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Determination of the multicomponent lightweight mixture optimal composition

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Abstract. Designing of multi-component building compositions is usually done by an empirical method, which is a laborious and time-consuming process. The article deals with the method of designing the optimal building mixture composition based on the criterion of minimal costs. In this study, the effect of composition factors (water demand, binder content, fillers and additives) on compressive and flexural strengths of the gypsum-based mortar and the expanded lightweight filler was determined. According to the results of experiments, the corresponding mathematical models of mortar properties were obtained. Such models have made it possible to develop a design methodology for the mixture composition, which uses methods of mathematical programming. It allows obtaining the optimal composition of the construction mixture, which provides the required properties of the mortar at its minimal cost.

1. Introduction

The object of research is the methodology for designing the optimal compositions of multicomponent building materials, which include modern concretes and mortars. The designing determination of their compositions is complicated by the lack of common dependencies and requires significant experimental work. For example, about 180 trial mixes were tried before arriving at the final mix composition for roller compacted concrete [1]. Therefore for the design of compositions of multicomponent mixtures, experimentally-statistical models are especially promising. Such models incorporate the effects of specific factors that determine the mixture properties and effects of their interaction on material properties. For obtaining models effectively using the methods of mathematical experiment planning.

The experiment planning methodology is shown in the works [2–4]. Experimentally-statistical (mathematical) models allow to solve some practical problems. V. Voznesenskiy [4] has formulated and developed a methodology for ten typical problems that can be solved individually or jointly based on a polynomial models. These are interpolation, extrapolation problems, problems of achieving the minimum or maximum value of the output parameters, etc.

One of the important problems solved by the mathematical models is optimization of compositions with different optimality criterions [5]. One of the most important criteria is the minimum cost criterion while ensuring the necessary quality indicators of the composite materials. In this case, the problem of designing the building mixture composition while providing a complex of specified properties and minimizing its total cost becomes much more complicated. Along with differential analysis using canonical and isoparametrical analysis as well as linear programming and alternative methods for getting optimal solutions is also possible [2, 6, 7].

In most works performed by the traditional experimental approach, it is proposed to reduce the cost of the mixture by reducing the content of the most expensive component. At the same time, the cost of other components that significantly affect the properties of the mixture is not taken into account. Examples of solved problems for designing of the optimal composition of building mixtures are given in works [8–14]. Parveen and Singhal [15] proposed a method for designing a mixture fof geopolymer concrete. In this work, the authors suggest to minimize the cost of the mixture by reducing the number of the most expensive

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component, but in doing so not to take into account the costs of other components that determine the cost of the mixture.

To take into account the characteristics of aggregates and additives in the designation of concrete compositions, a number of calculated and experimental dependencies have been proposed [16–20]. But these dependencies did not take into account the criterion of optimality.

Some authors proposed methods, based on mathematical experiment planning methodology, but these works excluded cost aspect [21–26]. The compositions' design usually is limited to minimize the cost of binder and additives. At that, only one of the required indicators of the material quality is ensured, for example, compressive strength. The dependence between concrete cost and its strength was determined [27]. It allowed to propose a method for composition design. However, this method did not allow to find the optimal composition, providing both the specified strength value and the minimum cost. Furthermore, the method requires to use a nomogram of concrete properties which significantly complicates the procedure of designing the composition. In the work [11], the optimum self-compacting mortar mix is defined as that mixture which maximizes durability while minimizing cost. But in this case, the total cost of all components of the mixture is not calculated.

The method of dynamic optimization in the process of designing concrete composition was used [28]. The optimization of the mixture components was performed using a nonlinear dynamic model to study the behavior of the variables. However, the cost criterion was not considered in this study.

The problem of finding the optimal building mixture composition with designed quality parameters can be formulated as follows: find the values of mixture factors $x_1 \dots x_n$, allowing to minimize its cost:

$$E_c = E_1 C_1 + E_2 C_2 + \ldots + E_n C_n \rightarrow \min \tag{1}$$

While the necessary quality parameters are provided

$$P_1 \geq f\left(x_1, x_2, ..., x_n\right) \tag{2}$$

$$P_2 \geq f\left(x_1, x_2, ..., x_n\right)$$

$$P_m \geq f\left(x_1, x_2, ..., x_n\right)$$

While

$$x_1 ..., x_n \in \left[a...b\right] \tag{3}$$

where,

$E_1, E_2, \dots, E_n$ are the cost (expenses) of mixture's components, \$/kg;

$C_1, C_2, \dots, C_n$ are the consumptions of dry mixture's components, respectively, kg/t (kg/m^3) of mixture;

P_1 ... P_m are the given quality mixture parameters;

x_1 ... x_n are the composition factors;

a, b are the limitations of factors’ possible values.

The solution of this problem can be found using mathematical programming methods [4]. These methods may be implemented using originally developed algorithms in the MS Excel Solver software. This problem is especially relevant for dry building mixtures containing many components that simultaneously affect both the cost and properties of the mixture.

The aim of the research was to develop a calculation method for designing the gypsum-perlite dry mixture composition (GPM) with specified values of compressive strength, flexural strength, and density.

Achieving this aim required the solution of the following tasks:

1. To obtain a complex of polynomial models of the normalized mixture properties depending on the composition parameters.
2. To analyze the influence of the main composition factors on the mixture properties.
3. To solve the task of GPM designing using MS Excel Solver software.

2. Methods and Materials

A series of experiments, based on algorithm according to the five-factor experiment plan (type Ha_5) [1], were implemented in order to determine the GPM design parameters under the planning conditions given in Table 1.

Table 1. Experiment Planning Conditions.

No.	Factors		Varying levels			Interval
	Coded	Parameter	-1	0	+1	
1	x_1	Perlite-gypsum ratio (P/G)	0.02	0.04	0.06	0.02
2	x_2	Limestone powder – gypsum ratio (L/G)	0.4	0.6	0.8	0.2
3	x_3	Hydrated lime – gypsum ratio (HL/G)	0.17	0.37	0.57	0.2
4	x_4	Cellulose ether content (CE), % by weight of mixture	0.23	0.25	0.27	0.02
5	x_5	Starch ether content (SE), % by weight of mixture	0.03	0.05	0.07	0.02

As components of the GPM were used construction gypsum marked with A1 in accordance with EN 13279-1:2009, limestone powder (0–3 mm), hydrated calcium lime (marked with CL 90 in accordance with EN 459-1:2010), cellulose ether, starch ether and expanded perlite sand (0.16–1.25 mm). Granulometry of perlite sand within the specified fraction was selected with the maximum packing density of grains. The density of perlite sand was 100.4 kg/m^3.

Table 2. Technical and chemical characteristic of limestone powder.

Technical characteristic:	- Humidity to 1 %, - Grinding fineness (the rest on a sieve of 0.2 mm) no more than 30 %, - Fraction of 0-3 mm.
Chemical composition, %:	- CaO – 53.0-54.0 - Al_2O_3 – 0.1 - Fe_2O_3 – 0.14 - MgO – 0.62 - SiO_2 – 0.5 - The insoluble rest – 1.97 - Loss of ignition – 44.0-45.0

Table 3. Characteristics of cellulose ether.

Indicators	Value
Methoxy group content, %	19.0 – 24.0
Hydroxypropyloxy group content, %	4.0 – 12.0
Gel formation temperature, °C	70 – 90
Moisture content, %, max.	5.0
Ash content, %, max.	1.0
pH (1 % solution at 25 ºC)	5.0 – 8.0

Table 4. Characteristics of starch ether.

Indicators	Value
Moisture content, %, max.	10
Ash content, %, max.	10
pH (1 % solution)	7.0 – 10.0
Hydroxypropyloxy-group content, %	19.0 – 24.0
Viscosity (5 % solution) by Brookfield rotational viscometer, mPa·s	300 – 500

Water consumption was determined experimentally to ensure the mixture workability of 8 cm. After measuring the mixture workability, it was used for casting the test specimens. Three identical specimens were prepared for each mixture composition. The specimens had a prismatic form with dimensions of 40×40×160 mm. The specimens were kept in the forms during 24 h and after that they had hardened in special chambers with humidity no more than 60 % and the temperature was 18±2 °C. The specimens were tested in 7 days age to obtain their compressive and flexural strengths.

The strength characteristics were measured using a test machine FP 100/1 with a 100 kN load capacity and accuracy S = ±1.0 %. To obtain the flexural strength the specimens were located on two cylindrical supports. The distance between the supports was 100 mm. The load was applied in the middle of the span (ISO 679:2009 [29]). The parts of the tested specimens were further used to measure their compressive strength using a standard method (ISO 679:2009 [29]).

3. Results and Discussion

Experimental results are presented in Tables 5-6. For this data corresponding statistical characteristics was obtained and coefficients of regression equation was calculated [2]. After the significance of the coefficients were estimated, the adequacy of equations were checked by calculating the adequacy dispersion, design value of Fishcer's criterion (F-criterion) and comparing the last with a normalized one [30].

Table 5. Planning matrix and compositions of gypsum-perlite mixture.

No.	Natural values of factors					Component consumption					
	P/G	L/G	HL/G	CE	SE	Gypsum (G), kg/t	Expanded perlite sand (P), kg/t	Limestone powder (L), kg/t	Hydrated calcium lime(HL), kg/t	Cellulose ether (CE), %	Starch ether (SE), %
1	0.06	0.8	0.057	0.27	0.07	520	31.2	416	29.6	2.7	0.7
2	0.02	0.4	0.057	0.27	0.07	675	13.5	269	38.5	2.7	0.7
3	0.02	0.8	0.017	0.23	0.03	543	10.9	434	9.2	2.3	0.3
4	0.06	0.4	0.017	0.23	0.03	675	40.5	270	11.5	2.3	0.3
5	0.02	0.8	0.017	0.27	0.07	543	10.9	434	9.2	2.7	0.7
6	0.06	0.4	0.017	0.27	0.07	675	40.5	269	11.5	2.7	0.7
7	0.06	0.8	0.057	0.23	0.03	520	31.2	416	29.7	2.3	0.3
8	0.02	0.4	0.057	0.23	0.03	675	13.5	270	38.5	2.3	0.3
9	0.02	0.8	0.057	0.27	0.03	531	10.6	424	30.3	2.7	0.3
10	0.06	0.4	0.057	0.27	0.03	657,	39.4	262	37.5	2.7	0.3
11	0.06	0.8	0.017	0.23	0.07	531	31.9	424	9.0	2.3	0.7
12	0.02	0.4	0.017	0.23	0.07	693	13.9	277	11.8	2.3	0.7
13	0.02	0.8	0.057	0.23	0.07	531	10.6	424	30.3	2.3	0.7
14	0.06	0.4	0.057	0.23	0.07	657	39.4	262	37.5	2.3	0.7
15	0.06	0.8	0.017	0.27	0.03	531	31.9	424	9.0	2.7	0.3
16	0.02	0.4	0.017	0.27	0.03	693	13.9	277	11.8	2.7	0.3
17	0.06	0.6	0.037	0.25	0.05	587	35.3	352	21.7	2.5	0.5
18	0.02	0.6	0.037	0,25	0.05	601	12.0	361	22,3	2.5	0.5
19	0.04	0.8	0.037	0.25	0.05	531	21.2	424	19,7	2.5	0.5
20	0.04	0.4	0.037	0.25	0.05	675	27.0	270	25.0	2.5	0.5
21	0.04	0.6	0.057	0.25	0.05	587	23.5	352	33.5	2.5	0.5
22	0.04	0.6	0.017	0.25	0.05	601	24.1	361	10.2	2.5	0.5
23	0.04	0.6	0.037	0.27	0.05	594	23.8	356	22.0	2.7	0.5
24	0.04	0.6	0.037	0.23	0.05	594	23.8	356	22.0	2.3	0.5
25	0.04	0.6	0.037	0.25	0.07	594	23.8	356	22.0	2.5	0.7
26	0.04	0.6	0.037	0.25	0.03	594	23.8	356	22.0	2.5	0.3
27	0.04	0.6	0.037	0.25	0.05	594	23.8	356	22.0	2.5	0.5

Table 6. Experimental values of the properties of the gypsum-perlite mixture mortar.

No.	Strengths of the mortar at 7 days, MPa		Density ρ_o, kg/m^3	Water consumption W, %
	compressive f_m	flexural f_{tf}		
1	2	1.5	1031	54
2	3.1	2	1064	49
3	3	2.2	929	57
4	1.95	1.6	1080	50
5	2.9	2.05	910	58
6	1.75	1.46	976	54
7	1.9	1.7	1087	54
8	2.6	2.1	1093	55
9	2.3	2.2	931	57
10	1.95	1.6	936	54
11	1.8	1.53	1084	53
12	2.9	2	1093	50
13	3.3	1.95	942	58
14	1.8	1.55	960	54
15	2.3	1.46	1085	52
16	3	2.1	938	56
17	1.9	1.8	1088	49
18	2.9	1.8	980	53
19	2.4	1.92	973	52
20	2.3	1.82	975	53
21	2.3	1.82	976	52
22	2.1	1.92	974	53
23	2.3	1.87	975	52
24	2.4	1.86	976	52
25	2.3	1.85	977	52
26	2	1.87	975	52
27	2.5	1.86	978	52

Adequate experimental-statistical models of GPM compressive and flexural strengths (f_m and f_{tf}, MPa, respectively) and also density (ρ_o, kg/m^3) at 7 days, in the terms of the coded variables are as follows:

- compressive strength

$$\begin{gathered} f_m = 2.46 - 0.05x_1 + 0.04x_2 + 0.01x_3 - \\ - 0.11x_4 + 0.18x_5 + 0.16x_1^2 + 0.21x_2^2 + 0.56x_3^2 - \\ - 0.34x_4^2 - 0.44x_5^2 - 0.14x_1x_2 - 0.07x_1x_3 - 0.16x_1x_5 - \\ - 0.07x_2x_3 - 0.16x_2x_5 + 0.27x_3x_5 \end{gathered} \tag{4}$$

- flexural strength

$$\begin{gathered} f_{tf} = 1.86 - 0.02x_1 + 0.02x_2 - 0.04x_3 - \\ - 0.01x_4 + 0.02x_5 + 0.13x_1^2 + 0.13x_2^2 - 0.02x_3^2 - \\ - 0.07x_4^2 + 0.08x_5^2 + 0.01x_1x_2 + 0.03x_1x_4 - \\ - 0.09x_1x_5 - 0.04x_2x_4 + 0.08x_3x_4 + 0.014x_3x_5 \end{gathered} \tag{5}$$

- density

$$\begin{aligned}\rho_o = {} & 999 - 67.9x_1 + 9.45x_2 + 3.55x_3 - \\ & - 3.39x_4 - 4.49x_5 + 37.19x_1^2 + 0.69x_2^2 - \\ & - 0.31x_3^2 + 0.19x_4^2 + 0.19x_5^2 + 4.81x_1x_2 + \\ & + 5.18x_1x_3 + 6.18x_2x_3 + 6.81x_2x_4 + 2.56x_2x_5 - \\ & - 6.56x_3x_4 - 2.06x_3x_5 - 5.43x_4x_5. \end{aligned} \quad (6)$$

To determine the optimal water consumption the following mathematical model was developed, % of the mixture mass:

$$\begin{aligned} W = {} & 53.31 + 2.06x_1 - 1.22x_2 + 0.49x_3 + \\ & + 0.33x_4 - 0.19x_5 + 0.79x_1^2 + 0.79x_2^2 + 0.29x_3^2 + \\ & + 0.29x_4^2 - 0.22x_1x_2 - 0.53x_1x_3 - 0.28x_1x_4 + \\ & + 0.41x_2x_4 - 0.28x_2x_5 + 0.22x_3x_4 - 0.34x_3x_5. \end{aligned} \quad (7)$$

Using the models (Eqs. 4–7), graphic dependences of the output parameters on two influence factors were obtained (see Fig. 1–3). The other factors, which are not shown in each of the graphs, was fixed at main (zero) level.

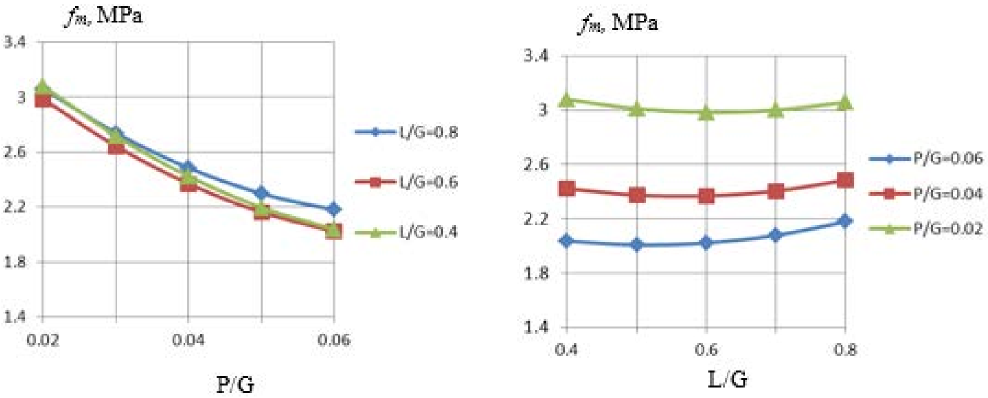

Figure 1. Dependence of the GPM mortar compressive strength on perlite content (P/G) and limestone powder (L/G).

Analyzing the obtained experimental and statistical compressive strength model (Eq. 4), it can be noted that, the most significant influencing factor is the perlite-gypsum ratio (x_1). Varying it from -1 to +1 (from P/G = 0.02 to P/G= 0.06) leads to a decrease in strength of 35 % (see Fig. 1).

The character of the flexural strengths dependences based on the model (Eq. 5) is not significantly different (see Fig. 2). The factor that has the most influence on flexural strength is also the content of perlite in GPM (x_1). Increasing the limestone powder content (x_2) and other variable factors in varied limits did not significantly influence to the mortar strength.

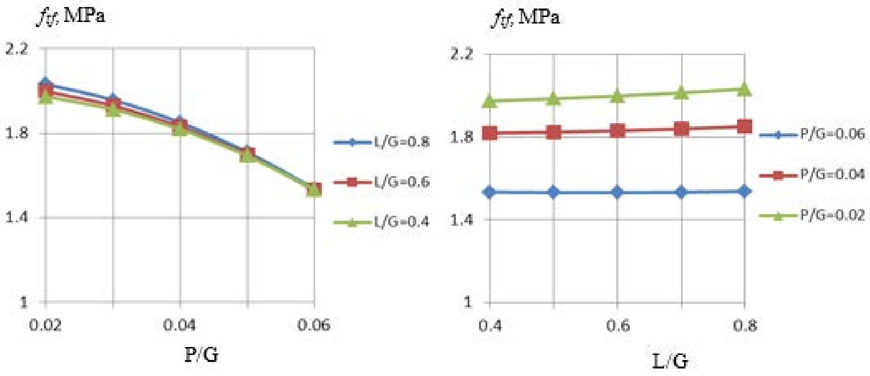

Figure 2. Dependence of the GPM mortar flexural strength on perlite content (P/G) and limestone powder (L/G).

Analyzing the obtained density model (Eq. 6) and related graphical dependencies (Fig. 3), it can be noted that the most influential factor that reduces the density of the GPM is also the content of perlite P/G (x_1). The value of its linear coefficient in the regression (Eq. 6) significantly exceeds than those of four other factors.

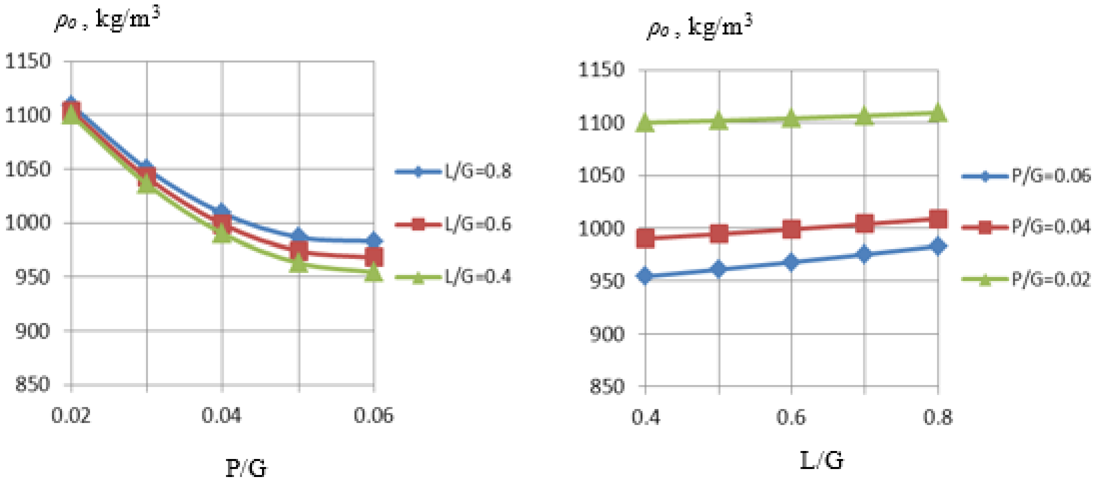

Figure 3. Dependence of the GPM mortar density on perlite content (P/G) and limestone powder (L/G).

The experimental and statistical water consumption model (Eq. 7) and graphical dependencies (Fig. 4) show that, with an increase in the perlite content (P/G) from 0.02 to 0.06, the water consumption increases on average by 10 %. The influence of other investigated factors can be considered insignificant.

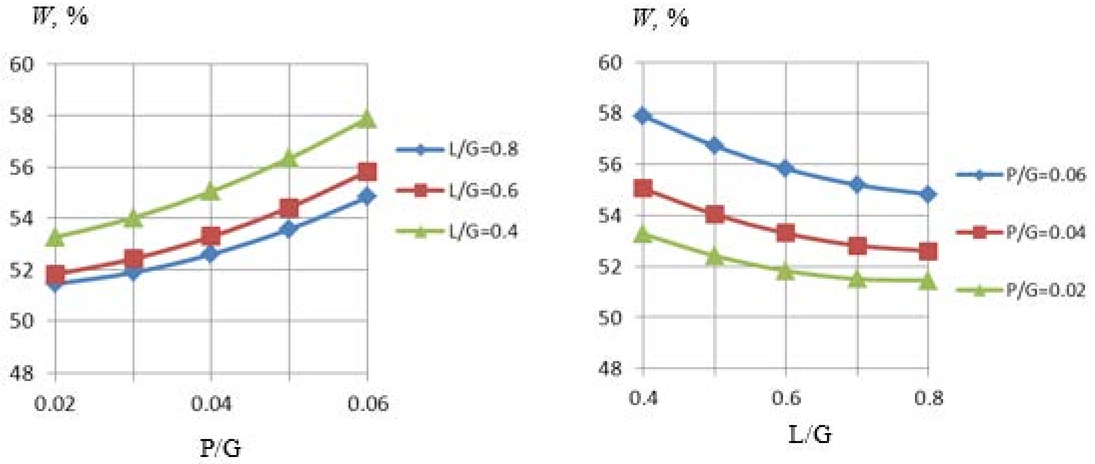

Figure 4. Dependence of the GPM mortar water consumption on perlite content (P/G) and limestone powder (L/G).

In order to obtain the optimal composition of the gypsum-perlite mixture, it is necessary to solve the problem of mathematical programming with the following formulation: to find such a composition of mixture which would allow to provide the necessary quality parameters of the GPM mortar and allowing to minimize the mortar cost in limits of admissible factors' values.

For example, the defining parameters of the mortar quality are its compressive strength and its density. Moreover, the strength must be at least of a certain value, and the density of the mortar is not more than of a certain value. Then the mathematical setting of this problem can be formulated as follows

$$\begin{gathered} E_M = E_G \cdot G + E_P \cdot P + E_L \cdot L + \\ + E_{HL} \cdot HL + E_{CE} \cdot CE + E_{SE} \cdot SE \rightarrow min \end{gathered} \tag{8}$$

While the necessary quality parameters is provided

$$\begin{gathered} f_m \geq \left(x_1,\ x_2,\ldots,\ x_5\right) \\ \rho_o \leq \left(x_1,\ x_2,\ldots,\ x_5\right) \end{gathered} \tag{9}$$

While

$$x_1 \dots x_5 \in \left[-1 \dots +1\right] \tag{10}$$

Where,

$E_G, E_P, E_L, E_{HL}, E_{CE}, E_{SE}$ are the cost of gypsum binder, perlite sand, limestone powder, hydrated lime, cellulose ether and starch ether, respectively, \$/kg;

G, P, L, HL, CE, SE are the consumption of gypsum binder, perlite sand, limestone powder, hydrated lime, cellulose ether and starch ether, respectively, kg/m^3 of dry mixture.

The conversion of the GPM composition parameters into the codified form is carried out as follows:

$$x_1 = \frac{P/G - 0.04}{0.02}; \; x_2 = \frac{L/G - 0.6}{0.2}; \; x_3 = \frac{HL/G - 0.37}{0.2};$$

$$x_4 = \frac{CE - 0.25}{0.02}; \; x_5 = \frac{SE - 0.05}{0.02}. \tag{11}$$

The calculation sequence is as follows.

- Substitute in the models (Eq. 4 and 6) the values of strength and density to be ensured, and in Eq. (8) – the GPM components value.
- Set in Eq. (10) the limit values of factors (in coded values from –1 to 1).
- The MS Excel Solver software picking up various combinations of factors, providing at least the specified strength value and not more than the specified density value by Eqs. (4) and (6) while minimizing function (Eq. 8).

A result of such iterations is determining the optimal values of composition factors: P/G ratio, L/G ratio, HL/G ratio, cellulose ether content and starch ether content. Water demand can be calculated by Eq. (7).

The gypsum binder consumption can be founded by Eq. (12):

$$G = \frac{1000 - (CE + SE)}{P/G + L/G + HL/G + 1}. \tag{12}$$

3.1. Numerical example

Determine the gypsum-perlite mixture composition with the following properties at 7 days: compressive strength of 2.3 MPa; density of 950 kg/m^3 as well as with a mixture workability of 8 cm. Were used experimental-statistical models (Eqs. 4-6) Assume the cost of the main mixture components as follows, \$/kg: G = 2; P = 10; L = 1; HL = 4; CE = 190; SE = 104.

1. Substituting the compressive strength value ($f_m \geq$ 2,3) in Eq. (4) and density ($\rho_o \leq$ 950) in Eq. (5), obtain the restriction function (Eq. 9) of the problem:

$$\begin{gathered} f_m = 2.46 - 0.05x_1 + 0.04x_2 + 0.01x_3 - \\ - 0.11x_4 + 0.18x_5 + 0.16x_1^2 + 0.21x_2^2 + 0.56x_3^2 - \\ -0.34x_4^2 - 0.44x_5^2 - 0.14x_1x_2 - 0.07x_1x_3 - \\ - 0.16x_1x_5 - 0.07x_2x_3 - 0.16x_2x_5 + 0.27x_3x_5 \geq 2.3; \end{gathered}$$

$$\begin{gathered} \rho_o = 999 - 67.9x_1 + 9.45x_2 + 3.55x_3 - \\ - 3.39x_4 - 4.49x_5 + 37.19x_1^2 + 0.69x_2^2 - \\ -0.31x_3^2 + 0.19x_4^2 + 0.19x_5^2 + 4.81x_1x_2 + \\ + 5.18x_1x_3 + 6.18x_2x_3 + 6.81x_2x_4 + \\ + 2.56x_2x_5 - 6.56x_3x_4 - 2.06x_3x_5 - 5.43x_4x_5 \leq 950. \end{gathered}$$

2. Substitute the values of GPM components cost into Eq. (8), and specify the limitation of the factors values: from -1 to 1 (in coded form).
3. By varying different combination factors the values that satisfy the problem and minimize the total GPM cost. The following parameters were obtained by using originally developed routines implemented using the MS Excel Solver software:

$$x_1 = 0.89;\ x_2 = -0.44;\ x_3 = -1;\ x_4 = 0.6;\ x_5 = 1.$$

For the obtained factors' values, from Eqs. (4 and 6) follows that f_c = 2.4 MPa, which corresponds to the required compressive strength value, and ρ_o = 950 kg/m^3, which provides the required density value.

4. Determine the natural factors using Eq. (11):

$$P/G = 0.02 \cdot x_1 + 0.04 = 0.02 \cdot 0.89 + 0.04 = 0.057$$

$$L/G = 0.2 \cdot x_2 + 0.6 = 0.2 \cdot (-0.44) + 0,6 = 0.512$$

$$HL/G = 0.2 \cdot x_3 + 0.37 = 0.2 \cdot (-1) + 0.37 = 0.17$$

$$CE = 0.02 \cdot x_4 + 0.25 = 0.02 \cdot 0.6 + 0.25 = 0.26 \text{ kg/m}^3$$

$$SE = 0.02 \cdot x_4 + 0.05 = 0.02 \cdot 1 + 0.05 = 0.07 \text{ kg/m}^3.$$

5. The water consumption which provide a mixture workability of 8 cm according to Eq. (7) is, – by %:

$$\begin{gathered} W = 53.31 + 2.06x_1 - 1.22x_2 + 0.49x_3 + \\ + 0.33x_4 - 0.19x_5 + 0.79x_1^2 + 0.79x_2^2 + 0.29x_3^2 + \\ + 0.29x_4^2 - 0.22x_1x_2 - 0.53x_1x_3 - 0.28x_1x_4 + 0.41x_2x_4 - \\ - 0.28x_2x_5 + 0.22x_3x_4 - 0.34x_3x_5 = 57\% \end{gathered}$$

– by mass:

$$W = W' \cdot 1000 / 100 = 57 \cdot 1000 / 100 = 570\ l.$$

6. The gypsum binder consumption determined by Eq. (12):

$$G = \frac{1000 - (CE + SE)}{P/G + L/G + HL/G + 1} = \frac{1000 - (0.26 + 0.07)}{0.057 + 0.512 + 0.17 + 1} = 572.7kg.$$

7. The minimum possible cost value per 1000 kg of GPM is found during the iterations in the "Solver" application according to Eq. (8):

$$E_{GPM} = 10 \cdot 33.1 + 1 \cdot 293 + 4 \cdot 97.4 + 190 \cdot 2.63 + 104 \cdot 0.7 + 2 \cdot 573 = \$2731.5.$$

8. The final GPM composition, kg is:

$$G = 573;\ P = 33;\ L = 293;\ HL = 97;\ CE = 2.6;\ SE = 0.7.$$

Proposed methodology allows to design of the mixture composition with a minimum cost and at the same time takes into account a greater number of factors than in the well-known manual [31]. At problem formulation stage for finding the GPM composition the desired values of strength and density should be correctly set. Obviously, these values are supposed to within the minimum and maximum possible value of the output parameter, since it is within these limits that the polynomial model adequately describes studied parameter. Such values can be easily found using the above - mentioned routines implemented in MS Excel Solver. For the above example, the limit values of strength and density within the factors variation range will be as follows:

$$f_m(min) = 1.5 \text{ MPa};\ f_m(max) = 3.4 \text{ MPa};\ \rho_o\ (min) = 926 \text{ kg/m}^3;\ \rho_o\ (max) = 1120 \text{ kg/m}^3.$$

Some deviation way beyond the output parameters limits is also possible. In this case, along with the optimization problem, an extrapolation problem is also solved, allowing to take the factor`s values outside the variation range (for example, $x_1 \ldots x_3$ = 1.1; 1.2; 1.3). However, it should be borne in mind that extrapolation may be due to certain errors, and these errors become more significant, the farther beyond the variation range limits. Extrapolation is possible, if according to the research results there is no doubt that outside the factors variation region the function nature remains unchanged.

4. Conclusions

Adequate mathematical models gypsum-perlite mixture mortar properties (compressive and flexural strengths, water consumption required for achieving the desired mixture workability) were obtained using mathematical experiments planning methodology. The models consider the influence of such main factors as perlite-gypsum ratio, limestone powder-gypsum ratio, hydrated lime-gypsum ratio, cellulose ether content and starch ether content.

Analysis of the obtained models shows that gypsum-perlite mixture mortar compressive strength depends mainly on the content of cellulose ether and starch ether, but the influence of the perlite-gypsum ratio also affects. As well, the perlite content is a main factor affecting the density and water demand.

Based on the obtained mathematical models, a method of design for building mixture composition was proposed. This method allows taking into account the special properties of the investigated materials and provides the most simple possibility for mixture composition optimization by a given minimum cost criterion. An additional advantage of the proposed method is a possibility to add a certain number of limitations. It allows simultaneous satisfaction of many quality indexes according to the given value.

In the authors' opinion, application of the proposed method in the production of building mixtures will allow more efficient use of raw materials and ensure high-quality mortars.

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