях первого рода. Приведены результаты вычислений по рассмотренному алгоритму в зависимости от времени с момента начала теплового воздействия и их сопоставление с аналитическим решением для осуществления его идентификации. Представлены рассчитанные радиальные профили температуры в стенке цилиндра в пределах температурной волны и предложены аналитические зависимости для коэффициента ее затухания. Полученные результаты сопоставлены с имеющимся аналитическим решением в прямоугольных координатах и отмечены их различия, а также общность найденных результатов независимо от материала и геометрии цилиндра, а также температур внутренней и наружной среды. Представленные зависимости предложено применять для аналитической оценки амплитуды колебаний температуры на внутренней поверхности цилиндрических нагреваемых конструкций, что позволит использовать инженерные методы проверки выполнения требований промышленной безопасности.

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