Thermophysical properties of the soil massif

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Abstract. Calculations show that a significant percent of the heat losses of monolithic foundations consists of heat loss to the ground from concrete during construction. Therefore, ignoring heat losses to the ground (i.e., taking into account only the formwork and thermal insulation) leads to significant deviations between calculated and actual technological parameters. The existing methods for calculating the coefficient of heat transfer of enclosures are not suitable when calculating this same parameter for soil massifs. While finite thicknesses are used in this calculation for enclosures, thickness is infinite for soil massifs. To create a method for calculating heat losses to the ground, we solved a differential equation of heat conduction using integral transform methods. In the classical theory of heat transfer, for any material of finite thickness, the heat transfer coefficient is constant over time. However, for an array of soil, this parameter varies depending on period of time during which concrete loses heat to the soil. At the same time, the heat transfer coefficient increases with increasing soil density, which is explained by the growing contact area between particles in a unit volume of soil. Thus, the surface area through which the heat flux moves also increases. The article presents the results of the finite element calculation in the simulation software ELCUT, confirming the reliability of the obtained analytical dependencies.

1. Introduction

When calculating the technological parameters of curing foundation structures, especially massive structures or those cured at negative temperatures, it is often necessary to estimate heat losses from concrete to the soil massif and to the surrounding air [1–4]. The calculation of heat losses to the air through the formwork and thermal insulation does not cause any difficulties since the process of heat distribution is described by the classical theory of heat transfer from a more heated medium to a less heated one through a separating wall of a given thickness [5–10]. Moreover, the entire calculation concerns the heat transfer coefficient of the concrete enclosure, which is influenced by the properties of the enclosure and the environment.

When calculating heat losses to the ground, it is impossible to specify the thickness of the soil massif in light of its near infinite size. Consequently, the standard methods for calculating the soil heat transfer coefficient are not suitable here. To this end, designers often ignore heat losses into the soil massif, focusing on the formwork and thermal insulation. At the same time, the reduced heat transfer coefficient of a concrete enclosure used in the calculations does not only take into account the ratio of heat transfer coefficients of different enclosures of the same design, but also the ratio of their areas. Thus, ignoring heat losses in the soil massif in the calculations can lead to significant errors.

It should be noted that these errors can affect not only the estimated time of concrete curing, but also its structural properties, since heat losses through a concrete enclosure play a major role in the formation of its thermally-stressed state [11–14]. It has already been shown [15] that the thermal characteristics of enclosures lead to uneven distribution of temperatures along the cross section of a monolithic structure, which can cause (if the temperature gradients exceed the limit values) unacceptable temperature stresses in concrete and, as a result, cracking.
In the norms of the Republic of Belarus [16], a different approach was taken to account for the thermophysical characteristics of the substrates while maintaining concrete structures. Here, the calculation principle is based on the heat balance equation, and the concrete temperature is estimated after the concrete loses some of its heat to the reinforcement, embedded parts, formwork, and soil foundation.

A book by Finnish authors [17] indicates that the average temperature of concrete in the hardening process depends on the coefficient of heat permeability of the surface at the time of the calculation (Wm²°C, which corresponds to the heat transfer coefficient of the enclosure considered in this article). However, methods for calculating this coefficient are not given in the book. At the same time, the authors indicate that it is the formwork that affects heat loss, without mentioning the soil base.

The American standard [18] provides tables that indicate the minimum permissible outdoor temperatures when concreting slabs of different thicknesses with an insulated enclosure with a certain thermal resistance (m°C/W, i.e. the reciprocal of the value considered in this article for fence heat transfer coefficient). However, it is also implied here that only formwork and insulation can be a fence, ignoring the soil base.

A publication by Canadian scientists [19] considers temperature changes at various points of the soil mass when exposed to coolant. The soil heat transfer coefficient appears in the study, but its values are not determined analytically - they are set from the results of the experiment. Moreover, the values are given as constant, although the heat transfer coefficient of the soil varies over time for objective reasons, as will be discussed below.

The overwhelming majority of literature published in recent years on the thermophysical properties of soil masses only provide data on measured temperatures in the soil column under the influence of an external heat source [20–22]. That is, the values of soil temperature themselves are not determined analytically - they are only recorded during experiments. This approach makes it possible to effectively and accurately to determine the actual temperature change of a particular soil over time. However, at the same time, the issue of predicting temperature changes at the project stage under external conditions which differ from the experimental conditions is not being addressed. In addition, the use of such experimental data does not allow one to quickly switch to solving similar problems with other soils nor to change their thermophysical characteristics.

In a number of works [23, 24], in addition to experimental data (or instead of them), the authors provide computer calculations of changes in the temperature of specific soils under given conditions under the action of an external heat source. Scientific works in which only computer calculations are presented a priori have significantly less accurate results and are not confirmed experimentally. The use of computer programs operating on the basis of the finite element method for solving thermotechnical problems does not allow for performing on-line calculations of heat losses of concrete in the soil on construction sites without preliminary development of the design scheme. Moreover, the degree of accuracy of the calculations is determined by the degree of detail of this calculation scheme.

Thus, the aim of the study is to obtain a fairly simple mathematical dependence, based on an analytical conclusion, to calculate the heat transfer coefficient of the soil mass.

To achieve this goal it is necessary to solve the following tasks:
– solve the differential heat equation for given boundary conditions;
– analyze the result of the decision;
– perform a sample calculation based on the obtained mathematical dependence and compare the obtained data with the calculation in the ELCUT software package.

2. Methods

To estimate heat losses to the ground, let us use Fourier’s differential equation of heat conduction

\[
\frac{\partial T(x, \tau)}{\partial \tau} = \alpha \frac{\partial^2 T(x, \tau)}{\partial x^2},
\]

specifying a soil foundation in the form of a semi-infinite body. In this case, due to the concrete laid on the soil, at the initial moment of time \((\tau = 0)\), the surface temperature of the semi-infinite body is \(T_c\) and does not change during the entire time of its curing, \(\tau_v\). In addition, at the initial moment of time, the temperature at all points of the soil massif is constant and equal to \(T_0\), and there is no temperature drop at an infinitely distant point of the soil massif.
To solve equation (1), let us use the Laplace integral transform method and obtain:

\[
L \left[ \frac{\partial T(x, \tau)}{\partial \tau} \right] = L \left[ \frac{\partial^2 T(x, \tau)}{\partial x^2} \right].
\] (2)

If we apply the Laplace transform to the function of temperature distribution inside the body of the soil massif in time and in depth \( T(x, \tau) \) and to the left side of equation (2), we obtain an ordinary differential equation for the image \( T_L(x, s) \) (since \( T_L(x, s) \) does not depend on time \( \tau \)):

\[
T''_L(x, s) - \frac{s}{\alpha} T_L(x, s) + \frac{T_0}{\alpha} = 0
\] (3)

To solve equation (3), let us use the method of variation of constants [22] and obtain:

\[
T_L(x, s) = \frac{T_0}{s} + A_1 e^{\sqrt{\frac{s}{\alpha} x}} + B_1 e^{-\sqrt{\frac{s}{\alpha} x}}
\] (4)

where \( A_1 \) and \( B_1 \) are the constants determined from the boundary conditions.

If we apply the Laplace transform to the boundary conditions and substitute the results in (4), we find that the constants are equal to:

\[
A_1 = 0; \quad B_1 = -\frac{T_0 - T_c}{s}.
\]

Then, (4) can be written as follows:

\[
\frac{T_0}{s} - T_L(x, s) = (T_0 - T_c) \cdot \frac{1}{s} e^{\sqrt{\frac{s}{\alpha} x}}.
\]

Restore the original function from its modified state (image):

\[
T_0 - T(x, \tau) = (T_0 - T_c) \left[ 1 - \text{erf}\left( \frac{x}{2\sqrt{\alpha \tau}} \right) \right].
\]

After the transform, we obtain the solution of the heat transfer equation (1):

\[
\frac{T(x, \tau) - T_c}{T_0 - T_c} = \text{erf}\left( \frac{x}{2\sqrt{\alpha \tau}} \right).
\]

Now, according to Fourier’s basic law of heat conduction, we determine the heat losses over time \( d\tau \) through a unit of area:

\[
dQ = -\lambda \left( \frac{\partial T}{\partial x} \right)_{x=0} d\tau = -\lambda (T_0 - T_c) \left\{ \frac{\partial}{\partial x} \left[ \text{erf}\left( \frac{x}{2\sqrt{\alpha \tau}} \right) \right] \right\}_{x=0}.
\]

According to the main property of the error function (erfx), its derivative:

\[
\frac{\partial}{\partial x} \left[ \text{erf}\left( \frac{x}{2\sqrt{\alpha \tau}} \right) \right] = \frac{1}{\sqrt{\pi \alpha \tau}} e^{\left( \frac{x^2}{4\alpha \tau} \right)}.
\]

Wherein at \( x = 0 \) the value of the exponential function is 1.

Thus, we obtained a very important intermediate result - an expression for determining the density of the heat flux (W/m²) into the soil massif of the foundation from the concrete mixture cured at a specific moment of time:

\[
q = \frac{dQ}{d\tau} = -\lambda \cdot (T_0 - T_c) \cdot \frac{1}{\sqrt{\pi \alpha \tau}} \left( T_0 - T_c \right).
\] (5)
The total amount of heat $Q$ given by the cured concrete to the soil massif over a certain period of time $(\tau_v)$ is found by the integration from 0 to $\tau_v$:

$$Q = \int_0^{\tau_v} \frac{\lambda c \gamma}{\pi \tau_v} (T_0 - T_c) d\tau = 2 \sqrt{\frac{\lambda c \gamma}{\pi}} (T_0 - T_c) s \sqrt{\tau_v}.$$  \hspace{1cm} (6)

Then, we transform expression (6) so that to isolate the part acting as a heat transfer coefficient:

$$Q = 2 \sqrt{\frac{\lambda c \gamma}{\pi \tau_v}} (T_0 - T_c) \cdot s \cdot \tau_v.$$  \hspace{1cm} (7)

Thus, it can be seen that in (7) the parenthesized expression determines the average heat transfer coefficient ($\alpha_m$) during the concrete curing time. Therefore, the desired value is determined by the following analytical dependence:

$$\alpha_m = 2 \sqrt{\frac{\lambda c \gamma}{\pi \tau_v}} = 1.13 \sqrt{\frac{\lambda c \gamma}{\tau_v}},$$  \hspace{1cm} (8)

where $\lambda$, $c$, $\gamma$ are the coefficient of thermal conduction, the coefficient of specific heat and soil density, respectively.

In the classical theory of heat transfer, the heat transfer coefficient is constant over time for any material of finite thickness. However, for an array of soil, this parameter varies depending on the period of time during which concrete gives off heat to the soil. This fully complies with the method proposed by Arbeniev [25] for calculating the decreased temperature of concrete mixtures laid on a frozen base and indicates the non-stationary nature of heat transfer processes.

At the same time, it can be seen from (8) that the heat transfer coefficient increases with increasing soil density, which is explained by the growing contact area between particles in a unit volume of soil. Thus, the surface area through which the heat flux moves also increases. This correlates well with the works [20, 21].

Thus, for a typical monolithic reinforced concrete structure, we can write a formula for determining the reduced heat transfer coefficient of its enclosure, which allows us to take into account heat loss through all surfaces of the monolithic structure:

$$\alpha_r = \frac{\alpha_f A_f + \alpha_c A_c + \alpha_m A_m}{A_f + A_c + A_m},$$

where $\alpha_f$, $\alpha_c$, $\alpha_m$ are the heat transfer coefficients of the formwork and covering material of undecked surfaces, respectively;

and $A_f$, $A_c$, $A_m$ are the area of formwork surfaces, undecked surfaces and surfaces in contact with the soil, respectively.

3. Results and Discussion

The obtained results were verified using finite element modeling in the ELCUT software suite and then analyzing of the results.

Figure 1 shows the calculation results in ELCUT as an image of the temperature distribution in a soil block when concrete of a monolithic free-standing foundation is cured on its surface (the size of the slab of the foundation contacting the foundation soil is 1.5×1.5 m; the initial temperature of the concrete mixture and concrete curing temperature is +30 °C; the initial soil temperature is +3 °C; the outdoor air temperature is -7 °C, the curing time is 7 days). The soil itself is represented by loam from solid to semi-solid consistency, weakly eruptive, with the density of 2030 kg/m³, the humidity of 15%, the thermal conduction coefficient of 2.1 W/m.°C and specific heat capacity of 1530 J/kg.°C.

Due to the impossibility of obtaining the heat transfer coefficient of one or another body in ELCUT, we will verify using the heat flux values $F_{elc}$ [W] generated by the software by the contact area of the slab part of the foundation and the soil foundation at certain points in time (Table 1).

Let us compare our formula (5) with the classical formula of the theory of thermal conduction, which determines heat flux density,

$$q = \alpha \cdot (T_0 - T_c).$$
As the formula shows, heat flux density is determined by the instantaneous value of the heat transfer coefficient and the temperature difference at the interface between the bodies that are heat exchange participants. In turn, (8) determines the average value of the heat transfer coefficient of the soil massif over the whole curing time of concrete on a soil foundation.

Given that heat flux density is determined by the value of the heat transfer coefficient at a certain moment of time, and knowing the area of the slab portion of the foundation $s = 1.5 \times 1.5 = 2.25 \text{ m}^2$, we can analytically determine heat flux density $F_{an}$ [W] for the considered example,

$$F_{an} = - \frac{\lambda c \gamma}{\pi t_i} (T_0 - T_i) \cdot s.$$  

The most important condition for applying the method of mathematical modeling of complex physical processes is the proof of the reliability of the mathematical model. Table 1 and Figure 2 show a comparison of the heat flux values determined in ELCUT and according to the derived analytical dependencies over the concrete curing time of 7 days.
Table 1. Heat flux values.

<table>
<thead>
<tr>
<th># of item</th>
<th>Moment of time, s</th>
<th>$F_{elc}$ (W)</th>
<th>$F_{an}$ (W)</th>
<th>Divergence (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>33 300</td>
<td>366.41</td>
<td>479.93</td>
<td>-23.6</td>
</tr>
<tr>
<td>2</td>
<td>99 800</td>
<td>262.63</td>
<td>273.38</td>
<td>-5.26</td>
</tr>
<tr>
<td>3</td>
<td>133 000</td>
<td>232.61</td>
<td>239.96</td>
<td>-3.04</td>
</tr>
<tr>
<td>4</td>
<td>166 000</td>
<td>213.07</td>
<td>215.06</td>
<td>-0.85</td>
</tr>
<tr>
<td>5</td>
<td>200 000</td>
<td>199.22</td>
<td>195.62</td>
<td>1.86</td>
</tr>
<tr>
<td>6</td>
<td>233 000</td>
<td>188.80</td>
<td>182.25</td>
<td>3.67</td>
</tr>
<tr>
<td>7</td>
<td>266 000</td>
<td>180.60</td>
<td>169.49</td>
<td>6.45</td>
</tr>
<tr>
<td>8</td>
<td>299 000</td>
<td>173.94</td>
<td>160.38</td>
<td>8.33</td>
</tr>
<tr>
<td>9</td>
<td>333 000</td>
<td>168.39</td>
<td>151.88</td>
<td>10.8</td>
</tr>
<tr>
<td>10</td>
<td>366 000</td>
<td>163.67</td>
<td>144.59</td>
<td>13.03</td>
</tr>
<tr>
<td>11</td>
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<td>12</td>
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<td>153.07</td>
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<td>147.67</td>
<td>120.29</td>
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</tr>
<tr>
<td>14</td>
<td>605 000</td>
<td>142.71</td>
<td>120.59</td>
<td>27.0</td>
</tr>
</tbody>
</table>

Figure 2. A comparison of heat flux values using ELCUT and analytical dependencies. The orange curve is calculated by formula (9), the blue curve represents the results from ELCUT.

As the data shows, divergence between the calculation results based on finite-element modeling and based on analytical dependence derived in this article for heat losses of concrete cured in the soil massif of the foundation is insignificant (on average +6.85%). At the same time, higher values of the heat flux at the initial moment of concrete curing can be explained by the accumulation of heat in the soil, which has a temperature much lower than the temperature of the concrete mix [22]. It is clear that for different types of soils with a variety of thermophysical characteristics (thermal conductivity, specific heat, density), the amount of accumulated heat will be different. We compare our obtained graph of changes in the values of heat fluxes over time with the results obtained in [19] based on laboratory tests.

When the monolithic construction is 300 mm high, our calculations for the above problem show that heat losses into the soil massif are responsible for 54.6% of the total heat losses (i.e., 45.4% of losses occur at all other surfaces contacting the vertical formwork and horizontal thermal insulation). This value is significant, and ignoring it leads to considerable deviations between the calculated and actual technological parameters. Thus, the specific heat capacities needed to ensure isothermal heating of concrete of this design at a temperature of 30 °C, without and without taking into account heat losses to the ground, differ by almost 2 times.

4. Conclusions

Based on the solution of the Fourier differential heat equation, the research objective was realized - the assessment of the heat loss of concrete in the ground. The following results were obtained:

1. We obtained analytical dependencies that are convenient for calculations which allow us to determine heat losses of concrete structures into the soil foundation and to determine the heat transfer coefficient of the soil massif.
2. The results of calculations for the derived analytical dependencies and calculations using finite element modeling in ELCUT were compared. In the considered example, the calculation accuracy of heat losses according to the derived dependence and the results of computer modeling in ELCUT averaged +6.85 %.

3. The necessity of accounting for heat losses to the soil while maintaining monolithic concrete is proved. In the considered example, heat loss to the soil mass was 54.6 % of the total heat loss.

References


Теплофизические свойства массива грунта

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Аннотация. Расчеты показывают, что тепловые потери в грунт бетона монолитных фундаментов в процессе их возведения занимают значительную долю в общем объеме тепловых потерь таких конструкций. Поэтому игнорирование тепловых потерь в грунт (т.е. учет только опалубки и утеплителя) приводит к значительным отклонениям рассчитываемых технологических параметров от фактических. Существующие методы расчета коэффициента теплопередачи ограждения не подходят для использования при расчете данного параметра грунтовых массивов. Это объясняется тем, что в таких расчетах используются конечные толщины ограждений, а у грунтовых массивов она бесконечна. Для создания методики расчета тепловых потерь в грунт было решено дифференциальное уравнение теплопроводности с использованием методов интегрального преобразования. Получено, что в отличие от классической теории теплопередачи, когда для любого материала конечной толщины коэффициент теплопередачи является величиной постоянной во времени, для массива грунта этот параметр меняется в зависимости от времени, в течение которого бетон отдает тепло грунту. При этом коэффициент теплопередачи увеличивается при увеличении плотности грунта, что объясняется растущей площадью контакта между частицами в единице объема грунта. Таким образом, увеличивается и площадь поверхности, через которую движется тепловой поток. Приведены результаты конечно-элементного расчёта в программном комплексе ELCUT подтверждающие достоверность полученных аналитических зависимостей.

Литература
5. Головнев С.Г. Технология зимнего бетонирования. Оптимизация параметров и выбор методов. Челябинск: ЮУрГУ, 1999. 156 с.


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