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Stress-strain state of bending reinforced beams with cracks

Zh. Nuguzhinov[a*], N. Vatin[b], Zh. Bakirov[c], O. Khabidolda[d], S. Zholmagambetov[c], I. Kurokhtina[c]

[a] Kazakhstan Multidisciplinary Institute of Reconstruction and Development Republican State Enterprise on the Right of Economic Use, Karaganda, Republic of Kazakhstan
[b] Peter the Great St. Petersburg Polytechnic University, St. Petersburg, Russia
[c] Karaganda State Technical University, Karaganda, Republic of Kazakhstan
[d] al-Farabi Kazakh National University, Almaty, Republic of Kazakhstan

[*] E–mail: kazmirr@mail.ru

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Abstract. The work is dealing with studying the stress-strain state of bending pre-stressed concrete beams with cracks. The problems of determining preliminary stresses in a reinforced concrete element, determining the moment of crack formation, and determining the stress in a section with a crack are successively solved. The problems were solved for a general section with the vertical axis of symmetry, taking into account the non-linear relationship between strain and stress in concrete. The resolving system of two transcendental algebraic equations is obtained from the equilibrium conditions of a part of the beam on one side of the section with a crack. Analytical expressions have been obtained for determining preliminary stresses, the external bending moment at which a crack normal to the axis appears, as well as the stress state parameters in the section with a crack, including the crack height. The results obtained make it possible to predict the bearing capacity of reinforced concrete structures at the design stage by two groups of limiting states, and to evaluate the real technical condition of the structures in operation. These results can be used to determine the parameters of fracture mechanics and evaluate crack resistance of a reinforced concrete beam.

1. Introduction

A very common type of defects and damage to reinforced concrete structures are cracks [1−3]. They appear both at the manufacturing stage and at the operation stage. Causes of cracking may be excessive tension of the reinforcement in pre-stressed structures, an insufficient protective layer of concrete, shrinkage of concrete, a high temperature during welding of mating elements units. In bearing reinforced concrete structures the occurrence and development of cracks take place due to their deformation under the effects of loads, temperature fluctuations, uneven subsidence of buildings and structures.

The appearance of cracks in bending elements does not mean exhaustion of its carrying capacity. It leads to increasing efforts in the sections with a crack, which reduces the element strength. In addition, due to crack opening, the corrosion of reinforcement increases that reduces the structure durability. The norms regulate the extent of crack opening that depends on the stress state in the section with a crack. Since cracks exist in any reinforced concrete structures, determining the stress state in a cross section with a crack is very important for assessing the actual state of the structure being operated.

The emergence of cracks was studied by the NIIZhB, TsNIISK, in the Kharkov Promstroy NII proekt and other leading research and design organizations. A lot of scientists dealt with the conditions of forming cracks and assessing their effect on strength and deformability, a lot of monographs [1−4] and scientific articles were written. They developed analytical and numerical methods for calculating reinforced concrete structures with cracks.

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Designing reinforced concrete structures with cracks is a complex scientific problem. The issues of substantiating the computational models and defining the stress-strain state of beams with cracks normal to the axis were dealt with in articles [5−7]. One of the main criteria for the limiting state of reinforced concrete structures with cracks is the width of crack opening. Articles [8-14] are dealing with its definition. Studying the influence of the angle of inclination of cracks on the parameters of the stress state in the cross section is described in works [15, 16].

The issues of crack formation are considered in works [17−22]. Work [23] is dealing with the calculation of crack resistance and changing the beam rigidity and deflections during formation of cracks. Some researchers believe that with the help of organized cracks the deflection of a reinforced concrete beam can be reduced. This idea is being dealt with in work [24].

The above problems are dealt with in a number of experimental studies, the results of which are given in [25−30]. A lot of studies were carried out by numerical methods [6, 8, 9, 31−38].

Recently probabilistic methods of designing have been actively introduced into engineering practice. The use of these methods in predicting the parameters of crack formation and calculation of reinforced concrete structures with cracks reliability according to various criteria are described in [5, 34, 35, 39].

In connection with the widespread introducing of computer technologies into computational practice, it became possible to calculate numerically the parameters of fracture mechanics and to use them for assessing the bearing capacity of reinforced concrete structures with cracks. Article [40] is dealing with these approaches.

From this brief review it is clear that a lot of scientific articles are published on calculation of reinforced concrete structures with cracks, which indicate the problem urgency. At the same time, it should be noted that in the last period there are not so many works dealing with the analytical determination of the parameters of reinforced concrete elements with a crack stress state.

At present there is a generally accepted method for determining the moment of crack formation. The numerical calculations carried out by us using the finite element method show a significant difference between the analytical and numerical results. The method proposed in this paper removes these inconsistencies. A review of the works shows that not enough attention has been paid in the studies to determining stresses from the reinforcement pre-stressing. This operation is considered only as a way to increase crack resistance. In addition, these stresses affect the stress state of the element and through it the limiting state of the structure in use. In this regard we need computational dependencies that allow determining pre-stresses and taking them into account when assessing the real state of the structures.

The authors have been studying the problems of designing reinforced concrete structures with cracks for many years [2, 41].

2. Methods

To determine stresses in the beam, the section method is used. The beam is cut along the considered section and the equilibrium conditions of the cut-off part of the beam are made. For bending beams, these conditions are reduced to the equality to zero of the sum of the projection of all the forces on the axis of the beam and the equality to zero of the sum of the moments of all the forces relative to the transverse axis of the section.

When determining pre-stresses from the tension of the reinforcement and determining the moment of the crack formation, the hypothesis of flat sections is adopted. In the case of cracks, this hypothesis is applied to the average cross section between the cracks. In the section with a crack, the linear deformation diagram is transformed taking into account the uneven deformation along the length of the beam.

An exponential law is used to describe the nonlinear relationship between stress and strain in concrete.

$$\sigma = 1.1R_b\left[1-\exp(-0.9\varepsilon E_b / R_b\right]. \tag{1}$$

where E_b and R_b are the elasticity modulus and the concrete strength limit.

This law is one of the frequently used expressions for describing the deformation diagram [2, 42].

The main unknown tasks are the height of the compression zone of concrete and the length of the tension zone above the crack. In the absence of a crack, the maximum (regional) tensile stress in concrete is taken as the second unknown. These unknowns are found from two equilibrium equations. Through these parameters, using the hypothesis of flat sections and the law of deformation, all the parameters of the stress state are determined: maximum compressive stress in concrete, stress in reinforcement, crack length or maximum tensile stress in concrete, crack opening width.

The results of the study are determining pre-stresses in a reinforced concrete element, determining the bending moment of the crack formation, determining the stress state of a section with a crack during bending with determining the length of operational cracks.

2.1. Determining pre-stresses

Sometimes in reinforced concrete structures in the process of manufacturing the are especially developed significant compressive stresses in concrete by tension of high-strength reinforcement. The initial compressive stresses are developed in those areas of concrete that will subsequently experience tensile stress. Therefore, in a pre-stressed beam under load the concrete experiences tensile stresses only after the initial compressive stresses are extinguished. At the same time, the load that causes the appearance of cracks or their opening limited in width is much higher than the corresponding load in the beam without pre-stressing. In such beams the structure rigidity is higher, therefore, deflections will be less, and the stochastic nature of forming cracks will not have such a “destructive” effect of cracks, as it happens in beams without pre-stressing.

The magnitude of the pre-stress significantly affects the subsequent work of the elements under load. With small pre-stresses in the reinforcement and low compression of concrete the effect of pre-stress over time will be lost due to relaxation of stresses in the reinforcement, shrinkage and creep of concrete and other factors. At high pre-stresses in the reinforcement close to the standard resistance, there is a danger of rupture and the risk of significant residual deformations. Therefore, it is necessary to know pre-stresses in the section of the reinforced concrete element from the reinforcement stress in order to establish the rational controlled reinforcement stress when manufacturing.

At present, to determine these stresses, a linear calculation of the reduced concrete section is performed, in which the reinforcement area is replaced with the equivalent concrete area. A linear calculation of reinforced concrete elements can be performed at low stress levels, when they do not exceed 70 % of the calculated ones. If a crack does not occur in the section of the element, then you can be sure that this condition is satisfied for the compression zone. This situation is characteristic of determining the compression stresses. At the same time, inelastic deformations can occur in concrete of the tension zone. Therefore, for the deformation diagram in this zone, we adopt the exponential law (1).

In calculations we will proceed from the following aspects:

1. the cross sections remain flat during flattening;
2. in concrete of the compressed zone deformations are only elastic, the stress diagram is triangular;
3. in concrete of the tension zone the relationship between stress and strain is described by the exponential law (1).

Let us consider an I-section with a vertical axis of symmetry (Fig. 1, a). We will introduce the notation:

A_{ct}, A_c are the flanges overhangs areas in the tension and compressed zones;

h_t, h_c are the flanges thicknesses in the tension and compressed zones;

$a, a^/$ are the thicknesses of the concrete protective layer in the tension and compressed zones;

A_s, A_s' are the reinforcement areas in the tension and compressed zones;

$N_a, N_a^'$ are internal efforts in the reinforcement in the tension and compressed zones.

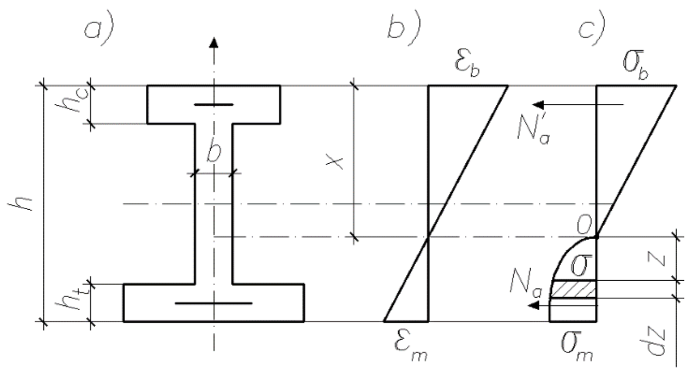

Figure 1. The beam cross section (*a*), deformation curves (*b*) and stress curves (*c*).

The basic unknown values of the problem the compressed zone height x and maximum tensile stress in concrete σ_m. According to the deformation diagram we will fine the corresponding deformation

$$\varepsilon_m = -\left(R_{bt}/0.9E_b\right)\ln\left[\left(1-\sigma_m\right)/1.1R_{bt}\right].$$

Let us introduce the notation

$$c = \sigma_m / 1.1R_{bt}, \;\; y = -\ln\left(1-c\right).$$

Then the boundary deformations will be equal

$$\varepsilon_m = 1.1yR_{bt}/E_b, \;\; \varepsilon_b = \varepsilon_m x/\left(h-x\right).$$

The maximum shearing stress in concrete

$$\sigma_b = E_b\varepsilon_b = 1.1yR_{bt}x/\left(h-x\right). \quad (2)$$

The mean stress on the compressed zone flange will be taken equal to the value at the level of the overhangs center of gravity:

$$\sigma_{bc} = 1.1yR_{bt}\left(x-h/2\right)/\left(h-x\right).$$

Due to the curvilinear nature of the curve, stress within the limits of the tension flange changes but little, and can be accepted as equal to the boundary value:

$$\sigma_{bct} = \sigma_m.$$

Due to the concrete pre-stress in the reinforcement there appear compressing stresses

$$\Delta\sigma_s = E_s\varepsilon_s = E_s\varepsilon_{b,s} = \alpha\sigma_{b,s},$$

where $\varepsilon_{b,s}$, $\sigma_{b,s}$ are deformation and stress in concrete at the level of the reinforcement center; $\alpha = E_s / E_b$ is the ratio of the elasticity modules of steel and concrete.

Taking into account this, we will determine stress in the reinforcement of the tension and compressed zones after the concrete pre-stressing

$$\sigma_s = \sigma_{sp} + \alpha\sigma_m, \;\; \sigma_s' = \sigma_{sp}' - 1.1y\alpha R_{bt}\left(x-a'\right)/\left(h-x\right), \quad (3)$$

where σ_{sp}, σ_{sp}' are pre-stresses in the reinforcement before pre-stress.

The height of the tension zone is designated as $z_p = h-x$, and the deformation of the layer at the distance of z from the zero line (Fig. 1,*b*) will be

$$\varepsilon = \varepsilon_m z / z_p.$$

Then the deformation diagram can be re-written as follows

$$\sigma = 1.1R_{bt}\left[1-\exp\left(-yz/z_p\right)\right].$$

Let us find the internal forces resultant in the tension zone of concrete (Fig. 1,*c*).

$$N_p = b\int_0^{z_p}\sigma dz = 1.1R_{bt}b\int_0^{z_p}\left[1-\exp\left(-yz/z_p\right)\right]dz.$$

After integrating we will obtain

$$N_p = 1.1bR_{bt}\left(h-x\right)\left(1-c/y\right). \quad (4)$$

Let us determine the moment of these forces relative to the zero line (Fig. 1, *c*)

$$M_{po} = b\int_0^{z_p}\sigma z dz = 1.1R_{bt}b\int_0^{z_p} z\left[1-\exp\left(-yz/z_p\right)\right]dz.$$

After integrating we will obtain

$$M_{po} = 1.1R_{bt}bz_p^2\left[0.5+(1-c)(1+y)/y^2-y^{-2}\right]. \tag{5}$$

The internal forces in the cross section from the preliminary reinforcement tension are self-balanced. Let us write the equation of the forces equilibrium in the cross section (Fig. 1, *c*):

$$N_p + A_{ct}\sigma_{bct} + \sigma_s A_s + 0.5\sigma_b bx - A_c\sigma_{bc} + \sigma_s^{'} A_s^{'} = 0.$$

By substituting here the determined above stresses and force (4), we will obtain

$$1.1R_{bt}\left\{b(h-x)(1-c/y)+A_{ct}c+\alpha A_s c - y\frac{bx^2/2+A_c(x-h_c/2)+\alpha A_s^{'}(x-a^{'})}{h-x}\right\}+$$

$$+\sigma_{sp}A_s+\sigma_{sp}^{'}A_s^{'}=0.$$

Let us write the sum of the internal forces moments relative to the zero line:

$$M_{po}+A_{ct}\sigma_{bct}\left(h-x-h_{ct}/2\right)+\sigma_s A_s\left(h-a-x\right)+$$

$$+\sigma_b bx^2/3+A_c\sigma_{bc}\left(x-h_c/2\right)-\sigma_s^{'}A_s^{'}\left(x-a^{'}\right)=0.$$

By substituting here new values of stresses, taking into account (5) we will obtain

$$1.1R_{bt}\begin{Bmatrix} b(h-x)^2\left[0.5+(1-c/y)-\dfrac{c}{y^2}\right]+A_{ct}c\left(h-x-h_{ct}/2\right)+ \\ +c\alpha A_s\left(h-a-x\right)+y\dfrac{bx^3/3+A_c(x-h_c/2)^2+\alpha A_s^{'}(x-a^{'})^2}{h-x}\end{Bmatrix}+$$

$$+\sigma_{sp}A_s\left(h-a-x\right)+\sigma_{sp}^{'}A_s^{'}(x-a^{'})=0.$$

Let us introduce dimensionless parameters:

$$\xi = x/h,\;\; a_0 = a/h,\;\; a_0^{'} = a^{'}/h,\;\; m_{ct} = A_{ct}/bh,$$

$$m_c = A_c/bh,\quad \mu = A_s/bh,\quad \mu' = A_s'/bh$$

and re-write these equations in the dimensionless form

$$\begin{gathered}(1-\xi)\left(1-\frac{c}{y}\right)+c\left(m_{ct}+\alpha\mu\right)-y\frac{\xi^2/2+m_c\left(\xi-h_c/2h\right)+\alpha\mu^{'}\left(\xi-a_0^{'}\right)}{1-\xi}+ \\ +\frac{\mu\sigma_{sp}+\mu^{'}\sigma_{sp}^{'}}{1.1R_{bt}}=0,\end{gathered} \tag{6}$$

$$\begin{gathered}(1-\xi)^2\left[0.5+\left(\frac{1-c}{y}\right)-\frac{c}{y^2}\right]+cm_{ct}\left(1-\xi-\frac{h_{ct}}{2h}\right)+c\alpha\mu\left(1-\xi-a_0\right)+ \\ +y\frac{\xi^3/3+m_c\left(\xi-h_c/2h\right)^2+\alpha\mu^{'}\left(\xi-a_0^{'}\right)^2}{1-\xi}+\frac{\mu\sigma_{sp}\left(1-\xi-a_0\right)-\mu^{'}\sigma_{sp}^{'}\left(\xi-a_0^{'}\right)}{1.1R_{bt}}=0.\end{gathered} \tag{7}$$

We have obtained the system of two transcendent equations relative to two unknown values $\xi,\; y\,(c)$. The solution of the system gives the possibility to determine the height of the compressed zone $x=\xi h$ and the maximum tension stress in concrete $\sigma_m = 1.1R_{bt}c$. Through these parameters there is determined the maximum compressing stress in concrete by formula (2) and stresses in the reinforcement by formulas (3).

Equations (6) and (7) are common for all the reinforced elements with pre-stress and without it, with different forms of cross section: I-shaped, T-shaped, rectangular. For the T-shaped cross section A_c or A_{ct}

is equal to zero. For the rectangular cross section the area of both overhangs are equal to zero. So, for the rectangular cross section with stressed reinforcement in the compressed zone we will obtain

$$(1-\xi)\left(1-\frac{c}{y}\right)-y\frac{\xi^2/2+\alpha\mu'\left(\xi-a_0'\right)}{1-\xi}+\frac{\mu'\sigma_{sp}'}{1.1R_{bt}}=0,$$

$$(1-\xi)^2\left[0.5+\left(\frac{1-c}{y}\right)-\frac{c}{y^2}\right]+y\frac{\xi^3/3+\alpha\mu'\left(\xi-a_0'\right)^2}{1-\xi}-\frac{\mu'\sigma_{sp}'\left(\xi-a_0'\right)}{1.1R_{bt}}=0.$$

There are a lot of ways to solve the system of nonlinear algebraic equations and the corresponding algorithms and programs for their implementation. In our case of two variables, the solution can be found graphically: to construct curves $\phi_1(\xi,y)=0$ and $\phi_2(\xi,y)=0$ on the plane $\xi,\ y$ and to find their intersection points.

This method of calculation is valid till there is no crack in the section of the element. Therefore, we find the pre-stressing value in the reinforcement at which a crack appears. A crack is formed when the maximum tensile stress in concrete reaches the tensile strength, i.e. $\sigma_m = R_{bt}$.

According to deformation diagram (1) we find the ultimate tensile deformation of concrete

$$\varepsilon_{ubt}=2.667R_{bt}/E_b. \tag{8}$$

Using formulas (2) and (9) we will find stress at the boundary of the compressed zone

$$\sigma_b=2.667R_{bt}x/(h-x).$$

Mean compression stresses on the flanges will be equal

$$\sigma_{bc}=2.667R_{bt}\left(x-h_c/2\right)/(h-x),\ \ \sigma_{bct}=R_{bt}.$$

Taking into account the limiting deformation (8) we will write stresses in the reinforcement

$$\sigma_s=\sigma_{sp}+2.667\alpha R_{bt},\ \ \sigma_s'=\sigma_{sp}'-2.667\alpha R_{bt}(x-a')/(h-x).$$

Similarly to the above-described distribution of stress on the height of the tension zone, we can write the following

$$\sigma=1.1R_{bt}\left[1-\exp\left(-2.4z/z_p\right)\right].$$

Then the internal forces resultant in the tension zone and their moment relative to the zero line can be determined by formulas (4) and (5) accepting $y=2.4,\quad c=0.909.$ As a result we will obtain:

$$N_p=0.683bR_{bt}\left(h-x\right),\ \ N_{po}=0.418R_{bt}b\left(h-x\right)^2. \tag{9}$$

Let us write down the equation of the internal forces equilibrium in the cross section and the sum of their moments relative to the zero line (Fig. 1).

$$N_p+A_{ct}R_{bt}+N_a-0.5\sigma_b xb-A_c\sigma_{bc}+N_a'=0,$$

$$M_{po}+A_{ct}R_{bt}\left(h-x-h_{ct}/2\right)+N_a\left(h-x-a\right)+\sigma_b bx^2/3+A_c\sigma_{bc}\left(x-h_c/2\right)-N_a'\left(x-a'\right)=0.$$

By substituting here the above-determined stresses, forces and the moment, we will write down

$$R_{bt}\left\{\begin{array}{l}0.683b\left(h-x\right)+A_{ct}+2.667\alpha A_s-\\ -\dfrac{1.333bx^2+2.667A_c\left(x-h_c/2\right)+2.667\alpha A_s'\left(x-a'\right)}{h-x}\end{array}\right\}+ \tag{10}$$
$$+\sigma_{sp}A_s+\sigma_{sp}'A_s'=0,$$

$$R_{bt}\left\{\begin{array}{l}0.4188b(h-x)^2+A_{ct}(h-x-h_{ct}/2)+2.667\alpha A_s(h-x-a)+\\+\dfrac{0.889bx^3+2.667A_c(x-h_c/2)^2+2.667A_s^{'}\left(x-a^{'}\right)^2}{h-x}\end{array}\right\}+ \quad (11)$$

$$+\sigma_{sp}A_s(h-x-a)-\sigma_{sp}^{'}A_s^{'}\left(x-a^{'}\right)=0.$$

These equations allow determining the limiting value of the pre-stress. Usually pre-stressed reinforcement is placed only in zones of tensile stresses under operational loads. Then $\sigma_{sp}^{'}=0$ and from equation (10) we can express the force $\sigma_{sp}A_{sp}$ in terms of x. Substituting this expression into (11), we obtain a cubic equation for determining the height of the compressed zone before the formation of a crack. Further, from (10), when x is known, the limiting pre-stressing is determined.

For the rectangular section with pre-stressed reinforcement in the compressed zone, the resolving equations are written as follows:

$$R_{bt}\left\{0.683b(h-x)-\left[1.333bx^2+2.667\alpha A_s^{'}\left(x-a^{'}\right)\right]\Big/(h-x)\right\}+\sigma_{sp}^{'}A_s^{'}=0,$$

$$R_{bt}\left\{0.4188b(h-x)^2+\left[0.889bx^3+2.667\alpha A_s^{'}\left(x-a^{'}\right)^2\right]\Big/(h-x)\right\}\sigma_{sp}^{'}A_s^{'}\left(x-a^{'}\right)=0.$$

From here for determining the height of the compressed zone we will obtain the following cubic equation:

$$x^3+\left(0.626h-3.63a^{'}\right)x^2+\left(3.19h^2-7.63ha^{'}\right)x+3.816h^2a^{'}-2.335h^3=0$$

2.2. Determining the ultimate moment of crack formation

Through this ultimate moment, the first category of requirements of the second group of the limit state is written. Therefore, such calculations are normative. They assume that stress in the tension zone of concrete is constant and equal to the tensile strength of concrete, R_{bt}. In this work the calculations have a real stress profile in this zone: the boundary stress is equal to R_{bt}, and in the other fibers they are determined according to the deformation diagram in accordance with the strain curve.

Let us consider the cross section of a general form, shown in Fig. 1, in which the boundary stress $\sigma_m=R_{bt}$. We consider fair the assumptions made in determining pre-stressing stress. Then all the arguments and calculations given in determination the maximum pre-stress in the reinforcement remain valid.

In contrast to the case considered here, an external load is added: the bending moment or an eccentrically applied force. If earlier the internal forces were self-balanced, here they are balanced by external load.

Therefore, an external force P is added to equilibrium equation (10):

$$R_{bt}\left\{\begin{array}{l}0.683b(h-x)+A_{ct}+2.667\alpha A_s-\\-\dfrac{1.333bx^2+2.667A_c(x-h_c/2)+2.667\alpha A_s^{'}\left(x-a^{'}\right)}{h-x}\end{array}\right\}+ \quad (12)$$

$$+\sigma_{sp}A_s+\sigma_{sp}^{'}A_s^{'}\pm P=0.$$

Here the force P is taken with the plus sign under the eccentric compression and the minus sign under the eccentric tension and during bending it is zero.

The sum of the moments of internal forces (11) is now equal to the external moment and determines the ultimate moment of the crack formation. The external moment is convenient to be determined relative to the center of gravity of the section. With respect to this axis, the moment of internal forces will be equal to:

$$M_m = R_{bt}\left\{\begin{aligned} & b(h-x)(0.076h+0.265x)+A_{ct}\left(h-h_{ct}/2\right)+2.667\alpha A_s(h/2-a)+ \\ & +\frac{0.222bx^2(3h-2x)+1.333A_c(x-h_c)(x-h_c/2)+2.667\alpha A_s^{'}\left(x-a^{'}\right)\left(h/2-a^{'}\right)}{h-x}+ \\ & +\frac{\sigma_{sp}A_s(h/2-a)-\sigma_{sp}^{'}A_s^{'}\left(h/2-a^{'}\right)}{R_{bt}} \end{aligned}\right\}= \tag{13}$$

$$=W_{pe}R_{bt},$$

where W_{pe} is the elastic-plastic moment of resistance of the pre-stressed section.

To determine the limiting moment from (12) by solving the quadratic equation there is determined the height of the compressed zone x. Substituting it into expression (13) we find the ultimate moment of the crack formation.

Equations (12) and (13) are universal: they are applicable for various forms of sections for different types of reinforcement. For special cases they are simplified. So, for a bending rectangular section with pre-stressed reinforcement in the tension zone, we obtain

$$0.683b(h-x)^2+2.667\alpha A_s(h-x)-1.333bx^2+\sigma_{sp}A_s(h-x)/R_{bt}=0,$$

$$M_m = R_{bt}\left\{b(h-x)(0.076h+0.265x)+2.667\alpha A_s\left(\frac{h}{2}-a\right)+\frac{0.222bx^2(3h-2x)}{h-x}+\sigma_{sp}A_s\left(\frac{h}{2}-a\right)\right\}.$$

Introducing dimensionless parameters:

$$\xi = x/h,\ \mu = A_s/bh,\ a_0 = a/h,\ S=\mu\sigma_{sp}/R_{bt} \tag{14}$$

We will re-write these equations in the following form:

$$\xi^2+2(1.05+2.05\alpha\mu+S/1.3)\xi-1.05-2.05\alpha\mu-1.54S=0,$$

$$\frac{M_t}{bh^2R_{bt}}=(1-\xi)(0.076+0.265\xi)+2.667\alpha\mu(0.5-a_0)+\frac{0.222\xi^2(3-2\xi)}{1-\xi}+S(0.5-a_0).$$

For non-reinforced rectangular section according to these equations, we obtain $\xi = 0.417$, $W_{pe}=0.252bh^2$, that coincides with the known formula of material resistance $W_{pe}=bh^2/4$.

2.3. Designing the stress state of a section with a crack

The first cracks along the length of the element appear in the most loaded section or due to the non-uniform strength of concrete in the weakest section. With the distance from the edges of the crack, due to adhesion with concrete, stress in the reinforcement decreases, and tensile stresses in concrete increase and where they reach the limit value, an adjacent crack appears located at the h_m distance from the first one. In the section with a crack the adhesion of the reinforcement to the concrete is broken, the tensile forces are generally taken up by the reinforcement. In the sections between the cracks both reinforcement and concrete take up tensile forces. Deformations and stresses in the reinforcement and concrete, as well as the height of the compressed zone between cracks vary. As it was noted above, for reinforced concrete structures without pre-stressing, the diagram-energy calculation method has been developed, but for pre-stressed reinforced concrete we will use below the classical formulation, when this unevenness in the calculations is taken into account by introducing special coefficients that are equal to the ratios of average values in the section between the cracks and values in the section with a crack.

$$\psi_s=\varepsilon_{sm}/\varepsilon_s,\ \psi_b=\varepsilon_{bm}/\varepsilon_b.$$

All the existing analytical methods for calculating stress in sections with a crack suggest the known crack length. It is believed that it exists initially, regardless of the actual loads, and its length is determined by measurement. The actual crack length depends on the applied load. Therefore, we must be able to predict it.

The proposed designing method is based on the following assumptions:

the middle sections located between the cracks remain flat after bending;

in the part of the tension zone of concrete, where stresses have reached the tensile strength, a crack is formed and it does not take the loads;

in concrete of the tension zone over a crack the relationship between stress and strain is described by exponential law (1);

the concrete of the compressed zone does not work elastically, the stress diagram is replaced by a rectangle here taking into account the coefficient of completeness of the curve ω**.**

Let us consider the section of a general form shown in Fig. 2,*a*. The deformation curve in the middle section and the stress curve in the section with a crack are shown in Fig. 2,*b* and 2,*c*.

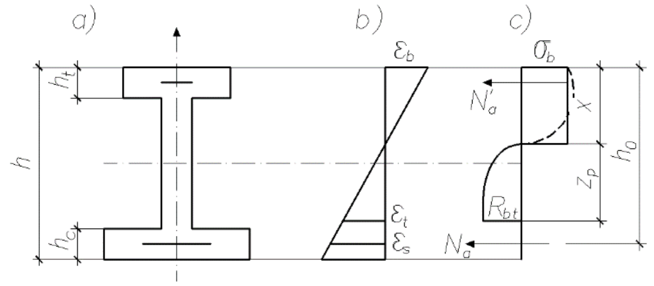

Figure 2. Towards designing a section with a crack.

By the deformation diagram, the strain at the tip of a crack will be equal

$$\varepsilon_t = 2.667 R_{bt} / E_b.$$

The strain at the boundary of the compressed zone is

$$\varepsilon_b = \varepsilon_t x / z_p = 2.66 \left(R_{bt} / E_b \right) x / z_p ,$$

where $z_p = h - x - l_m$ is the height of the tension zone of concrete above the crack; l_m is the crack length.

Stresses at the boundary of the compressed zone will be

$$\sigma_b = E_b^{'} \varepsilon_b = 2.667 R_{bt} \nu x / z_p, \quad \nu = E_b' / E_b.$$

Stresses on the reinforcement of the compressed zone

$$\sigma_s^{'} = \sigma_{sp}^{'} - \varepsilon_t E_s \left(x - a^{'} \right) / z_p = \sigma_{sp}^{'} - 2.667 \alpha R_{bt} \left(x - a^{'} \right) / z_p .$$

Let us find stress in the reinforcement of the tension zone. In the middle section deformations at the level of the crack and the reinforcement are equal

$$\varepsilon_{tm} = \psi_b \varepsilon_t, \;\; \varepsilon_{sm} = \varepsilon_{tm} (h_0 - x) / z_p, \;\; h_0 = h - a.$$

Deformation at the level of the reinforcement in the section with a crack

$$\varepsilon_s = \varepsilon_{sm} / \psi_s = \varepsilon_t \psi_b / \psi_s \cdot \left(h_0 - x \right) / z_p .$$

Then stress in the reinforcement is equal

$$\sigma_s = E_s \varepsilon_s = 2.667 \alpha R_{bt} \left(\psi_b / \psi_s \right) \left(h_0 - x \right) / z_p . \tag{15}$$

The internal forces resultant in the tension zone and their moment relative to the zero line are determined by formulas (9) with substituting $(h - x)$ by z_p. These forces moment relative to the concrete center of gravity will be

$$M_{pc} = M_{po} - N \left(h/2 - x \right) = R_{bt} b z_p \left[0.418 z_p - 0.683 \left(h/2 - x \right) \right].$$

Let us write down the equation of the forces equilibrium in the section with a crack

$$\sigma_s A_s + N_p + \sigma'_{sp} A'_s - 2.667 \alpha R_{bt} A'_s \left(x - a'\right) / z_p - \omega \sigma_b \left(bx + A_c\right) \pm P = 0.$$

Substituting here the above-determined forces and stresses, after transformations we will write

$$0.256 b z_p^2 + \alpha A_s \frac{\psi_b}{\psi_s}\left(h_0 - x\right) + \frac{0.375 z_p \sigma'_{sp} A'_s}{R_{bt}} - \alpha A'_s \left(x - a'\right) - \\ -\nu \omega x \left(bx + A_c\right) \pm \frac{P z_p}{2.667 R_{bt}} = 0 \tag{16}$$

In this equation the product $\omega \nu$ weakly depends on the shape of the normal stress field of the compressed zone. For example, with a rectangular curve $\omega = 1,$ the coefficient ν can be taken 0.5 in connection with the appearance of significant inelastic deformations; consequently, $\omega \nu = 0.5.$ With a triangular curve (elastic deformations) $\omega = 0.5,$ and $\nu = 1;$ therefore, in this case $\omega \nu = 0.5.$ The real curvilinear diagram in this zone is replaced by a rectangular diagram for convenience of calculation.

We will write the sum of moments of all forces relative to the center of gravity of the section with a crack.

$$M_{pc} + \sigma_s A_s \left(\frac{h}{2} - a\right) + \sigma_b \omega b x \frac{h - x}{2} + \sigma_b \omega A_c \frac{h - h_c}{2} + 2.667 \alpha R_{bt} A'_s \frac{\left(x - a'\right)\left(\frac{h}{2} - a'\right)}{z_p} - \\ -\sigma'_{sp} A'_s \left(\frac{h}{2} - a'\right) = M_{bn}.$$

Since stress σ_s in the reinforcement of the tension zone occurs after the moment of the external forces M exceeds the moment of the pre-stress force M_{pr}, in this equation the total external moment is

$$M_{bn} = M - M_{pr} = M - \sigma_{sp} A_s \left(h/2 - a\right),$$

where the bending moment M is taken from the external forces relative to the concrete center of gravity.

By substituting in the equation of the moment the above-determined stresses, after transformations we will obtain

$$0.157 b z_p^3 - 0.256 b z_p^2 \left(h/2 - x\right) + \alpha A_s \left(\psi_b / \psi_s\right)\left(h_0 - x\right)\left(h/2 - a\right) + \\ + \nu \omega x \left[bx\left(h - x\right) + A_s \left(h - h_c\right)\right] / 2 + \\ + \alpha A'_s \left(x - a'\right)\left(h/2 - a'\right) - 0.375 z_p \left[\sigma'_{sp} A'_s \left(h/2 - a'\right) + M_{bn}\right] / R_{bt} = 0. \tag{17}$$

Equations (16) and (17) form a system of two nonlinear algebraic equations relative to two unknown values $(x,\ z_p).$ After solving this system, stress in the reinforcement is determined by expression (15). The boundary stress in the compressed zone concrete is determined by the deformation diagram by substituting deformation ε_b :

$$\sigma_b = 1.1 R_{bn} \left[1 - \exp\left(-2.4 x R_{bt} / R_{bn} z_p\right)\right].$$

The crack length is determined by the formula

$$l_m = h - x - z_p.$$

To make the calculations it is necessary to determine preliminarily the irregularity ratios. The norms [2, 3] recommend $\psi_b = 0.9,$ a

$$\psi_s = 1.25 - \varphi_{es} \varphi_m - \frac{1 - \varphi_m^2}{\left(3.5 - 1.8 \varphi_m\right) M / N_t e_{op}}, \tag{18}$$

where the total axial force

$$N_t = \pm P + P_0;$$

$P_0 = \sigma_{sp} A_s$ is the force of pre-stress; e_{op} is the eccentricity of the total force relative to the concrete center of gravity.

The φ_{es} coefficient characterizes the duration of the load action and the profile of the reinforcement bars. It is selected in the following way [3]: under short-term load for corrugated bars $\varphi_{es} = 1.1$; for plain bars and wire binders $\varphi_{es} = 1$; under long-term load independent on the reinforcement profile $\varphi_{es} = 0.8$.

The coefficient

$$\varphi_m = M_{b,m} / M_{bn} \le 1,$$

where $M_{b,m}$ is the moment taken up by the concrete non-reinforced section prior to the crack formation determined from equations (12) and (13). For the rectangular section $\xi = 0.417$, and $M_{b,m} = 0.252bh^2 R_{bt}$.

For binding elements without pre-stress formula (18) is simplified

$$\psi_s = 1.25 - \varphi_{es}\varphi_m.$$

The norms also allow using the simplified expression [2, 9]:

$$\psi_s = 1 - 0.85\varphi_m.$$

Let us write down resolving equations (16) and (17) for the rectangular section with the unilateral reinforcement in the tension zone with bending

$$0.256\lambda^2 - 0.5\xi + \alpha\mu\left(\bar{h} - \xi\right)\psi_b / \psi_s = 0, \quad (19)$$

$$0.157\lambda^3 - 0.256\lambda^2\left(0.5 - \xi\right) + \alpha\mu\left(\bar{h} - \xi\right)\left(\bar{h} - 0.5\right)\psi_b / \psi_s +$$
$$+\xi^2\left(1 - \xi\right)/4 - 0.357\lambda M_{bn} / R_{bt}bh^2 = 0.$$

Here there are used dimensionless parameters (14) and added

$$\lambda = z_p / h, \quad \bar{h} = h_0 / h.$$

From the first equation of this system

$$\lambda = \sqrt{1.95\xi^2 - 3.9\alpha\mu\left(\bar{h} - \xi\right)\psi_b / \psi_s}. \quad (a)$$

The integrand must be larger than zero:

$$\xi \ge -\alpha\mu\psi_b/\psi_s + \sqrt{\left(\alpha\mu\psi_b/\psi_s\right)^2 + 2\alpha\mu\bar{h}\psi_b/\psi_s}. \quad (b)$$

From the second equation of the system we will write

$$y = \frac{M_{bn}}{R_{bt}bh^2} = 0.418\lambda^2 - 0.683\lambda\left(0.5 - \xi\right) +$$
$$+\frac{2.667\alpha\mu\left(\bar{h} - \xi\right)\left(\bar{h} - 0.5\right)\psi_b/\psi_s + 0.667\xi^2\left(1 - \xi\right)}{\lambda}. \quad (c)$$

The system of equations (a), (c) is convenient to be solved graphically. We will take ξ according condition (b), according to (a) we determine λ; then we find y from expression (c). Repeating these procedures, we build the function graph $y = f\left(\xi\right)$. The graph crossing with horizontal $y = M_{bn} / R_{bt}bh^2$

gives the ξ value that satisfies the system being solved. Then from (a) we obtain λ. The further determining of the stress state parameters is carried out according to the above-described methodology.

3. Results and Discussion

At present pre-stresses are determined by the elastic model for the transformed concrete section. Replacing the real section with the transformed one is, in our opinion, not entirely justified. The modeling of pre-stressing using the finite element method shows that the linear relationship between stress and strain is confirmed only for the compressed zone of concrete; in the tension zone the stress profile is not linear.

To estimate the error of the existing designing method, a calculation was performed using the LIRA program for the rectangular cross section with dimensions of $b = 15.3\,cm$, $h = 30.1cm$ with pre-stressed single reinforcement in the compressed zone: $A_s^{'} = 4\,cm^2,\ \ a^{'} = 2.8\,cm,\ \ \sigma'_{sp} = 112\,MPa.$ The material characteristics are as follows: B40 class concrete, grade TB ($R_{bt} = 2.1\,MPa$, $E_b = 3.2 \cdot 10^4\,MPa$); $A-III$ class reinforcement ($R_s = 370MPa$, $E_s = 2.1 \cdot 10^5\,MPa$). The difference between the numerical results and the results of the linear calculation was 37 % for tensile stresses and 7.5 % for compressive stresses. This was the basis for developing a nonlinear method for calculating pre-stress, which reduced the indicated error to 5 % for tensile stresses and to 2.8 % for compressive stresses.

In addition, of independent significance is determining pre-stress in the formation of a crack σ_{pr}. This allows reasonable selecting the value of the controlled pre-stressing of the reinforcement σ_{con} in manufacturing pre-stressed concrete structures. At present the norms recommend that $\sigma_{con} \le 0.8R_s$. We believe that this stress should be limited to 80 % of the stress in the formation of cracks ($\sigma_{con} \le 0.8\sigma_{pr}$).

Determining the moment for the crack formation refers to regulatory calculations. In the normative model it is assumed that in the tension zone of concrete, stress is constant and equal to the tensile strength R_{bt}. But logic dictates that in order for the crack appearance it is enough for the maximum tensile stress to reach the limit value. It follows that the normative calculations give overestimated values of the moment for the crack formation. These arguments confirmed the numerical calculations by the finite element method. Therefore, the problem of determining the moment of the crack formation was set and solved on the basis of the actual strain diagram of the tension zone of concrete. According to the developed method, the moment of the crack formation is much smaller than that in the existing method. So, for the above section with non-stressed reinforcement by our method it has been obtained

$$\xi = 0.53,\ \ M_m = 0.318 R_{bt} b h^2 = 0.318 \cdot 2100 \cdot 0.153 \cdot 0.301^2 = 9.24\,kNm.$$

According to the existing methodology

$$\xi = 0.526,\ \ M_m = 0.68 R_{bt} b h^2 = 19.4\,kNm.$$

To check the proposed model, a nonlinear calculation of the stress state of the reinforced concrete element considered above for pure bending was performed using the finite element method using the ANSYS program. The bending moment was increased with a certain step and the appearance of a crack controlled. It appeared at $M = 10.8\,kNm,$ which exceeds the moment calculated by us by 8.3 %, and the position of the zero line almost coincides. Taking this into account, the proposed model should be recognized as adequate and suitable for engineering calculations.

An important result of the work is the developed analytical method for designing the stress state in the cross section with a crack, which makes it possible to determine the crack length. According to this method, the stress state of the previously considered rectangular section without pre-stressing from the bending moment $M = 18\,kNm$ has been calculated and there has been obtained

$$\sigma_s = 189.8\,MPa,\ \ \sigma_b = 9.2\,MPa,\ \ l_m = 15.8\,cm.$$

To assess the accuracy of the method this problem was solved by the numerical method using the numerical method using the ANSYS program. There were obtained the following results (in brackets there is indicated % of the results discrepancy):

$\sigma_s = 184.1\, MPa$ (3 %), $\sigma_b = 9.2\, MPa$ (5.4 %), $l_m = 16.5\, cm$ (4.2 %).

Comparing the calculation results indicates good accuracy of the proposed method of designing the stress state of reinforced concrete beams with cracks.

Resolution equation (19) gives the relationship between the dimensionless parameters of the compression zone height and the length of the crack. A plot of the experimental relationship between these parameters is given in [1] (Fig. 8.11). For comparison, we built this graph according to equation (19) with $\alpha\mu = 0.038$, $\psi_b/\psi_s = 1.2$, $\bar{h} = 0.907$. In Fig. 3 it is shown by the solid line. In the same place, a dashed line shows this dependence obtained by the finite element method. In addition, the experimental points from the aforementioned graph are plotted with a cross (the other points are absent). The maximum deviation of our results from the numerical experiment is 9.1 %, and from the full-scale experiment 9.2 %. This discrepancy in results can be considered satisfactory.

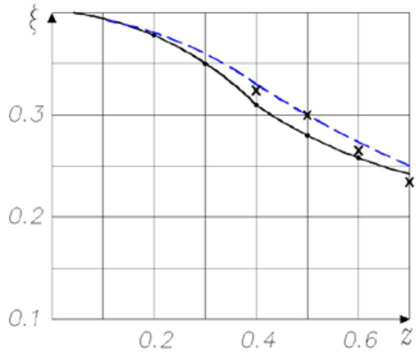

Figure 3. Compression zone height dependence on the crack length.

In [27] experimental values of the maximum compressive deformations in a bent reinforced concrete beam of rectangular cross section under various external loads are given. For comparison with the experimental value, we performed a calculation with the force F = 400 kg using the described methodology. In this case

$$y = \frac{Fl}{4R_{bt}bh^2} = \frac{400 \cdot 9.8 \cdot 0.9}{4 \cdot 1.4 \cdot 10^6 (7 \cdot 14^2) 10^{-6}} = 0.459;$$

$$\alpha\mu\psi_b / \psi_s = (20/2.05)0.005 \cdot 0.9 / 0.68 = 0.065; \quad \bar{h} = 0.9.$$

Solution of system (19) gives $\xi = 0.307$, $\lambda = 0.182$. The maximum strain in concrete

$$\varepsilon_b = 2.667(R_{bt}/E_b)\xi/\lambda = 2.667(1.4/20500)0.307/0.182 = 3.07 \cdot 10^{-4}.$$

The strain experimental value is 2.96×10^{-4}. The results discrepancy makes 3.6 %.

This calculation method allows determining all the parameters of the stress state in the section with a crack. These parameters can be used to determine the step and width of the crack opening (criterial parameter) according to the normative method [1–3].

4. Conclusion

The paper proposes an analytical method for determining stresses in reinforced concrete elements when mounting reinforcement with preliminary stress. From the obtained dependences, the value of the ultimate stress of the reinforcement is determined at which a crack appears in the element. This allows reasonable selecting the value of the controlled pre-stressing of the reinforcement.

In the proposed method of assessing the bearing capacity of bent reinforced concrete beams determining the stress-strain state of these beams taking into account operational cracks is of great importance. The proposed method of calculating reinforced concrete beams with a crack, in contrast to the existing analytical calculation methods, allows determining the length of a crack in a section. This is very important for determining the parameters of fracture mechanics in the future and assessing on this basis the crack resistance of the structures by the force criterion.

Based on the results of the work, the following conclusions can be made.

1. A methodology for nonlinear analytical calculation of the stress state of reinforced concrete elements with pre-stressing of reinforcement has been developed.

2. The limiting value of the preliminary stress of the reinforcement is determined, at which a crack appears at the manufacturing stage.

3. A new technique is proposed for determining the external moment from crack formation.

4. A new analytical method has been developed for calculating bending reinforced concrete beams with a crack, which allows determining all the parameters of the stress state: maximum compressive stress in concrete, stresses in reinforcing bars, crack length and height of the compression zone.

The method is qualitatively confirmed by comparing with experimental data of determining the relationship between the crack length and the height of the compression zone [1] and determining the maximum compressive deformations in a bending reinforced concrete beam with a crack [27]. In addition, the proposed analytical method agrees well quantitatively with numerical calculations by the finite element method. The maximum deviation in the calculations to determine pre-stresses in rectangular elements and the stress state parameters in cross sections with a crack did not exceed 5.4 %.

The calculation method is valid for bending reinforced concrete beams of arbitrary section with working rod reinforcement.

References

1. Zaytsev, Yu.V. Mekhanika razrusheniya dlya stroiteley [Destruction mechanics for builders]. Moscow: Higher school, 1991. 288 p. (rus)
2. Nuguzhinov, Zh.S. Teoriya rascheta zhelezobetonnykh konstruktsiy s povrezhdeniyami [Theory of calculating reinforced concrete structures with damage]. Karaganda: KSTU Publishing house, 2012. 203 p. (rus)
3. Baykov, V.N., Sigalov, E.Ye. Zhelezobetonnyye konstruktsii: Obshchiy kurs [Reinforced concrete structures: General course]. Moscow: Stroyizdat, 1991. 767 p. (rus)
4. Travush, V.I., Konin, D.V., Krylov, A.S. Strength of reinforced concrete beams of high-performance concrete and fiber reinforced concrete. Magazine of Civil Engineering. 2018. 77(1). Pp. 90−100. DOI: 10.18720/MCE.77.8
5. Desnerck, P., Lees, J., Morley, C. Bond behaviour of reinforcing bars in cracked concrete. Construction and Building Materials. 2015. Vol. 94. Pp. 126−136. DOI: 10.1016/j.conbuildmat.2015.06.043
6. Chernin, L., Val, D.V. Prediction of corrosion-induced cover cracking in reinforced concrete structures. Construction and Building Materials. 2011. 25 (4). 1854−1869. DOI: 10.1016/j.conbuildmat.2010.11.074
7. Kolchunov, V.I., Dem'yanov, A.I. The modeling method of discrete cracks in reinforced concrete under the torsion with bending. Magazine of Civil Engineering. 2018. 81 (5). Pp. 160–173. DOI: 10.18720/MCE.81.16
8. Kwan, A.K.H., Ma, F.J. Crack width analysis of reinforced concrete under direct tension by finite element method and crack queuing algorithm. Engineering Structures. 2016. Vol. 126. Pp. 618−627. DOI: 10.1016/j.engstruct.2016.08.027
9. Ma, F.J., Kwan, A.K.H. Crack width analysis of reinforced concrete members under flexure by finite element method and crack queuing algorithm. Engineering Structures. 2015. Vol. 105. Pp. 209−219. DOI: 10.1016/j.engstruct.2015.10.012
10. Ng, P, Ma, F, Kwan, A. Crack analysis of reinforced concrete members with and without crack queuing algorithm. Structural Engineering and Mechanics. 2019. 70 (1). Pp. 43−54. DOI: 10.12989/sem.2019.70.1.043
11. Oliver-Leblond, C., Delaplace, A., Ragueneau, F. Modelling of three-dimensional crack patterns in deep reinforced concrete structures. Engineering Structures. 2015. Vol. 83. Pp. 176−186. DOI: 10.1016/j.engstruct.2014.10.040
12. Tan, R., Eileraas, K., Whispers, O., Zirgulis, G., Hendriks, M.A.N., Geiker, M., Brekke, D.-E., Kanstad, T. Experimental and theoretical investigation of crack width calculation methods for RC ties. Structural Concrete. 2018. 19 (5). Pp. 1436−1447. DOI: 10.1002/suco.201700237
13. Yang, S.T., Li, K.F., Li, C.Q. Numerical determination of concrete crack width for corrosion-affected concrete structures. Computers & Structures. 2018. Vol. 207. Pp. 75−82. DOI: 10.1016/j.compstruc.2017.07.016
14. Amin, A., Gilbert, R.I. Instantaneous Crack Width Calculation for Steel Fiber-Reinforced Concrete Flexural Members. ACI Structural Journal. 2018. 115 (2). Pp. 535−543. DOI: 10.14359/51701116
15. Bashirov, Kh.Z., Chernov, K.M. Razrusheniye zhelezobetonnykh sostavnykh konstruktsiy ot poteri stsepleniya v zone zaankerivaniya po naklonnym treshchinam tretyego tipa [Tangential section failure of anchorage zone in composite reinforced concrete elements]. Stroitelnaya mekhanika i raschet sooruzheniy. 2013. 1 (246). Pp. 02−06.
16. Travush, V.I., Krylov, S.B., Konin, D.V., Krylov, A.S. Ultimate state of the support zone of reinforced concrete beams. Magazine of Civil Engineering. 2018. 83 (7). Pp. 165−174. DOI: 10.18720/MCE.83.15
17. Hua, Zhu. Crack formation of steel reinforced concrete structure under stress in construction period. Frattura ed Integrita Strutturale. 2016. No. 36. Rp. 191−200. DOI: 10.3221/IGF-ESIS.36.19
18. Salnikov, A., Kolchunov, Vl., Yakovenko, I. The computational model of spatial formation of cracks in reinforced concrete constructions in torsion with bending. Applied Mechanics and Materials. 2015 Vol. 725-726. Pp. 784–789. DOI.org/10.4028/www.scientific.net/AMM.725-726.784
19. Dem'Yanov, A., Salnikov, A. The estimation of spatial cracks formation in reinforced concrete structures under the action torsion with bending. IOP Conference Series: Materials Science and Engineering. 2018. 365 (5). DOI: 10.1088/1757-899X/365/5/052019
20. Visintin, P., Sturm, A., Oehlers, D. Long- and short-term serviceability behavior of reinforced concrete beams: Mechanics models for deflections and crack widths. Structural Concrete. 2018. 19 (2). Pp. 489−507. DOI: 10.1002/suco.201700022
21. Deng, Zh., Wang, Y., Yang, H., Qian, J. Research on Crack Behavior of Recycled Concrete Beams under Short-term Loading. KSCE Journal of Civil Engineering. 2018. 22 (5). Pp. 1763−1770. DOI: 10.1007/s12205-017-0678-7
22. Shafigullin, L.N., Treschev, A.A., Telichko, V.G., Erofeev, V.T. Calculation of reinforced concrete shell of positive Gaussian curvature, given different resistantance of concrete and cracking. Astra Salvensis. 2017. Pp. 77–91.

23. Sheng, J., Yin, S.P., Xu, S.L., Jin, Z.Y. Experimental and theoretical investigations on crack spacing and stiffness of textile-reinforced concrete-strengthened reinforced concrete beams. Advances in Structural Engineering. 2018. 21 (11). Pp. 1696−1707. DOI: 10.1177/1369433218754333
24. Mitasov, V.M., Logunova, M.A., Statsenko, N.V. Novyye podkhody k resheniyu zadach deformirovaniya zhelezobetonnykh konstruktsiy s treshchinami [New approaches to the solution of the tasks of deformation of reinforced framings with cracks]. Izvestiya vuzov. Investitsii. Stroitelstvo. Nedvizhimost. 2017. Vol. 7, No. 1. Pp. 77−82.
25. Koyankin, A.A., Mitasov, V.M., Deordiev, S.V. The compatibility of deformation of the hollow-core slab with beams. Magazine of Civil Engineering. 2019. 87 (3). Pp. 93−102. DOI: 10.18720/MCE.87.8
26. Travush, V.I., Konin, D.V., Krylov, A.S. Strength of composite steel and concrete beams of high-performance concrete. Magazine of Civil Engineering. 2018. 79 (3). Pp. 36−44. DOI: 10.18720/MCE.79.4
27. Mikhaylova, N.S. Eksperimentalnyye issledovaniya zhelezobetonnykh balok bez treshchin i zaraneye namechennoy treshchiny [Experimental Investigation of Reinforced Concrete Beams without Cracks and with Previously Intended Crack]. Izvestiya vuzov. 2007. Stroitelstvo. No. 4. Pp. 110−113. (rus)
28. Shardakov, I., Bykov, A., Shestakov, A. et al. Crack control in concrete using shock wave techniques. Procedia Structural Integrity. 2017. Vol. 5. Pp. 210−216. DOI: 10.1016/j.prostr.2017.07.114
29. Fantilli, A.P., Chiaia, B. Golden Ratio in the Crack Pattern of Reinforced Concrete Structures. Journal of Engineering Mechanics. 2013. Vol. 139. Pp. 1178−1184. DOI: 10.1061/(ASCE)EM.1943-7889.0000548
30. Bykov, A.A., Matveenko, V.P., Shardakov, I.N., Shestakov, A.P. Shock wave method for monitoring crack repair processes in reinforced concrete structures. Mechanics of Solids. 2017. Vol. 52. Pp. 378−383. DOI: 10.3103/S0025654417040033
31. Bílý, P., Kohoutková, A., Jedlinský, P. FEM simulation of static loading test of the Omega beam. IOP Conference Series: Materials Science and Engineering. 2017. 236 (1). Art. No. 012066, DOI: 10.1088/1757-899X/236/1/012066
32. Morozov, V.I., Pukharenko, Yu.V., Yushin, A.V. The numerical investigations of double-span concrete beams strengthened with fiber reinforced plastics across the oblique section. Materials Physics and Mechanics. 2017. 31 (1−2). Pp. 40−43.
33. Ma, F.J., Kwan, A.K.H. Finite element analysis of concrete shrinkage cracks. Advances in Structural Engineering. 2018. 21 (10). Pp. 1454−1468. DOI: 10.1177/1369433217746346
34. Rossi, P., Daviau-Desnoyers, D., Tailhan, J.L. Analysis of cracking in steel fibre-reinforced concrete (SFRC) structures in bending using probabilistic modeling. Structural Concrete. 2015. Vol. 16. Pp. 381−388. DOI: 10.1002/suco.201400081
35. Cohen, M., Monteleone, A., Potapenko, S. Finite element analysis of intermediate crack debonding in fibre reinforced polymer strengthened reinforced concrete beams. Canadian Journal of Civil Engineering. 2018. 45 (10). Pp. 840−851. DOI: 10.1139/cjce-2017-0439
36. Nagy, N., Mohamed, M., Boot, J. Nonlinear numerical modelling for the effects of surface explosions on buried reinforced concrete structures. Geomechanics and Engineering. 2010. 2 (1). Pp. 1−18. DOI: 10.12989/gae.2010.2.1.001
37. Sainov, M.P., Kotov, F.V., Nazarov, N.V. Serviceability of rockfill dam with reinforced concrete face and grout curtain. Magazine of Civil Engineering. 2018. 79 (3). Pp. 77–85. DOI: 10.18720/MCE.79.8.18720/MCE.79.8
38. Chen, G., Teng, J., Chen, J. Finite-element modeling of intermediate crack debonding in FRP-plated RC beams. Journal of Composites for Construction. 2011. 15 (3) Pp. 339−353. DOI: 10.1061/(ASCE)CC.1943-5614.0000157
39. Utkin, V.S., Solovyev, S.A. Raschet nadezhnosti zhelezobetonnoy balki po kriteriyu dliny treshchiny [Calculation of reinforced concrete beam reliability on the crack length criterion]. Stroitelnaya mekhanika inzhenernykh konstruktsiy i sooruzheniy. 2016. No. 5. Pp. 24−33.
40. Klyuyeva, N.V., Kolchunov, Vl.I., Yakovenko, I.A. Problemnyye zadachi razvitiya gipotez mekhaniki razrusheniya primenitelno k raschetu zhelezobetonnykh konstruktsiy [Problem tasks of development of hypotheses of mechanics of destruction in relation to calculation of ferroconcrete designs]. Izvestiya KGASU. 2014. No. 3 (29) Pp. 41−45. (rus)
41. Korsun, V., Vatin, N., Franchi, A., Korsun, A., Crespi, P., Mashtaler, S. The strength and strain of high-strength concrete elements with confinement and steel fiber reinforcement including the conditions of the effect of elevated temperatures. Procedia Engineering. 2015. 117 (1). Pp. 970−979. DOI: 10.1016/j.proeng.2015.08.192
42. Almazov, V.O. Proyektirovaniye zhelezobetonnykh konstruktsiy po yevronormam [Designing reinforced concrete structures according to Euro-norms]. Moscow: ACB Publ. House, 2011. 216 p. (rus)

Contacts:

Zhmagul Nuguzhinov, kazmirr@mail.ru
Nikolai Vatin, vatin@mail.ru
Zhetpisbai Bakirov, bakirov_50@mail.ru
Omirkhan Khabidolda, oka-kargtu@mail.ru
Syrlybek Zholmagambetov, syrlybekzh@mail.ru
Irina Kurokhtina, kurohtina.ira@mail.ru