Shear lag phenomenon in the tubular systems with outriggers and belt trusses

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Keywords: framed tube system; tube-in-tube system; trussed tube system; bundled tube system; shear lag.

Abstract. Development of technology facilitates construction of tall buildings. One of the common kinds of them is Tubular systems, divided into different types: framed tube, tube-in-tube, trussed tube and bundled tube systems. The main problem of tubular systems is the shear lag phenomenon that decreases the bending rigidity and moment resistance of the structures. In this paper, the phenomenon of shear lag in all kinds of steel tube systems is investigated analytically. In order to reach this objective, sixteen steel multi-storey tubular structures with the same plan, but a with a different number of stories and different tubular systems were designed by ETABS software based on AISC. Then the shear lags of each structure in different elevations are calculated by using the linear response spectrum analysis. The results show that nearly in the upper half of the structures the negative shear lag happens. Besides all, the formula was derived for each system with regard to the analyses data with linear regression examine by SPSS software, which showed that there is a significant relation between shear lag and three independent variables: story number, height ratio and distance from the web of the structures.

1. Introduction

For centuries, mankind have been mesmerized by the notion of building tall structures, which developments in the domain of civil engineering made this dream feasible. By passing the time, different types of lateral resistant systems for high-rise buildings were introduced, as in: tubular systems (tube-in-tube, braced tube and others), braced tall buildings, moment resisting systems, suspended tall buildings with a concrete core and so on. The lateral-resisting system is an important part of the structural system that supports the building against the lateral loads including: wind and earthquake loads. The majority of the lateral-resisting systems can be categorized into three types: 1) shear wall systems, 2) frame systems, and 3) the combination of the previous two systems. Shear wall systems are rather common choices in many earthquake-prone countries. They provide high strength and stiffness simultaneously in the direction of their placement. These structural elements are vertical ones which endure lateral loads in their plane. Core wall systems (shear walls) can reduce the lateral displacement by the core bending resistance and their displacement mode is flexural. However, these systems have a conspicuous problem which is putting the lateral resisting elements close to the neutral axes. This reduces their efficiency because they do not absorb a considerable amount of tensions and their bending rigidity index is low. Then, Fazlur Khan suggested the resisting core had to be put on the perimeter of a structure in which the normal stresses are greater, instead of being close to neutral axes [1]. Besides, their bending rigidity index increases by putting resisting elements far from the central axes. This new system was called a tubular (tube) system. The layout of this system can be rectangular, triangular or square. This system can be recognized as an evolved form of the flexural frames. By passing the time, this system changed gradually into the bundled tube, braced tube and tube-in-tube ones to get rid of the problems of the first generation system of tubular systems. In simple terms, a tube system can be defined as a three-dimensional system that utilizes the entire building perimeter to resist lateral loads [1].

One of the most important deficiencies of the tubular systems is the shear lag phenomenon. The influence of shear lag is to increase axial stresses in the corner columns and reduce the same ones in the inner columns of both the flange and the web panels [2], as shown in Figure 1. This decreases the moment resistance and
bending rigidity of structures. Furthermore, the designers intend to design all the exterior columns as typically as the same as the critical one and this leads to losing materials and money. There are many strategies to overcome this problem: using bundled tube systems, mega bracings, deep spandrel beams, and mega columns at the corner of the structures. However, the most efficient is using the spandrel beams (shear rigidity index), because they distribute forces between columns more uniformly. Then the more shear rigidity index, the less shear lag is observed. It is worth mentioning that the belt trusses (along with the outriggers) can be used to prevent the rotation of the internal tube, decreasing its drift and moment, diminishing shear lag and so on, they also connect exterior columns to each other. The outriggers connect them to the columns of internal tube. These two can distribute forces among columns more uniformly.

Mazinani et al. investigated the shear lag effect of on the braced tube and framed tube systems under the wind load and the efficiency of each structure was evaluated using the linear response spectrum analysis to obtain the shear lag [3]. They show there is a relatively less shear lag in all the braced tube configurations compared to the framed tube structural system and the shear lag is not proportional to the lateral displacement. With respect to the results, optimum braced tube configuration in term of lower shear lag caused by lateral loads is presented. A simple formula for shear lag is suggested based on the cantilever method modification by Kazemiia and Khoshnudian [4]. They also realized that although by increasing the number of stories shear lag factor decreases, this index gets constant (independent of the storey number) in tall buildings more than 30 stories. Naderpour and Kheyroddin investigated the effect of increasing the stiffness of the columns and spandrel beams on decreasing the shear lag index in the concrete tubular systems, they concluded that increasing the dimension of the columns is not as effective as increasing the stiffness of spandrels [5]. Kheyroddin and Zahiri-Hashemi investigated the influence of geometric configurations of the multi-story bracing on the shear lag behavior of the braced tube structures and they finally proposed empirical equations to provide the optimum number of the stories that should be braced, in order to exert minimum shear lag on the structures [6]. Gaur and Goliya researched the mitigating shear lag in tall buildings [7]. Thanh Dat et al. investigated on the shear lag effect on the design of high-rise buildings [8]. Also, Sreevalli and Priya studied the effect of the shear wall area factor decreases, this index gets constant (independent of the storey number) in tall buildings more than 30 stories.

With regard to the results, optimum braced tube configuration in term of lower shear lag caused by lateral loads is presented. A simple formula for shear lag is suggested based on the cantilever method modification by Kazemiia and Khoshnudian [4]. They also realized that although by increasing the number of stories shear lag factor decreases, this index gets constant (independent of the storey number) in tall buildings more than 30 stories. Naderpour and Kheyroddin investigated the effect of increasing the stiffness of the columns and spandrel beams on decreasing the shear lag index in the concrete tubular systems, they concluded that increasing the dimension of the columns is not as effective as increasing the stiffness of spandrels [5]. Kheyroddin and Zahiri-Hashemi investigated the influence of geometric configurations of the multi-story bracing on the shear lag behavior of the braced tube structures and they finally proposed empirical equations to provide the optimum number of the stories that should be braced, in order to exert minimum shear lag on the structures [6]. Gaur and Goliya researched the mitigating shear lag in tall buildings [7]. Thanh Dat et al. investigated on the shear lag effect on the design of high-rise buildings [8]. Also, Sreevalli and Priya studied the effect of the shear wall area on the seismic behavior of the multi-storied building tube-in-tube structures [9]. Patels compared different types of the tubular systems in terms of different aspects such as: the time period, the lateral displacements, the base shear and the steel consumption [10]. Finally, Salehi and Khaloo studied the shear lag factor in the long structures with the pipeline system in tubes under the wind load [11].

1.1. Tube systems

The general behavior of tall buildings is similar to a cantilever column with the medium slenderness ratio. With regard to the high shear flexibility, their behavior is different from the ordinary structural columns which have flexural behavior. Then the general probabilistic buckling mode of the tall buildings is not only the flexural mode, but also shear mode or even a combination of them. Besides, not just these modes appear in the buckling modes, yet they also appear in torsion or transverse torsion modes.

One of the most challengeable issues in the domain of tall building designs is choosing the lateral resisting system. There are different types of lateral resisting systems such as: bracings, moment resisting systems, suspended tall buildings with concrete or steel braced core and so on. There are two criteria in choosing the lateral resisting tools: 1) bending rigidity index (BRI), 2) shear rigidity index (SRI). The BRI is achieved by calculation of the inertia moment of the columns about the geometric central axes, and then the farther the columns from the central axes, the more BRI is reached. The SRI asserts the amount of continuous performance between the columns, in other words, columns should be connected well with each other through the structure. An ideal SRI = 100 is shear walls without openings. Obviously, lateral resisting systems like tall buildings with central core seem inefficient in terms of having low bending rigidity indices. Besides, because of the distribution of stress through the structure, putting resistant elements near to the neutral axes which have less normal stresses leads to decreasing efficiency of them. Accordingly, for the first time, Fazlur Khan suggested the resisting core (or elements) had to be put in the perimeter of a structure in which the normal stresses and flexural rigidity indices are higher [1]. These systems called tube systems which have different kinds: framed tube system, tube-in-tub system, trussed tube systems and bundled tube systems, which will be discussed in the paragraphs below.

1.1.1. Framed tube systems

The notion behind the framed tube systems is to create a completely three-dimensional structural system that involves the entire building inertia to resist lateral loads. The main objective is to obtain higher efficiency for lateral load resistance in tall buildings. Then the suggested system for a framed tube one is closely spaced exterior columns and deep spandrel beams rigidly connected together. The closer the distribution to that of a rigid box cantilevered at the base, the more efficient the system is considered to be. The requirements necessary to create a wall-like tube structure is to place columns on the exterior approximately close to each other and to use deep spandrels joined to those columns [12]. With regard of the height and plan dimensions of the structure, the spacing of exterior columns is usually 3–4.6 m, as an example: a spacing as close as 1.0 m was used for the 110-story World Trade Center twin towers, New York. Figure 2 shows the plan and isometric views of the framed tube systems.
The efficiency of the system is directly related to the building height-to-width ratio, the plan dimensions, the spacing, and the size of the columns and spandrels. It is worth saying that generally the exterior columns have to bear up under the gravity and lateral loads and the contribution of the interior columns to lateral load resistance is negligible. Moreover, the interior columns should not be spaced closely to each other like the exterior ones. It is suggested that the floor system should be designed as a rigid diaphragm, distributes the loads to all elements in proportion to their stiffness. In the tube structures, usually, the strong bending axis of the columns are placed toward the face of the building to have the most efficient bending action. The frames parallel to the lateral load act as the web of the performed tube, while the frames normal to the loads act as flanges. In the case of being under bending out of the lateral forces, the framed tube acts as a cantilever beam in which the columns on the opposite sides of the neutral axis like the flanges are subjected to the normal (tensile and compressive) forces. In addition, the frames parallel to the direction of the lateral load like the webs are subjected to in-plane bending, leads to a shear rocking action associated with an independent rigid frame [13]. These systems were the first generation of the tube systems and accordingly had some disadvantages like the shear lag and closely spacing columns which limited architectural choices.

1.1.2. Tube-in-tube systems

In these systems, there are two resistant systems: two exterior and interior tubes which consist of closely spaced columns and deep spandrels that connect them. The interior tube which acts as a sort of core usually occupies 30 to 40 % of the whole area of the structural plan. This tube is the place where staircases and elevators are put in. Columns between these tubes are designed under the gravity loads, and only columns of tubes should be resistant to lateral loads. The roof should be rigid so that either of the tubes act in accordance with each other completely. The advantage of the systems is the architectural aspect because there are a few columns between the tubes and this gives vast space for many purposes [2]. The exterior tube acts like a frame and the interior tube acts like a shear wall, then they have the same interaction as the frames and shear walls. Namely, the interior tube supports the exterior tube against the lateral displacement at the bottom, and the exterior tube supports the interior one at the top. Accordingly, at the top of the structure, the interior tube has a negative effect and creates negative shear and moment which leads to a strengthening of the exterior tube so that it takes additional shear and moment. Then, the stiffness of the interior tube (the dimension of columns and spandrel beams) should be decreased to prevent this repercussion. The schematic plan of the tube-in-tube systems is demonstrated in Figure 3.

1.1.3. Trussed tube systems (TTS)

For super-tall buildings, the dense grid of beam and column members of a framed tube has a drastic effect on the façade architecture. Trussed tube systems have the least number of diagonals on each façade and making the diagonals intersect at the same point of the corner column (The number of spans and stories covered by diagonals should be at the same). The system is known as a tubular system because the fascia diagonals aren’t just creating a truss in the plane, but also interact with the trusses on the perpendicular faces to affect the tubular behavior. In order to optimize the trussed tube action, the vertical columns should be replaced with closely spaced diagonals in both directions. The diagonally braced tube is by far the most usual method of increasing the efficiency of the framed tube [6]. The diagonals connected to the columns at each intersection, virtually eliminate the effects of shear lag in both the flange and web of frames. As a result, the structure behaves more like a braced frame, with greatly diminished bending of the exterior columns and girders. Accordingly, the columns can be spaced at larger distances and the size of the columns and spandrels.
get smaller than in the framed tube systems, which lead to allowing larger size windows. Moreover, in the trussed tube systems, bracings contribute to improving the performance of the tube in carrying the gravity load. Figure 4. indicates a trussed tube building.

Figure 3. The schematic plan of tube-in-tube systems.  
Figure 4. Trussed tube building.

1.1.4. Bundled tube systems (BTS)

One of the solutions to control the shear lag is to add web and flange frames as the tubular cells, which contribute to distribute forces more uniformly between columns. These systems are called bundled tubes which consist of multi tubes. Moreover, tubular systems are applicable to prismatic profiles, including a variety of non-rectilinear plans, such as circular, hexagonal, triangular, and other polygonal shapes. However, for buildings with significant vertical offsets, the discontinuity in the tubular forms introduces serious inefficiencies. A bundled tube that can be configured with the multiple cells, on the other hand, provides for vertical offsets without much loss in efficiency. Additionally, it allows for wider column spacing that would be possible with a single cell tube [12]. In principle, many building shapes can be configured using bundled tubes. The structural principle behind the bundled tube concept is that the interior rows of columns and spandrels act as interior webs minimizing the shear lag effects. Without their beneficial effect, the exterior columns in a framed tube toward the center of the building would play a negligible role in resisting the overturning moment. Figure 5. shows a bundled tube building.

1.2. Shear lag phenomenon

Although a framed tube is quite an efficient system for the tall buildings, it does have outstanding idiosyncrasies due to the shear lag effects. The shear lag phenomenon is to increase axial stresses in the corner columns and reduce them in the inner columns of both the flange and the web panels which lead to decreasing moment resistance and their efficiency [15].

The shear lag effects in tubular buildings are as similar as the solid-wall hollow tube. The major resistance comes from the web panels which deform in a manner that some columns are in tension and others are in compression. The web frames are exposed to the in-plane flexural and racking action associated with an independent rigid frame. The primary action is modified by the flexibility of the spandrel beams which increases the axial stresses in the corner columns and decreases those in the interior columns. The main interaction between the web and flange frames occurs via the axial displacements of the corner columns. The more flexible spandrel beam, the less the deformation. Every successive interior column will face a smaller deformation and hence a lower stress than the outer ones. The stresses in the corner column will be greater than those from the pure tubular action, and those in the inner columns will be less. The stresses in the inner columns lag behind those in the corner columns (shear lag). Because the column stresses are distributed less effectively than in an ideal tube, the whole capacity of the structure is not used and this makes the moment resistance and the flexural rigidity be reduced [16].

There are many strategies in order to solve this irregularity: using the bundled tube systems (increasing the number of the web and flange frames to decrease the distance of columns from them), mega bracings (they assist to distribute forces better between columns), the deep spandrel beams, the mega columns at the corner of the structures (because of having more stresses at the corners and less ones in interior columns), the belt trusses and outriggers (they assist to distribute forces better between columns) and so on. However, the most efficient solution is the spandrel beams, because they have a key role in the uniform distribution of forces between columns. The more shear rigidity index (SRI), the less the shear lag. It is worth mentioning that belt trusses (along with the outriggers) can be used to prevent the rotation of the internal tube, decreasing its moment and drift, diminishing the shear lag and so forth. The Belt trusses connect exterior columns to each other and the outrigger connect them to the columns of internal tube. These can distribute forces among columns uniformly.
It is expected that the shear lag decreases through moving to the top of the structure. Nearly, at the three-fourths of the height of structures, a shear lag index (defined as the proportion of the axial force of the tubular columns to the axial force of the central column of the tube) gets 1.0. In higher spots than the mentioned spot, this ratio gets inverted (negative shear lag happens). As will be shown in this paper, the belt trusses and outriggers draw down this point to approximately the half of the height of the structure. The positive and negative shear lag phenomena are shown in Figure 6.

![Figure 5. Trussed tube building.](image)

![Figure 6. The positive and negative shear lags phenomena [1].](image)

In this study the shear lag in different types of the tube systems is studied. To reach this goal, 16 models of the steel tube, tube-in-tube, bundled tube and braced tube systems (43, 54, 67, 79 stories) are designed by the ETABS software. Their geometric characteristics are regarded as the same. It is also assumed that earthquake loads are more dominant than the wind loads. By the linear dynamic analysis, the shear lag of each columns in different levels is obtained and compared with each other. The results showed that the belt trusses and outriggers drew down the point that positive shear lag turned into negative (which is at the half of the general height). Also, it is shown that by moving up to the middle of the structures, shear lag decreased and got close to 1.0, and then it progressed inversely to the top of the structures. Namely, the axial force of the central column got bigger than the others. Lastly, it was tried to investigate statistically to find a formula for each of the systems with multi-variable linear regression examines with SPSS version 23.0 software based on the results of the linear dynamic analyses (to reach a general estimation of the weight of effective variables on the shear lag phenomenon). These formulations showed that there was a significant relation between the shear lag and some dependent variables such as the story number, the height ratio and the distance from the web of the structures. The statistical analyses done in this research, indicated that the aspect ratio of the structures had not a significant role in the shear lag index.

**2. Methods**

**2.1. Structural Modelling**

In this study, 16 models: four types of tubular systems (the framed tube, tube-in-tube, trussed tube and bundled tube system) with four numbers of stories (43, 54, 67, 79 stories) have been designed based on AISC [17] to investigate on shear lag effect. Figures 7–10 demonstrate the plan view of different types of tubular models used in this study. It is worth mentioning that the green points show the location of the columns, and the blue lines with the white borders around them show the location of the outriggers which connect the tubular columns to each other. Moreover, the ordinary blue lines indicate the beams, which connect none-tubular columns to each other. The models are 60m × 60m in their plan and the overall height of the structures are 172, 216, 268 and 316 m, respectively. Therefore, the aspect ratios of structures (calculated by Eq. (1)) are much greater than 1.5π and then they are considered super tall buildings [12].

\[
\frac{a}{H} \geq 1.5\pi, \tag{1}
\]

where \(a\) is the dimension of the core structure and \(H\) is the overall height of the structure.

Furthermore, the belt trusses and outriggers are used in the infrastructure stories of the structures to limit their lateral displacement as discussed in the paragraphs above. The outriggers are created by connecting the exterior columns with their counterpart columns in the interior or cell tubes by bracings. Also, the belt trusses are created by connecting exterior columns with each other by bracings either. In tall buildings, usually, an infrastructure story is assigned to every 20 stories. Yet in this study, for 43, 54, 67 and 79-story structures,
the belt trusses and outriggers are put respectively in every 13, 17, 21 and 25 stories. It is worth mentioning that the optimum location of the outriggers for the optimum performance is at \((\frac{1}{n} + 1), (\frac{3}{n} + 1), (\frac{4}{n} + 1)\)… and \((\frac{n}{n} + 1)\) height locations, where \(n\) is the number of outriggers. The highest floor of the structures is not an appropriate place to put the belt trusses on, because it has less efficiency in order to decrease drift and moment amounts. All of the models have only three stories with belt trusses and outriggers, so that their shear lag results can be compared with each other. The spaces between tubular columns are 3 m and between the non-tubular columns are 6 m. The former are designed under lateral and gravity loads, then they should connect rigidly with each other. The latter is designed for just the gravity loads and they should not connect rigidly to each other. The overall height of each story is 4 m. All of the tube-in-tube systems have an interior tube with the dimension of 20 m×20 m located in the middle of their plan. The cell tube dimensions in the bundled tube systems are 20 m×20 m too. In the braced tube systems, 4 pairs of mega bracings are used on each edge of exterior tubes. The floors are regarded rigid and the alternative loading is used for slabs.

As shown in Figures 11 and 12, the trussed tube system is considered as the tube-in-tube system which has mega braces. These mega braces are installed in the tube-in-tube systems in order to decrease their lateral displacements. Modeling and analyses are done by ETABS software based on the instructions of the AISC [17]. The shear lag factors in the first floor, the floors corresponding to a fourth and three fourths of structure height, and the highest floor is calculated, because at the bottom of the structure the positive shear lag factor and at the top of structures, the negative shear lag factor is critical. The shear lag factor (SF) is defined in Eq. (2).

\[
SF = \frac{P_u}{P_c},
\]

where \(P_u\) is the axial force of tubular columns and \(P_c\) is the axial force of the central columns of tubes. One of the assumptions in this paper is that the earthquake loads are more critical than the wind loads. Therefore, the comparison of the models is done based on the seismic analysis in the following sections. Lastly, the stress ratios of the elements in designing are considered between 0.7 and 0.9, so that the results of models could be comparable.
2.2. Definition of parameters of analyses

The ST37 steel with yield strength of 2400 kg/cm² for the elements. Gravity loads consist of 3.1 KN/m² as dead load and 1.5 KN/m² as a live load for the roof, and for all of the other floors: 4.4 KN/m² as dead load and 3.4 KN/m² as a live load.

The design-seismic load is calculated by using the Iranian Code of Practice for Seismic Resistant Design of Buildings [18]. According to this code requirement, dynamic analysis using the design spectrum of this code is necessary for buildings which are taller than 50 m height. In addition, the base shear force due to the dynamic analysis must be scaled to the base shear resulted from the linear static analysis, if that is smaller than the static base shear. The base shear force is calculated by Eq. (3).

\[ V = (\frac{A B I}{R}) \times W, \]  

where \( I \) is the importance factor,
\( R \) is the response modification factor,
\( A \) is the design spectral acceleration,
\( B \) is the response factor and \( W \) is the seismic weight of the structures.

The following factors are chosen for all structures to compute the base shear force: the \( I \) factor in these structures was considered as 1.0. It is also assumed that the structures are located on a site with soil classification of type II (375 < \( V_s \) < 750 m/s, in which \( V_s \) is the velocity of shear waves) and in a region with a very high level of seismicity risk (\( A = 0.35 \)). With regard to the fact that the \( R \) factor for tubular structures has not been mentioned in this code, in this study \( R \) factor set equal to 9.0 (which is similar to the response modification factor of the special steel moment-resisting frames with bracings). This amount is reasonable because the tubular systems are the developed format of the moment resisting systems. Moreover, the \( R \) factor of these structures obtained around 10.0 based on the research of Kim et al. [19]. Because of having outriggers and belt trusses which decrease the natural period of structures, our modification factor should be less than 10.0. Besides, Zahiri-Hashemi and Kheyroddin considered the amount of \( R \) factor of the braced tube systems equal to 7.0 [6] which is too low based on researches of Kim et al. [19].

By the linear response spectrum analysis and static analyses, the shear lag of each column in different levels was found and compared with each other. It is worth saying that the calculation of the flange shear lag factor of levels where negative shear lag happens with the spectral dynamic analysis is meaningless, because of its negative sign, the spectral dynamic analysis could not show levels which signify the changing axial force (because of the SRSS method of modal combination). In order to verify the ETABS software and calibration, the models analyzed by Mashhadialii et al. [20] are taken into consideration.

3. Results and Discussion

In this part, it is decided to consider the geometry specifications of the plan of the structures as the same and use the same number of belt trusses and outriggers, with regard to our objective (the investigation on the trend of shear lag in different tubular systems and comparing them to each other). Firstly, the maximum
amounts of the axial forces in the columns of the tubes or cell tubes were acquired by the linear dynamic (modal) analysis. Secondly, the shear lag factors are calculated by Eq. (2) and their diagrams are depicted as demonstrated in the figures below. In this study, only the shear lag factor of the tubular columns in between the web panels (flange shear lag factor) is calculated.

3.1. Studying shear lag of models

As mentioned before, one of the shortcomings of the tubular systems is shear lag, which decreases the moment resistance and efficiency of these systems against lateral loads. Generally and expectedly, the shear lag factor decreases with moving up towards the top of the structures. One of the reasons is the decreasing of the shear forces by moving up to the top of the structures. Usually, at the three-fourths of the height of the structures, the shear lag index reach to 1.0 and the distribution of forces between columns become uniform. On the other hand, higher than this point, the shear lag trend gets inverted by moving to the top of the structures. Namely, the axial force of the central column of each tube gets greater than the surrounding ones closer to the web and flange panels in that tube. As shown in Figures 13–16, this point in these structures is drawn down and located in the middle of the height of the structures. This may be out of the presence of the belt trusses and outriggers which draw down the mentioned point to that point. Accordingly, for positive shear lag factor, the more critical location is at the bottom of the structure and for the inverted on is at the top of the structures. In order to design economically, stronger columns should be put in the corners (web or flange panels) at the bottom and the central columns should be more reinforced than other ones at the half top. Figures 13-16 show the shear lag factor of the tubular columns in the flange of the structures along with the IDs of the columns. The column IDs are the number of columns in the plan from left to right.

![Figure 13. The flange shear lag factor in different stories of 43-story building: a) shear lag index at 1st story, b) shear lag index at 11th story, c) shear lag index at the 33rd story and d) shear lag index at 43rd story.](image-url)
As demonstrated in figures above, the trend and quantity of the shear lag indices in each type of the tubular systems remain similar to each other by increasing the number of stories. The number of stories indicates both the height and weight of structures, then the quantity of shear lag indices cannot be dependent on these two parameters. The results of the stochastic analysis with multi-variable linear regression method (as discussed in the paragraphs below) verify this fact too. It is also seen that the most uniform force distribution between columns is observed in tube-in-tube systems. In these systems, the interior and exterior tubes by the means of deep spandrels are connected and work with each other. This may be the reason for rather uniform distribution of forces. However, their distribution is not completely uniform and this uniformity is more located in the central area of the tubes. By moving towards the corners, drastic changes (increasing) in shear lag indices are seen. These differences are more at the lower stories. Generally, by increasing the number of the stories, a negligible improvement is seen in force distributions of the tube-in-tube systems. This phenomenon is expected, because increasing the number of the stories in buildings with more stories than 30 is ineffective on the shear lag improvement [4].

On the other hand, the worst force distribution among columns is observed in the framed tube systems. However, this system along with the trussed tube and tube-in-tube systems has similar amounts in the middle of the tubes and their amounts diverge by moving toward the corners. It is also observed that adding mega-braces to the tube-in-tube systems improves the uniform distribution of forces a little and the difference between them is more located at the corner of the tubes. Strengthening of mega-braces and using an effective number of mega braces may end up better improvement. The most irregular shape is shown in the bundled tube systems. In these systems, columns closer to the web panels have shear lag factor close to 1.0, but by getting distant, drastic changes happen. Except for the framed tube systems, the (positive) shear lag has more uniform distribution at the bottom of the structures than the shear lag factor at the top of them.
1.2. Formulation of the shear lag factor for different tubular systems

In this section, it is tried to find a relation between independent variables (the story number, the height ratio-the ratio of the height of section cuts to the overall height, the aspect ratio and distance from the web of the structures) and the shear lag index with multi-variable linear-regression examine by SPSS software. This examine tells us if there is a significant relationship between the independent (predictor) variables and it also can give weights (coefficients) to the variables to reach a linear formulation. In other words, it says how much the output is explained by the independent variables.

Firstly, the linear regressions indicated that there is a significant relationship between the mentioned variables (except for the aspect ratio, which was excluded from the regression). Because the significance area (Sig. factor) is nearly 0.0001 (less than 5 %) in all types of the tubular systems. The Sig. factor shows whether there is a significant relationship between the variables or not. If the Sig. factor is less than 0.0001, there is a significant relationship. The Sig. factor is equivalent to the P-values in the linear regression. The p-value is the level of marginal significance within a statistical hypothesis test representing the probability of the occurrence of a given event. The p-value is used as an alternative to reject points to provide the smallest level of significance at which the null hypothesis would be rejected. The linear regressions demonstrated that the Sig. factor of aspect ratio (a) is 0.744 then the software excludes this variable from the regression, because it hasn't a significant role in the shear lag amount.

Figure 15. The flange shear lag factor in different stories of 67-story building:

a) shear lag index at 1st story, b) shear lag index at 17th story,
c) shear lag index at 51st story and d) shear lag index at 67th story.
Secondly, the linear regressions showed that the dependence percentage of shear lag on these variables is about 30% ($R^2$ factor), it means that nearly 70% of the shear lag index are dependent on other factors that should be regarded in the regression such as the shear rigidity indices (rigidity of spandrels), the slenderness, the span lengths and so on. The $R^2$ factor shows how many percentages of shear lag as a dependent variable are explained by these independent variables (predictor variables). It is worth saying that $R^2$ is a statistical measure of how the data are close to the fitted regression line. It is also considered as the coefficient of determination, or the coefficient of multiple determination for multiple regression.

Finally, it is tried to find formulations for the shear lag indices with the linear regression for each type of the tubular systems. These formulas can give us a general view about the effect of the mentioned factors on the shear lag amounts. As shown in the regressions, the storey number (the overall height of the structures) has the least influence on the shear lag (which verifies the research of Kazeminia and Khoshndian [4], that asserts this parameter effect on the shear lag decreases in the structures with more than 30 stories). The number of stories is the representative of the height and weight of structures, and then the amount of the shear lag indices cannot be dependent on these two parameters. Figures 17-20 verify this issue too. Table 1 shows $Sig.$ and $R^2$ factors for each kind of tubular systems. Moreover, although it was expected that by increasing the aspect ratio and changing the displacement mode of the structures from shear to flexural, the shear lag increases, it was seen that the aspect ratio does not have any effect on the shear lag index and it was excluded from the linear regressions by the software based on the results. The bundled tube systems have the greatest $R^2$ factor and the tube systems have the least one, namely, the effect of the mentioned independent variables on the shear lag index of the bundled tube systems is the most and on the tube systems is the least.
Table 1. The parameters of tubular systems.

<table>
<thead>
<tr>
<th>Tubular systems</th>
<th>$R^2$ (R-squared)</th>
<th>Sig.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Framed tube buildings</td>
<td>0.294</td>
<td>**0/0001</td>
</tr>
<tr>
<td>Braced tube-in-tube buildings</td>
<td>0.325</td>
<td>**0/0001</td>
</tr>
<tr>
<td>Tube-in-tube buildings</td>
<td>0.307</td>
<td>**0/0001</td>
</tr>
<tr>
<td>Bundled tube buildings</td>
<td>0.307</td>
<td>**0/0001</td>
</tr>
</tbody>
</table>

The Eq. (4) shows the linear formulations for the framed tube systems:

\[ LT = -0.005S - 1.13H - 0.007D + 2.07, \]  

(4)

where \( L \) is the shear lag index amount, \( S \) is the story number, \( H \) is the height ratio (the ratio of the height of the story in which the section is cut to the overall height of the structure) and \( D \) is the distance between columns and the nearest web of the structures. The standard error of the estimate is 0.70957.

The Eq. (5) shows the linear formulations for the braced tube systems:

\[ L_{BT} = -0.004S - 0.83H - 0.004D + 1.78, \]  

(5)

where \( L, S, H, \) and \( D \) are as the same as the ones defined in Eq. (4). The standard error of this estimate is 0.47781.

The Eq. (6) shows the linear formulations for the tube-in-tube systems:

\[ L_{TT} = -0.005S - 0.9H - 0.006D + 1.9, \]  

(6)

where \( L, S, H, \) and \( D \) are as the same as the ones defined in Eq. (4). The standard error of this estimate is 0.55040.

The Eq. (7) shows the linear formulations for the bundled tube systems:

\[ L_{BuT} = -0.003S - 0.534H + 0.04D + 1.15, \]  

(7)

where \( L, S, H, \) and \( D \) are as the same as the ones defined in Eq. (4). The standard error of this estimate is 0.34643.

As shown in Eq. (7), the role of the story number and the column distances (from the web of the structures) in the braced tube systems is very negligible, which the amount of their Sig. factor verifies it. Because they are 0.054 and 0.612 respectively, which is too low, so they can be excluded from the formulation. There is another factor in the linear regression named \( \beta \)-effective which shows the approximate effect of each independent variable on the shear lag amounts. Figures 17–20 show the \( \beta \)-effective factor of the effective variables on the shear lag amounts. It is observed in Figure 20 that \( \beta \)-effective factor of the distance and story number in the braced tube system is negligible, this phenomenon can be because of the more uniform distribution of forces among columns in these systems. Also, due to the presence of the numerous webs and decreasing the distance of the columns from the webs, the role of \( D \) (the distance from the nearest webs of the structures) in the bundled tube system gets more considerable than the other tubular systems.

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It is worth saying that regressions in order to have more inclusive results need more data. These formulas can be regarded as approximate criteria to estimate the effects of the mentioned independent variables on the shear lag index.

4. Conclusion

In this paper, the shear lag indices in different types of the tube systems are perused and statistically analyzed. To reach this goal, 16 models of steel tube, tube-in-tube, bundled tube and braced tube systems (with 43, 54, 67, 79 stories) are created by the ETABS software. Their geometric characteristics are as the same. It was also assumed that earthquake loads are more dominant than wind loads. By linear response
spectrum analysis, the shear lag of each column in different levels was found and compared with each other. The results show that:

1) Belt trusses and outriggers draw down the point that the shear lag amounts get 1.0, to the half of the general height of the structures. By moving toward the top of the structure, shear lag decreases and gets 1.0, and then it progresses inversely to the top of the structures.

2) Then, it is tried to investigate statistically the data reached from the linear response spectrum analysis, to find formulations for all four types of tube systems by multi-variable linear regression examine with entering method (by SPSS version 23.0). This showed a significant relation between the shear lag index and the independent variables (the storey number, the aspect ratio of structures, the height ratio and the distance from the web of the structures).

3) It is observed that the most important independent variable is the height ratio and the least important one is the storey number (or the distance from the nearest web in the braced tube systems) in the linear regressions.

4) Although it was expected that by increasing the aspect ratio and changing the displacement mode of structures from the shear mode to the flexural one, the shear lag increases, it is seen that the aspect ratio does not have any effect on the shear lag index and was excluded from the linear regressions by the software.

References
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