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The location of supports under the monolithic reinforced concrete slabs optimization

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Abstract. We consider the problem of finding the optimal location of point supports under a monolithic reinforced concrete floor slab, which provides the minimum of the objective function. The maximum deflection, potential strain energy, and reinforcement consumption are selected as the objective function. The load and plate configuration can be arbitrary. A restriction on the number of supports is introduced. The solution is performed using stochastic and deterministic optimization methods in combination with the finite element method to determine the objective functions An assessment of the proposed methods for a different number of supports n is made. Particular solutions are presented for n = 3, 4, 5.The optimal relations between the marginal and middle spans are established for buildings with a rectangular grid of columns with large n. It is shown that only the pitch of the columns of the marginal rows can act as a variable parameter, and the steps of the middle rows at the optimal arrangement are equal to each other. The developed methods were tested for the real object. It is established that of the three criteria used, the criterion of the minimum potential strain energy is preferable. It was also revealed that in most of the considered problems, the selected criteria give very close results. The plate thickness and material characteristics do not affect the optimal arrangement of columns.

1. Introduction

The issues of reinforced concrete structures optimal design, including reinforced concrete floor slabs of buildings for various purposes, are the subject of a large number of papers including [1–12]. The inverse method is widely used for optimization, the essence of which is to find such laws of changing the characteristics of a material within a structure in which its stress-strain state is given [1–5]. In some publications, for example [13], models of equal-strength plates of variable thickness are constructed, however, the practical implementation of this model is associated with great difficulties, as well as the creation of artificial heterogeneity of the structure. As a method of optimizing reinforced concrete slabs, the most common one is the selection of rational reinforcement [6–12].

Optimization problems by varying the location of the supports are solved mainly for beams [14–16]. There are relatively few publications on determining the optimal arrangement of supports for slabs [17–26]. The criterion of optimization in these publications is deflection, the frequency of natural vibrations, and the value of the breaking load. They are distinguished by a simple statement of the problem, for example, in [17] a round axisymmetrically loaded plate is considered, in [18–24] the problem for a rectangular plate on four point supports is solved. The presented solutions are not applicable for real objects. The publications [27–28] optimize the real object, but not by changing the position of the supports, but by partially replacing the existing columns with modernized ones.

The goal of this work is the development and testing of the methods for determining the optimal location of point supports under the reinforced concrete slabs with arbitrary configuration and load.

In the framework of the goal, the following tasks are solved:

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1. Development of the methodology for determining the optimal arrangement of supports based on a stochastic approach.
2. Development of optimization methodology using the deterministic approach.
3. Comparative evaluation of the effectiveness of stochastic and deterministic methods with a different number of supports.
4. Testing optimization techniques at a real construction object.

2. Methods

Let us first consider the application of the stochastic method to this problem. The following values will be selected as the objective function f:

1. The maximum value of the deflection of the slab $w_{\max}$, mm;
2. The value of the potential strain energy W, kJ;
3. Reinforcement consumption m_s, t.

The choice of the potential strain energy (PSE) as the objective function is explained by the fact that it is an integral measure that determines the level of the stress-strain state. The smaller the value of W, the better the system resists external influences.

While using the values of w_{max} and W as target values, we assume that the rigidity of the slab does not depend on reinforcement. The calculation will be based on the theory of elastic thin plates. Supports are considered as restraints in nodes along the z axis. The reinforcement consists of rods located near the upper and lower surfaces of the slab along the x and y axis. As a result of the selection of reinforcement for each finite element, the values of cross-sectional areas in m^2 per linear meter for tensile reinforcement $A_{sx,i}$, $A_{sy,i}$ and compressed reinforcement $A'_{sx,i}$, $A'_{sy,i}$ are determined.

To determine the optimal arrangement of the columns, we will use the Monte Carlo method in combination with the finite element method. At the first stage, the slab, depending on its geometry, is meshed by triangular or rectangular finite elements of plate with three degrees of freedom at the node: deflection along the z axis and two angles of rotation relative to the x and y axis. The load on the slab, as well as the number of point supports n are considered to be given and constant. The structural stiffness matrix and the load vector are calculated taking into account stationary supports, but excluding columns, the position of which can vary. Then, using a random number generator, n uniformly distributed random values are generated that determine the numbers of the nodes in which the columns are installed. A check is made for the absence of duplicate node numbers, and it is also controlled so that the minimum distance between the supports is greater than the specified value. Otherwise, random numbers are generated repeatedly.

Then, boundary conditions are imposed on the stiffness matrix and the load vector, taking into account the selected arrangement of columns, the system of equations of FEM is solved:

$$[K]\{U\}=\{P\}, \quad (1)$$

where $[K]$ is stiffness matrix, $\{U\}$ is nodal displacement vector, $\{P\}$ is load vector.

The implementation of the FEM calculation was carried out by the authors personally in the Matlab software package.

The potential strain energy is determined by the formula:

$$W=\frac{1}{2}\{U\}^T[K]\{U\}. \quad (2)$$

The selection of reinforcement in the slab is based on Russian standards for the design of reinforced concrete structures from the conditions:

$$\left(M_{x,ult}-M_x\right)\left(M_{y,ult}-M_y\right)-M_{xy}^2\geq 0;$$
$$M_{x,ult}\geq M_x;$$
$$M_{y,ult}\geq M_y;\qquad (3)$$
$$M_{xy,ult}\geq M_{xy},$$

where M_x, M_y are bending moments acting on a flat selected element; M_{xy} is torque; $M_{x,ult}$, $M_{y,ult}$, $M_{xy,ult}$ are ultimate bending moments and torques perceived by a flat selected element.

The total consumption of reinforcement in tons for the slab can be determined by the formula:

$$m_s=\rho\sum_{i=1}^{k}\left(A_{sx,i}+A_{sy,i}+A'_{sx,i}+A'_{sy,i}\right)A_i,\qquad (4)$$

where ρ = 7.8 t/m^3 is the density of steel, A_i is the area of the i-th finite element, k is total number of finite elements.

The value of the objective function f is compared with the value f_0, for which a very large number is initially taken. If $f<f_0$, then f_0 is assigned the value f. The calculation with a random arrangement of columns is repeated a large number of times (by us, the number of tests k was taken equal to 10^6 or more). The block-scheme of the calculation is shown in the Fig. 1.

Due to the large number of possible combinations of the arrangement of columns, the variant obtained as a result of the calculation may not be the most optimal. However, for large k, it will be rational and can be further implemented in practice.

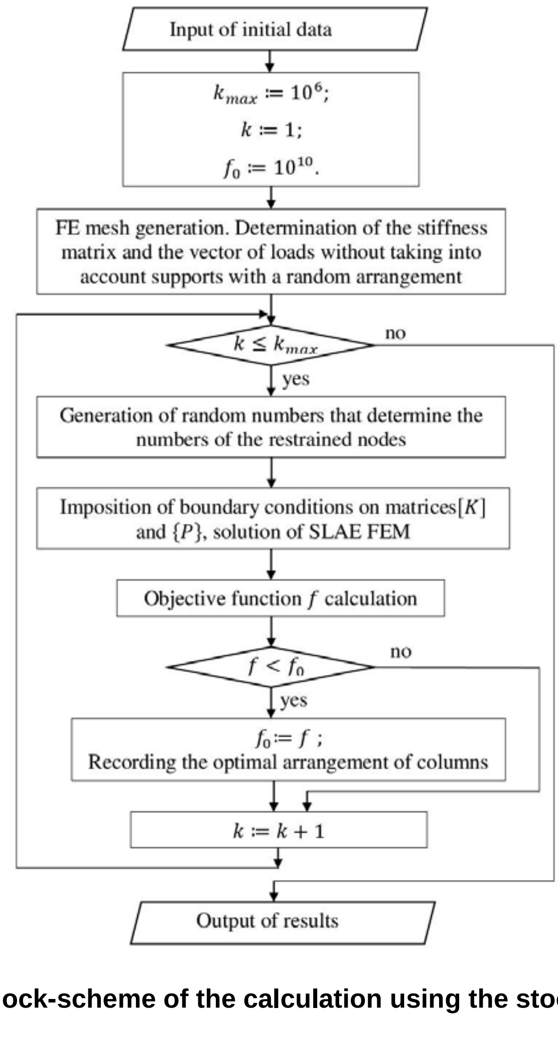

Figure 1. The block-scheme of the calculation using the stochastic method.

The presence of reinforcement leads to the change in the rigidity of the structure and a redistribution of internal forces, therefore, the task of selecting reinforcement taking into account changes in the rigidity of the slab is nonlinear. This non-linearity is called “engineering”, and it is implemented in some software systems, for example, LIRA-SAPR. When using the Monte Carlo method, the calculation is repeated many times, so taking non-linearity into account is impractical because of sharp increase in the calculation time.

Also, the proposed algorithm can be slightly modified. In the modified algorithm, the initial arrangement of the columns is set regularly with a given step. Then, for each column, two random values are generated that determine their offset relative to the initial position in x and y so that the new position of the column coincides with some node of the finite element mesh. The maximum offset should be less than the half of the initial column pitch. The calculation is also performed 10^6 times and the most optimal variant is selected.

When using the deterministic method to achieve the best result, the number of varied parameters must be minimized. To do this, optimization is performed on a regular grid of columns with varying steps. We perform calculations in the *Matlab* environment using the *fmincon* function of the *Optimization Toolbox* non-linear optimization package. As an optimization method, the internal point method is chosen. The calculation of objective functions is based on the subprogram developed by the authors based on the finite element method.

3. Results and Discussion

Using the Monte Carlo method, a series of test problems for a square slab was solved for various values of n. The calculation was performed on the action of a uniformly distributed over the area load with a rectangular grid of finite elements 10×10. The optimal location of the three point supports is shown in Fig. 2. According to the criteria of minimum consumption of reinforcement and minimum potential strain energy, the same result is obtained.

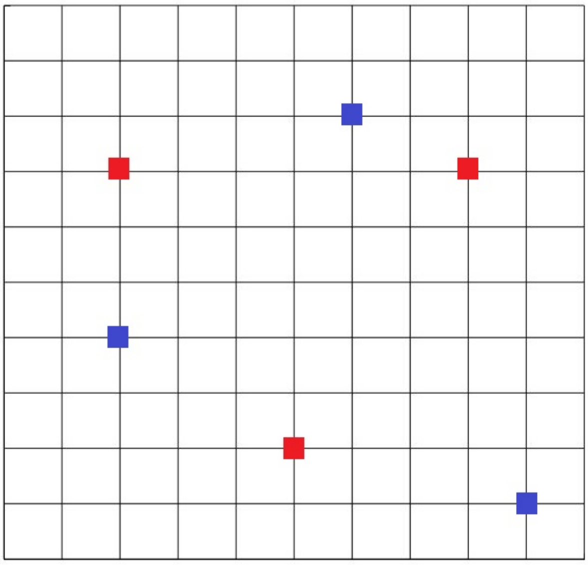

Figure 2. The optimal location of the three supports:
■ – from the condition of minimum reinforcement consumption and potential strain energy,
■ – from the condition of minimum deflection.

The optimal arrangement of four supports is shown in Fig. 3. From the symmetry of the problem for $n = 4$ it follows that the optimal arrangement of columns should be symmetrical with respect to the center of the slab. However, in the variants shown in Fig. 3, symmetry is not observed due to restrictions on the location of the supports (columns can only be located at the nodes of the finite element mesh). In the practical realization of the obtained supports arrangement, it is advisable to place them symmetrically relative to the center of the slab (see below Fig. 6).

Note that the arrangement of the supports corresponding to the minimum deflection is similar to the arrangement of supports obtained in the paper [20] from the condition of maximum fundamental frequency of the plate (Fig. 4).

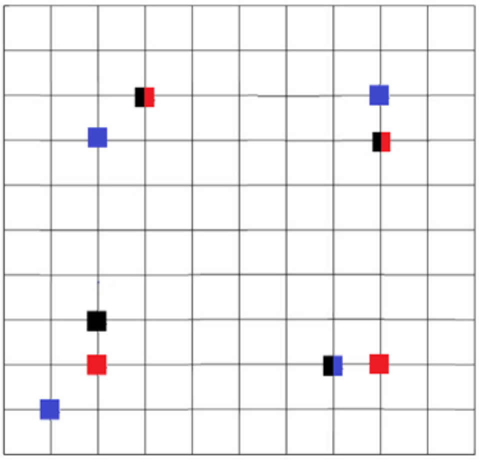

Figure 3. The optimal location of four supports:
■ – from the condition of minimum potential strain energy,
■ – from the condition of minimum deflection,
■ – from the condition of minimum reinforcement consumption.

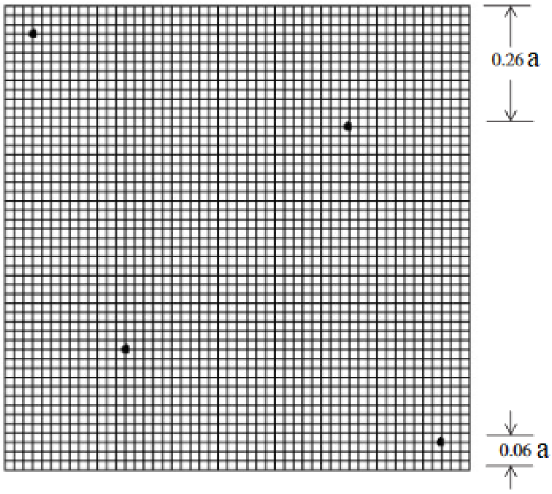

Figure 4. The optimal arrangement of four supports from the condition of maximum natural frequency obtained in [20].

At $n = 5$, the location of the supports was found, satisfying both the minimum of displacements, potential strain energy, and the reinforcement consumption (Fig. 5).

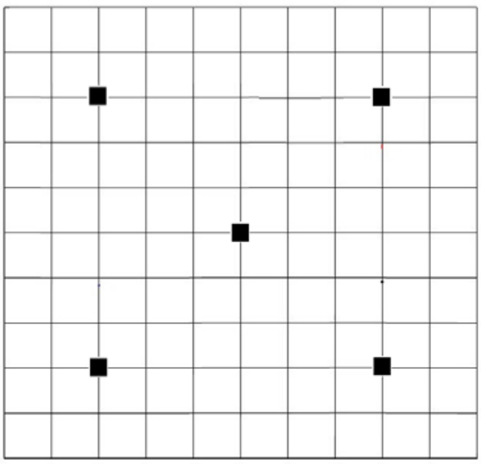

Figure 5. The optimal location of the five columns.

For n = 4 and n = 5, the problem was also solved by the deterministic method. Four columns were located symmetrically relative to the center of the plate, and the ratio a_1/a was used as a variable parameter (Fig. 6). The restriction on size a_1 was $0 \leq a_1 < a/2$. The calculation of the objective functions was performed by the finite element method. For each iteration, the FE grid was automatically regenerated taking into account the size a_1. Because of symmetry, a quarter of the structure was considered. At n = 5, the fifth column was placed in the center. The optimal a_1/a ratios for n = 4 were 0.224 from the condition of minimum deflection, 0.228 from the condition of minimum potential strain energy, and 0.233 from the condition of minimum reinforcement consumption. At n = 5, these ratios turned out to be 0.2028, 0.1989, and 0.2034, respectively. The obtained values are consistent with the results based on the Monte Carlo method.

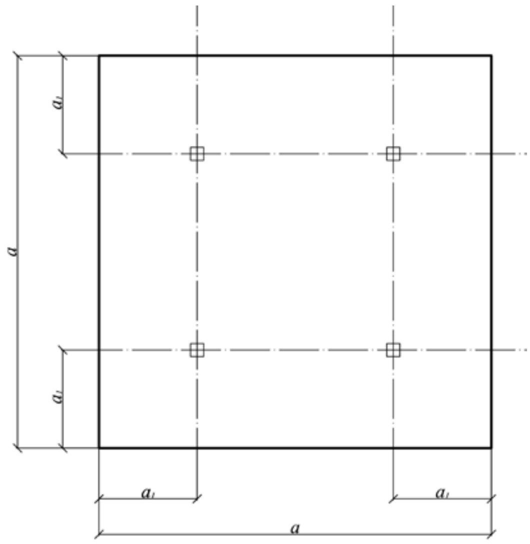

Figure 6. To optimization of the slab by deterministic method.

The presented solutions for n = 3, 4, 5 are mainly illustrative and are unlikely to be encountered in the design of real objects. If the position of most of the supports is predetermined and it is required to find how to optimally place the small number of the remaining supports, the task can be easily solved using proposed approach.

We pass on to a large number of supports n. Fig. 7 shows the arrangement of 25 columns obtained using the Monte Carlo method and basic algorithm as a result of 10^6 tests for a 24×24 m slab. The maximum deflection at $E_b = 3 \times 10^4$ MPa, q = 50 kPa, and h = 20 cm was 58.4 mm. For the same slab with a uniform column pitch of 6 m: w_{max} = 43.6 mm. Thus, with such a number of columns, the proposed algorithm was ineffective. The result of the search for the optimal support location using a modified algorithm is shown in Fig. 8. For the supports location presented on Fig. 8, the maximum deflection was 25.4 mm, which is lower by 42 % in comparison with the result for a regular step of the columns. However, in comparison with the regular arrangement of columns, inconveniences may arise with the arrangement of premises in the building.

A stochastic calculation was also performed for a 36×36 m building with an initial column pitch of 6 m. In this case, the efficiency of the modified algorithm turned out to be relatively low, the maximum deflection decreased by only 5.7 %.

With an increase in the number of supports, the effectiveness of the Monte Carlo method decreases due to the large number of possible combinations. We proceed further to the optimization of the grid of columns using deterministic methods. Consider a slab 48×36 m with a column pitch of 6 m (Fig. 9). By virtue of symmetry, a quarter of the structure is calculated. The following initial data are used: concrete B25, reinforcement A400, load q = 50 kPa, plate thickness h = 20 cm. Column steps a_1, a_2, a_3, b_1, b_2 are used as variable parameters.

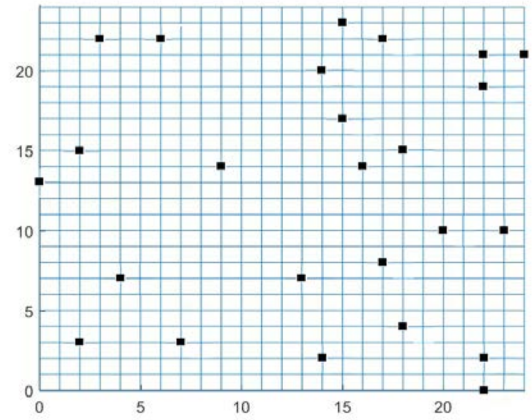

Figure 7. The result of the search for the optimal supports location using a basic algorithm.

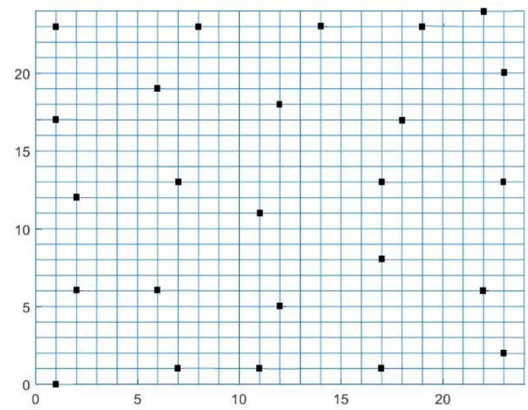

Figure 8. The result of the search for the optimal supports location using a modified algorithm.

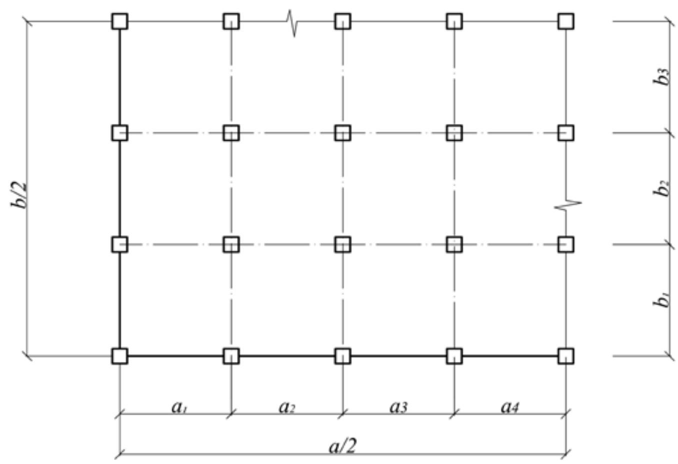

Figure 9. A quarter of the optimized slab 48×36 m.

With regular column pitch, maximum deflections and forces occur in marginal spans. Contour plot of vertical displacements is shown in Fig. 10. The values of the objective functions are w_{max} = 43.8 mm, W = 233.36 kJ, m_s = 17.01 t. The results of optimization of the slab based on three criteria are presented in Table 1. The effect of optimization was 48 % for deflection, 29 % for potential strain energy and 27 % for reinforcement consumption. When optimizing for deflection, alignment of displacements in the middle of the marginal and middle spans occurs, as can be seen from Fig. 11. Table 1 shows that for alignment of displacements and internal forces in all spans, it is possible to take only the steps of the marginal rows as a variable parameter, and put the remaining steps equal among themselves. In addition to the slab 48×36 m, the calculation of slabs 48×48 m, 36×36 m and 24×24 m was carried out. It was found that the optimal column steps along the x axis do not depend on the steps along the y axis, and the ratio between the steps of the columns of the marginal and middle row is determined by the number of spans.

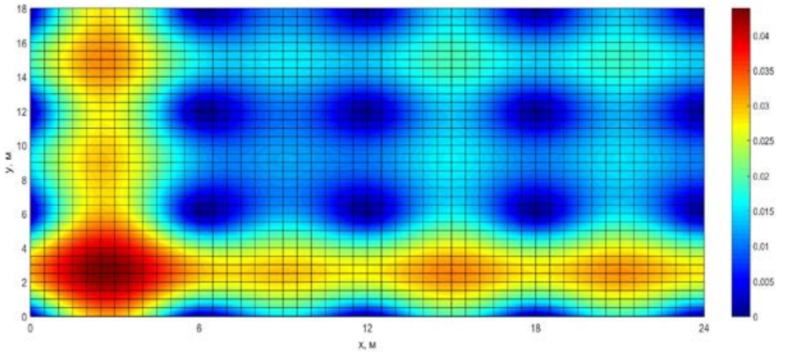

Figure 10. Contour plot of vertical displacements (m) for a 48×36 slab with a basic arrangement of columns.

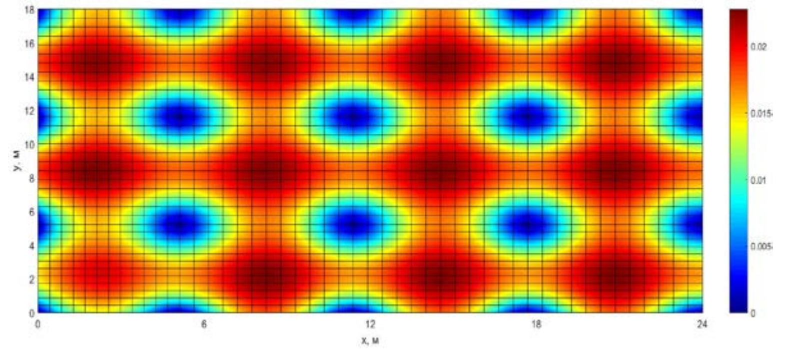

Figure 11. Contour plot of vertical displacements (m) for a 48×36 slab with an optimal arrangement of columns.

Table 1. Optimization results for the 48×36 m slab.

	a_1, m	a_2, m	a_3, m	a_4, m	b_1, m	b_2, m	b_3, m	w_{max}, mm	W, kJ	m_s, t
From a minimum of a deflection	5.11	6.27	6.3	6.32	5.22	6.39	6.39	22.8	164.99	12.389
From a minimum of PSE	5.16	6.28	6.28	6.28	5.24	6.38	6.38	23.2	164.97	12.392
From the minimum consumption of reinforcement	5.06	6.3	6.32	6.32	5.14	6.42	6.44	23.5	165.19	12.385

Our program allows to optimize floor slabs with more complex configurations. With its use, we performed the optimization of the project of the scientific and laboratory complex of the Maritime State Academy named after Admiral F.F. Ushakov in the city of Rostov-on-Don. In the original design, the building consisted of two parts, separated by an expansion joint. Optimization of each part was carried out separately. The offsets a_1 and a_2 of the middle row columns were used as variable parameters. The position of some columns, as well as stiffness diaphragms, was left unchanged. Performed optimization allowed to preserve the initial dimensions of the premises. A schematic representation of the part of the floor slab located to the right of the expansion joint is shown in Fig. 12.

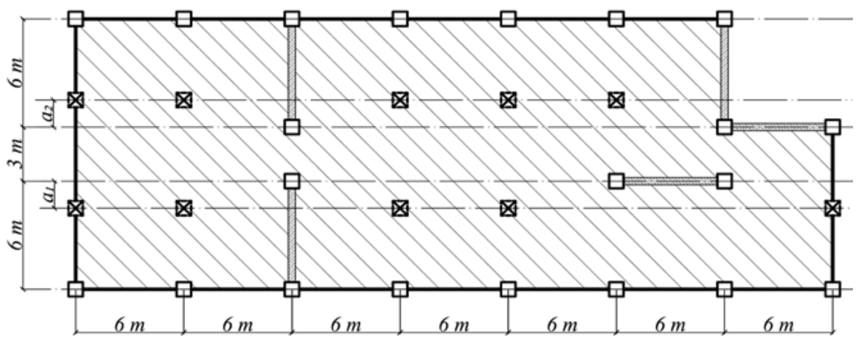

Figure 12. Schematic representation of part of the floor slab: ☐ – columns whose position does not change, ☒ – displaceable columns.

As before, the maximum deflection, the potential strain energy, and the reinforcement consumption were used as optimization criteria. The calculation was performed with the following initial data: concrete class B25, reinforcement class A400, plate thickness h = 20 cm, load uniformly distributed over an area with intensity q = 20 kPa. Fig. 13 shows the contour plot of vertical displacements for the right part of the slab with the basic arrangement of columns ($a_1 = a_2 = 0$). The values $\{w_{max}, W, m_s\}$ were {1.96 mm, 4.37 kJ, 6.57 t}.

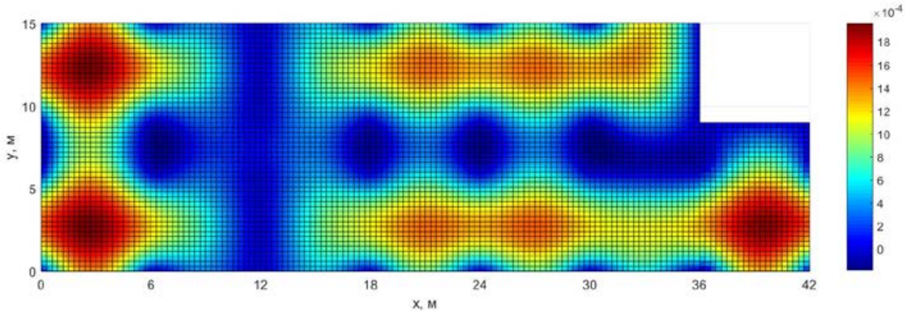

Figure 13. Deflections contour plot (m) at the basic arrangement of columns.

The optimal values of the parameters a_1 and a_2 obtained using three criteria, as well as the corresponding values $\{w_{\max}, W, m_s\}$ are given in Table 2.

Table 2. Building optimization results.

Criterion	a_1, m	a_2, m	w_{max}, mm	W, kJ	m_S, t
Minimum deflection	2.34	0.848	1.52	3.19	5.41
Minimum PSE	1.35	1.51	1.61	3	5.18
Minimum reinforcement consumption	1.38	1.65	1.61	3.01	5.18

From the Table 2 it can be seen that the results based on the criteria of minimum PSE and minimum consumption of reinforcement are quite close. Finally, we accepted the variant corresponding to the minimum PSE, since it also provides a minimum reinforcement consumption. The reinforcement consumption has reduced by 21.1 %, the potential strain energy by 31.4 %, and the maximum deflection by 17.9 %. The contour plot of vertical displacements corresponding to this arrangement of columns is shown in Fig. 14.

Note that optimization based on the criterion of minimum PSE requires less machine time, since there is no need to determine internal forces and perform the calculation of reinforcement.

When using the criterion of the minimum deflection, it is also not necessary to calculate the internal forces, but the deflection, unlike the PSE and the total reinforcement consumption, is not an integral characteristic of the structure efficiency.

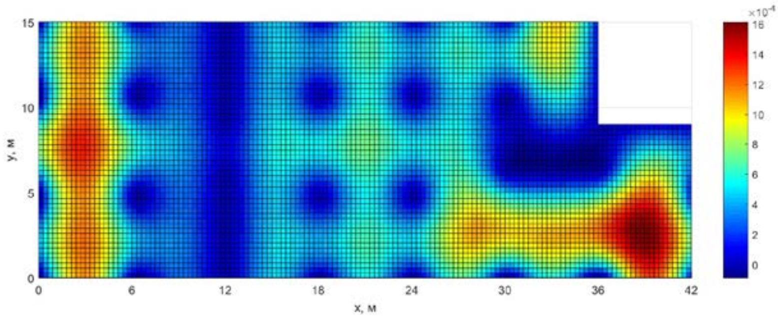

Figure 14. Deflections contour plot (m) at the optimal arrangement of columns.

4. Conclusions

1. The methodology has been developed for determining the optimal arrangement of supports for a given number of them using stochastic approach based on three optimization criteria: minimum deflection, minimum potential strain energy and minimum reinforcement consumption. In most of the problems considered, these criteria give very close results. In all the examples considered, the calculation was carried out for a uniformly distributed load. Since the optimization methodology is based on the finite element method, the load can be arbitrary.

2. The methodology for optimizing columns location by deterministic approach is proposed. For buildings with a rectangular grid of columns, it was found that only the pitch of the columns of the marginal rows can act as a variable parameter.

3. It is shown that the stochastic method is effective with a small number of columns, and with an increase in their number, the efficiency decreases due to a large number of possible combinations. With a large number of columns, deterministic method should be used and the number of variable parameters should be minimized.

4. The solution of the optimization problem for a real object is presented. During the optimization process, compared to the initial project, the reinforcement consumption was reduced by 21.1 %, the potential strain energy by 31.4 %, and the maximum deflection by 17.9 %. It is shown that of the three optimization criteria used, the criterion of the minimum potential strain energy is the most preferable, since its calculation requires less machine time and at the same time, potential energy is an integral characteristic of the design efficiency in contrast to the maximum deflection. It was also found that plate thickness and material characteristics do not affect the optimal arrangement of columns. This is because these parameters, if they are constant within the structure, do not affect the character of the internal forces distribution in the slab.

References

1. Andreev, V.I., Barmenkova, E.V., Potekhin, I.A. Way of optimization of stress state of elements of concrete structures. Procedia Engineering. 2016. 153. Pp. 37–44. DOI: 10.1016/j.proeng.2016.08.077
2. Yazyev, S., Bekkiev, M., Peresypkin, E., Turko, M. Task for a Prestressed Reinforced Concrete Cylinder with External Reinforcement and Cylinder Optimization by Varying the Modulus of Elasticity. Energy Management of Municipal Transportation Facilities and Transport. Springer, Cham, 2017. Pp. 869–876. DOI: 10.1007/978-3-319-70987-1_93
3. Andreev, V.I. About one way of optimization of the thick-walled shells. Applied mechanics and materials. Trans Tech Publications, 2012. 166. Pp. 354–358. DOI: 10.4028/www.scientific.net/AMM.166-169.354
4. Andreev, V.I. Optimization of thick-walled shells based on solutions of inverse problems of the elastic theory for inhomogeneous bodies. Computer Aided Optimum Design in Engineering. 2012. Pp. 189–202. DOI: 10.2495/OP120171
5. Andreev, V.I., Chepurnenko, A.S., Yazyev, B.M. Model of equal-stressed cylinder based on the Mohr failure criterion. Advanced Materials Research. 2014. 887–888. Pp. 869–872. DOI: 10.4028/www.scientific.net/AMR.887-888.869
6. Alcalá, J. et al. Embodied energy optimization of prestressed concrete slab bridge decks // Technologies. 2018. 6(2). DOI: 10.3390/technologies6020043
7. Aldwaik, M., Adeli, H. Cost optimization of reinforced concrete flat slabs of arbitrary configuration in irregular highrise building structures. Structural and Multidisciplinary Optimization. 2016. 54(1). Pp. 151–164. DOI: 10.1007/s00158-016-1483-5
8. Ahmadi-Nedushan, F., Hojjatli, A. Optimum cost design of reinforced concrete slabs using neural dynamics model. Engineering Applications of Artificial Intelligence. 2005. 18 (1). Pp. 65–72. DOI: 10.1016/j.engappai.2004.08.025
9. Fraile-Garcia, E. et al. Optimization based on life cycle analysis for reinforced concrete structures with one-way slabs. Engineering Structures. 2016. 109. Pp. 126–138. DOI: 10.1016/j.engstruct.2015.12.001
10. Mohammed, A.H. et al. Finite element analysis and optimization of bonded post-tensioned concrete slabs. Cogent Engineering. 2017. 4(1). DOI: 10.1080/23311916.2017.1341288
11. Yepes, V., Albiñana, M., García-Segura, T. Design optimization of precast-prestressed concrete road bridges with steel fiber-reinforcement by a hybrid evolutionary algorithm. International Journal of Computational Methods and Experimental Measurements. 2017. 5(2). Pp. 179–189. DOI: 10.2495/CMEM-V5-N2-179-189
12. Vatin, N.I., Ivanov, A.Yu., Rutman, Y.L., Chernogorskiy, S.A., Shvetsov, K.V. Earthquake engineering optimization of structures by economic criterion. Magazine of Civil Engineering. 2017. 76(8). Pp. 67–83. DOI: 10.18720/MCE.76.7
13. Moita, J.S. et al. Material distribution and sizing optimization of functionally graded plate-shell structures. Composites Part B: Engineering. 2018. 142. Pp. 263–272. DOI: 10.1016/j.compositesb.2018.01.023
14. Hauser, B.R., Wang, B.P. Optimal design of a parallel beam system with elastic supports to minimize flexural response to harmonic loading using a combined optimization algorithm. Structural and Multidisciplinary Optimization. 2018. 58(4). Pp. 1453–1465. DOI: 10.1007/s00158-018-1973-8
15. Aydin, E. et al. Optimization of elastic spring supports for cantilever beams. Structural and Multidisciplinary Optimization. 2020. Pp. 1–27. DOI: 10.1007/s00158-019-02469-3
16. Kozikowska, A. Multi-objective topology and geometry optimization of statically determinate beams. Structural Engineering and Mechanics. 2019. 70(3). Pp. 367–380. DOI: 10.12989/sem.2019.70.3.367

17. Xie, C.N., Li, X.F. Optimal location of ring support for heavy Mindlin plates under axisymmetric loading. Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science. 2018. 232(7). Pp. 1270–1279. DOI: 10.1177/0954406217700182
18. Won, K.M., Park, Y.S. Optimal support positions for a structure to maximize its fundamental natural frequency. Journal of Sound and Vibration. 1998. 213(5). Pp. 801–812. DOI: 10.1006/jsvi.1997.1493
19. Park, Y.H., Park, Y. Optimal Support Positions to Enhance Structure Natural Frequency. Proceedings of the KSME Dynamics and Control Division Summer Annual Meeting. KSME Dynamics and Control Division, 1999. DOI: 10.1006/jsvi.1997.1493
20. Wang, D., Jiang, J.S., Zhang, W.H. Optimization of support positions to maximize the fundamental frequency of structures. International journal for numerical methods in engineering. 2004. 61(10). Pp. 1584–1602. DOI: doi.org/10.1002/nme.1124
21. Wang, D. Optimization of support positions to minimize the maximal deflection of structures. International Journal of solids and structures. 2004. 41(26). Pp. 7445–7458. DOI: 10.1016/j.ijsolstr.2004.05.035
22. Wang, D. Optimal design of structural support positions for minimizing maximal bending moment. Finite Elements in Analysis and Design. 2006. 43(2). Pp. 95–102. DOI: 10.1016/j.finel.2006.07.004
23. Wang, C.M. et al. Optimization of internal line support positions for plates against vibration. Journal of Structural Mechanics. 1993. 21(4). Pp. 429–454. DOI: 10.1080/08905459308905196
24. Son, J.H. Kwak, B.M. Optimization of boundary conditions for maximum fundamental frequency of vibrating structures. AIAA journal. 1993. 31(12). Pp. 2351–2357. DOI: 10.2514/3.11935
25. Wang, B.P. Chen, J.L. Application of genetic algorithm for the support location optimization of beams. Computers & structures. 1996. 58(4). Pp. 797–800. DOI: 10.1016/0045-7949(95)00184-I
26. Kwon, Y.D. et al. Convergence enhanced genetic algorithm with successive zooming method for solving continuous optimization problems. Computers & Structures. 2003. 81(17). Pp. 1715–1725. DOI: 10.1016/S0045-7949(03)00183-4
27. Seo, H., Kim, J., Kwon, M. Optimal seismic retrofitted RC column distribution for an existing school building. Engineering Structures. 2018. 168. Pp. 399–404. DOI: 10.1016/j.engstruct.2018.04.098
28. Papavasileiou, G.S., Charmpis, D.C. Earthquake-resistant buildings with steel or composite columns: Comparative assessment using structural optimization. Journal of Building Engineering. 2020. 27. Pp. 100988. DOI: 10.1016/j.jobe.2019.100988

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