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Structural reliability analysis using evidence theory and fuzzy probability distributions

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Abstract. A structural element failure probability (as a structural reliability measure) is an indicator of the structural safety. Approaches for structural reliability analysis with incomplete statistical data are a special scientific problem. In the development of this scientific direction, the article proposed a method for structural reliability analysis based on a combination of evidence theory and fuzzy probability distributions when the problem of reliability analysis involves quantitative and qualitative uncertainty at the same time. The article presents an experimental study of reliability analysis for a steel truss by the truss members strength criterion based on various approaches to reliability analysis. The reliability interval [0.99272; 1] of the proposed method covers the FOSM (First Order Second Moment) reliability value of 0.99354. From the experiment results, it follows that the proposed approach can be used in practice for a more cautious assessment of the structural reliability with incomplete statistical information. The proposed approach also allows reducing the number of tests and getting an operational (preliminary) assessment of the structural element reliability. The value of the acceptable reliability level in discrete or interval form should be set individually for each design situation taking into account the risk of economic and non-economic losses.

1. Introduction

Evaluation the safety of structural elements during manufacture, construction and operation is one of the most important tasks for a civil engineer. The structural reliability theory is often used for the quantitative assessment of structural safety.

The origin of research on the structural reliability theory is considered to be the beginning of the twentieth century. John Tucker Jr. [1], M. Mayer, N.F. Hocialov presented researches on probabilistic analysis of the structures behavior. Later, soviet scientists A.R. Rzhanitsyn, N.S. Streletsky made a significant contribution to the formation of the existing theory of structural reliability.

Structural reliability analysis for incomplete statistical information is one of the important current tasks of reliability theory as a science. Relevance of this direction is emphasized by the major conference – 13th International Conference on Applications of Statistics and Probability in Civil Engineering (ICASP13) in Seoul, South Korea. The article [2] notes that the structural reliability theory has become one of the main methods of structural safety design in recent years. Reliability analysis has been increasingly applied to structural design and structural assessment due the uncertainties involved with material, load and geometric properties [3]. The research [4] also notes that it is well known that the inevitable uncertainties inherent in both load and resistance of the structure will seriously affect its safety and serviceability, and hence, reliability analysis plays an important role in structural engineering. The subject of this paper is the reliability of a structural element which is expressed as a failure probability or a probability of non-failure (safety).

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FORM (First Order Reliability Method) [5], SORM (Second Order Reliability Method) [6] and Monte Carlo simulation (MSC) [7] are most common approaches for structural reliability analysis. These methods are based on complete statistical information when the cumulative distribution function (CDF), the parameters of CDF, information about the dependence / independence of random variables, etc. are known. The problem of a limited statistical data often arises in practical problems of reliability analysis. A quantitative limitation of statistical data is a small sample size. A qualitative limitation of statistical data is an inaccurate (interval) estimate of a single value from a general or sample population.

Special methods of structural reliability analysis were developed to take into account the factor of statistical data fuzziness. Methods based on the provisions of fuzzy set theory and the theory of possibilities has received great development. H. Li and X. Nie [8] present a novel structural reliability analysis method with fuzzy random variables from the perspective of error propagation. Fuzzy numbers are used in research [9] to define an equivalence class of probability distributions compatible with available data and corresponding upper and lower cumulative density functions. A procedure is then proposed to perform reliability analysis using extended fuzzy operations. It gives estimates of small and large fractiles of output variables which are conservative with respect to probability. The paper [10] presents a novel algorithm for obtaining membership function of fuzzy reliability with interval optimization based on Line Sampling (LS) method. Methods for structural reliability analysis were developed based on the evidence theory (or Dempster-Shafer theory) and random set theory for modeling random variables with limited statistical data. Evidence theory employs a much more general and flexible framework to quantify the epistemic uncertainty, and thereby it is adopted to conduct reliability analysis for engineering structures recently [11]. The paper [11] proposes a response surface (RS) method to evaluate the reliability for structures using evidence theory, and hence improves its applicability in engineering problems. The article [12] presents the evidence theory model based on the copula function and the related structural reliability analysis method. It is an effective tool for uncertainty modeling and reliability analysis with dependent evidence variables. Random set theory is used for calculating the upper and lower bounds on the failure probability in the research [13]. The method allows the modulation of dependence between the input variables by means of copulas.

The relevance of this research is due to the great interest of the scientific community to probabilistic methods of structural mechanics and a reliability theory for evaluating the structural safety. In addition, there are separate methods for reliability analysis based on evidence theory and fuzzy set theory, but there is no generalized approach to reliability analysis based on a combination of evidence theory and fuzzy probability distributions taking into account the impact on reliability of uncertainties in the form of a small number of intervals and the degree of confidence in expert estimates, instruments, methods, etc.

In this regard, the purpose of this research is to develop a method for structural reliability analysis based on the combination of the evidence theory and fuzzy probability distributions.

2. Methods

As noted above, some random variables can be represented by a subset of intervals as $x \in x_i^I = \left\{ \left[\underline{x}_i ,\ \overline{x}_i \right] \right\}$, where $\underline{x}_i$ and $\overline{x}_i$ are the lower and upper value of the i-th interval of a random variable. In practical problems, these can be intervals of measured values in different time periods, a set of confidence intervals when using different measurement tools and methods, etc. Random set theory and evidence theory can be used for statistical analysis of a subset of interval values. In the evidence theory, an interval random variable can be described by two boundary distribution functions. According to researches [14, 15], the upper boundary distribution function is called the “belief function” and denoted as $Bel(A)$ and the lower boundary distribution function is called the “plausible function” and denoted as $Pl(A)$, where A is a set consisting of subsets (A_i) in Ω (set of all values).

If random value X, then x_i^I is the subset of the x set. Let C_i is a number of observed subsets A_i. Then in according to [10]:

$$Bel(A) = \sum_{A_i : A_i \subseteq A} m(A_i), \tag{1}$$

$$Pl(A) = \sum_{A_i : A_i \cap A \neq \varnothing} m(A_i), , \tag{2}$$

where $m(A_i) = C_i / N$ with N is a number of measurements (a number of intervals) and C_i is a number of observed subsets A_i.

Hence, $Bel(A)$ and $Pl(A)$ can be considered as the lower and upper probability of A as $Bel(A) \le P(A) \le Pl(A)$.

The limit state mathematical model for reliability analysis can be written in the form:

$$X \le Y. \tag{3}$$

where X is a generalized load (load, stress, bending moment, etc.); Y is a generalized strength (maximum allowable stress, ultimate deformations, etc.).

The probability of non-failure P [16] of the condition $X \le Y$ is defined by the equation:

$$P(X \le Y) = \int_0^\infty f_Y(y) F_X(y) dy, \tag{4}$$

where $f_Y(y)$ is a probability density function of the random variable Y by variable y; $F_X(y)$ is a probability density function of the random variable X by variable y.

Let X is described by a subset of intervals $\{[\underline{x}_i, \overline{x}_i]\}$, and Y is described by some fuzzy probability distribution functions. Conditional view of distribution function X and Y graphs is shown on Fig. 1.

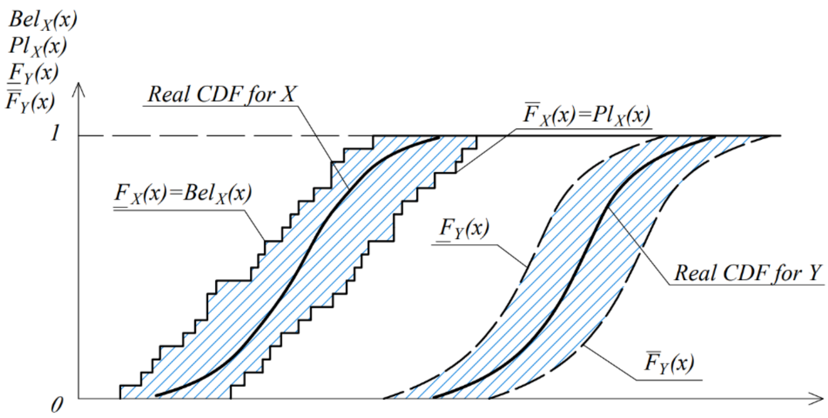

Figure 1. Graphs of the belief $Bel_X(x)$ and plausible $Pl_X(x)$ functions of the random value X and the probability distribution functions of the fuzzy value Y; CDF – cumulative distribution function.

Graphically, as well as analytically from dependencies (1), (2) and (4), the equations can be figured out to calculate the lower $\underline{Q}$ and upper $\overline{Q}$ boundaries of the failure probability by (3) in general form:

$$\Pr(X \le Y) = \begin{cases} \underline{Q} = n^{-1} \cdot \sum_{i=1}^{n} \overline{F}_Y(\underline{x}_i) \\ \overline{Q} = n^{-1} \cdot \sum_{i=1}^{n} \underline{F}_Y(\overline{x}_i) \end{cases}. \tag{5}$$

where n is a number of intervals $\{[\underline{x}_i, \overline{x}_i]\}$.

The non-failure probability interval is related to the failure probability interval by the following equation $\left[\underline{P};\ \overline{P}\right]=\left[1-\overline{Q};\ 1-\underline{Q}\right]$. Structural reliability will be characterized by an interval $\left[\underline{P};\ \overline{P}\right]$.

Such approach is also called a p-boxes (probability boxes) approach.

Let us consider the case where the “strength” Y is described by the fuzzy distribution functions (based on the possibility theory [17]) with the analytical form:

$$\overline{F}_Y(y)=\begin{cases}1-\exp\left[-\left(\dfrac{y-a_y}{b_y}\right)^2\right] & if\ \ y>a_y\\ 0\ \ otherwise\end{cases},\tag{6}$$

$$\underline{F}_Y(y)=\begin{cases}\exp\left[-\left(\dfrac{y-a_y}{b_y}\right)^2\right] & if\ \ y<a_y\\ 1\ \ otherwise\end{cases},\tag{7}$$

where a_y is a “mean” value which is calculated as $a_y=0.5\left(Y_{\max}+Y_{\min}\right)$; b_y is a measure of variance which is calculated as $b_y=0.5\left(Y_{\max}-Y_{\min}\right)\big/\sqrt{-\ln\alpha}$, where $Y_{\max}$ and $Y_{\min}$ is a maximum and a minimum value in the set of $\{Y\}$; α is a cut (risk) level [18, 19].

Other fuzzy distributions can be used to model an uncertainty as p-box. For example, the distribution functions based on the Chebyshev's inequality [20] with the analytical form:

$$\underline{F}_Y(y)=\begin{cases}\dfrac{S_Y^2}{\left(m_Y-y\right)^2+S_Y^2}, & for\ \ y<m_Y\\ 1, & for\ \ y\geq m_Y\end{cases},$$

$$\overline{F}_Y(y)=\begin{cases}0, & if\ \ y<m_Y\\ 1-\dfrac{m_Y}{y}, & if\ \ m_Y\leq y\leq m_Y+\dfrac{S_Y^2}{m_Y}\\ \dfrac{\left(m_Y-y\right)^2}{\left(m_Y-y\right)^2+S_Y^2}, & if\ \ y>m_Y+\dfrac{S_Y^2}{m_Y}\end{cases},$$

where m_Y, S_Y is an expected value and a standard deviation for random variable Y.

For a more convenient using of the proposed approach, let us consider the case of reliability analysis for a small number of intervals in subset. The research [15] is proposed to use the extended belief and plausible functions based on the generalized Dirichlet model as a type of robust models. The upper and the lower bound of the non-failure probability can be written as:

$$\underline{P}\left(A\middle|c,s\right)=\frac{N\cdot Bel(A)}{N+s}=\chi Bel(A)\ \text{and}\ \overline{P}\left(A\middle|c,s\right)=\frac{N\cdot Pl(A)+s}{N+s}=1-\chi\left[1-Pl(A)\right],\tag{8}$$

where N is a number of tests (measurements); s is a parameter that characterizes the measure of uncertainty; $\chi=\left(1+s/N\right)^{-1}$, $\chi\in[0;1]$. Some recommendations for assigning the s parameter are given in the paper [21].

In a simplified form, the expressions for reliability based on the robust Dirichlet model and the above approach can be written as:

$$\left[\underline{P}';\ \overline{P}'\right]=\left[\chi\underline{P};\ 1-\chi\underline{Q}\right]. \tag{9}$$

In accordance with research [15], the values of the expectation bounds of an interval random variable X also can be found by the equations:

$$\underline{\mathrm{E}}X=\int_{\Omega}\omega d\overline{F}\left(\omega|c,s\right)=\left(N+s\right)^{-1}\left(s\cdot\Omega_{*}+\sum_{i=1}^{n}c_i\inf A_i\right),$$

$$\overline{\mathrm{E}}X=\int_{\Omega}\omega d\underline{F}\left(\omega|c,s\right)=\left(N+s\right)^{-1}\left(s\cdot\Omega^{*}+\sum_{i=1}^{n}c_i\sup A_i\right).$$

For more information on this issue, see [14].

3. Results and Discussion

P-box approaches can be successfully applied in practical problems of reliability analysis. The random variable values were generated by the PTC MathCAD program with statistical parameters: expected value $m_X=300$ MPa and standard deviation $S_X=15$ MPa. The obtained values are the result of a numerical experiment to assess the steel strength of some structural element. The following values were obtained from data generation results: $X\in${303.58; 289.73; 275.78; 321.17; 314.57; 282.66; 302.16; 325.46} MPa. Fig. 2 shows the graphs of the empirical probability distribution functions $F_X^{emp}\left(x\right)$ based on 8 values and the normal probability distribution graph $F_X\left(x\right)$ if full statistical information is available.

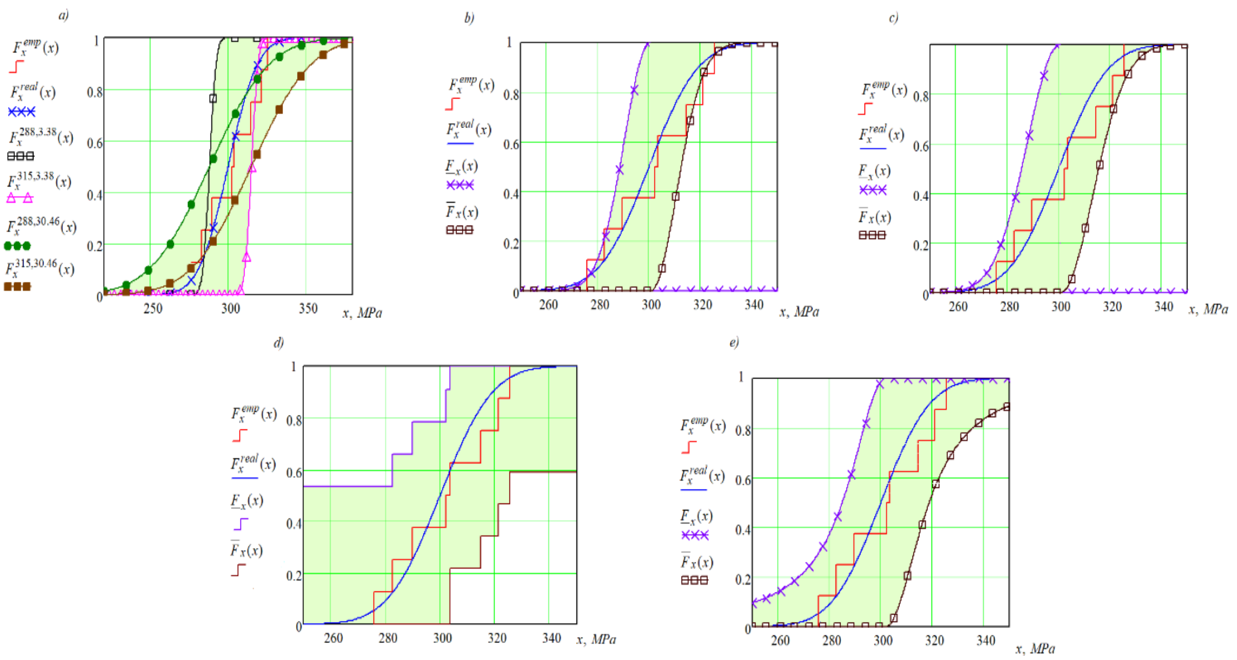

Figure 2. Different p-box cases of incomplete statistical data:
a) Combinations of normal distribution interval parameters; b) Fuzzy distribution functions with $\alpha=0.05$; c) Fuzzy distribution functions with $\alpha=0.15$; d) Kolmogorov-Smirnov bounds; e) Boundary distribution functions based on Chebyshev's inequality [20].

As can be seen from Fig. 2, there are various p-box approaches for modeling the incompleteness of statistical data in reliability analysis problems. Each p-block is applied depending on the quantity and quality of the initial statistical data about the random variable.

The proposed approach is considered on the example of the steel planar truss reliability analysis. The design scheme of the truss is shown in Fig. 3.

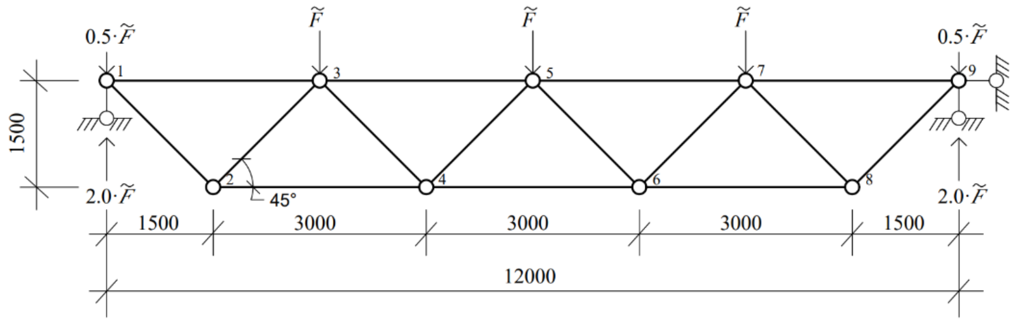

Figure 3. Planar truss design scheme.

The mathematical model of the limit state can be formed for any truss member (bar) in the form:

$$\tilde{N}_{i-j}\left(\tilde{F}\right) \leq \tilde{N}_{i-j,ult}, \tag{10}$$

where $\tilde{N}_{i-j,ult}$ is an ultimate longitudinal force for the $i-j$ truss member (bar).

The ultimate force $\tilde{N}_{i-j,ult}$ for the $i-j$ truss member can be determined by various criteria of limit states.

For example, according to the criterion of the strength of a steel truss member:

$$\tilde{N}_{i-j}\left(\tilde{F}\right) \leq \tilde{N}_{i-j,ult} = \tilde{\sigma}_{s,ult} \cdot A, \tag{11}$$

where A is a cross-sectional area of the truss member; $\tilde{\sigma}_{s,ult}$ is an ultimate stress in the steel of the truss member (random value).

Another important limit state criterion is a buckling of truss compressed members. In that case, design mathematical model can be written as: $\tilde{N}_{i-j}\left(\tilde{F}\right) \leq \tilde{N}_{i-j,ult} = \tilde{\sigma}_{s,ult} \cdot A \cdot \phi\left(\tilde{\sigma}_{s,ult}\right)$, where ϕ is a buckling factor. In this research, the application of the proposed approach is presented in the case of the truss reliability analysis by the criterion of the truss member's strength using mathematical model (11).

Since the loads included in the mathematical model (11) are described by different probability distribution functions, the model (11) can be written as:

$$\tilde{N}_{i-j}\left(\tilde{F}_{snow}\right) \leq \tilde{N}_{i-j,ult} - \tilde{N}_{i-j}\left(\tilde{F}_{sw}\right), \tag{12}$$

where $\tilde{N}_{i-j}\left(\tilde{F}_{snow}\right)$ is a force in $i-j$ truss member from the snow load; $\tilde{N}_{i-j}\left(\tilde{F}_{sw}\right)$ is force in $i-j$ truss member from the truss and the structural cover self-weight load.

In a generalized form, inequality (12) can be written as $X \leq Y$, which is analogous to inequality (3).

Table 1. The forces in the truss members by Fig. 3.

Member	Force	Member	Force
1-2, 8-9	$\tilde{N}_{1-2} = +\frac{3 \cdot \tilde{F}}{\sqrt{2}}$	3-4, 6-7	$\tilde{N}_{3-4} = +\frac{\tilde{F}}{\sqrt{2}}$
1-3, 7-9	$\tilde{N}_{1-3} = -1.5 \cdot \tilde{F}$	3-5, 5-7	$\tilde{N}_{3-5} = -3.5 \cdot \tilde{F}$
2-3, 7-8	$\tilde{N}_{2-3} = -\frac{3 \cdot \tilde{F}}{\sqrt{2}}$	4-5, 5-6	$\tilde{N}_{4-5} = -\frac{\tilde{F}}{\sqrt{2}}$
2-4, 6-8	$\tilde{N}_{2-4} = +3 \cdot \tilde{F}$	4-6	$\tilde{N}_{4-6} = +4 \cdot \tilde{F}$

The maximum force in the truss with the design scheme according to Fig. 3 will occur in the member 4-6: $\tilde{N}_{4-6} = +4 \cdot \tilde{F}$. The mathematical model of the limit state will take the form for analyzing the reliability of this truss member:

$$\tilde{N}_{4-7}\left(\tilde{F}_{snow}\right) \leq \tilde{N}_{4-7,ult} - \tilde{N}_{4-7}\left(\tilde{F}_{sw}\right). \tag{13}$$

The force from snow load $\tilde{N}_{4-7}\left(\tilde{F}_{snow}\right)$ can be described by the belief and plausibility functions in the conditions of real snow load data analysis. Data on the maximum snow cover heights of Vologda [22] will be used for reliability analysis example of the truss member (bar). The snow density on the truss surface is one of the uncertainty factors in the reliability analysis. The density of accumulated snow can vary within interval [200; 400] kg/m^3, in accordance with Russian State Standard GOST 53613-2009 “Influence of environmental conditions appearing in nature on the technical products. Overall performance. Precipitation and wind”. By multiplying the interval of possible snow density on the maximum snow, cover heights over the last 50 years, boundary empirical cumulative distribution functions can be constructed for the distribution of snow cover weight $\tilde{S}$ (Fig. 4).

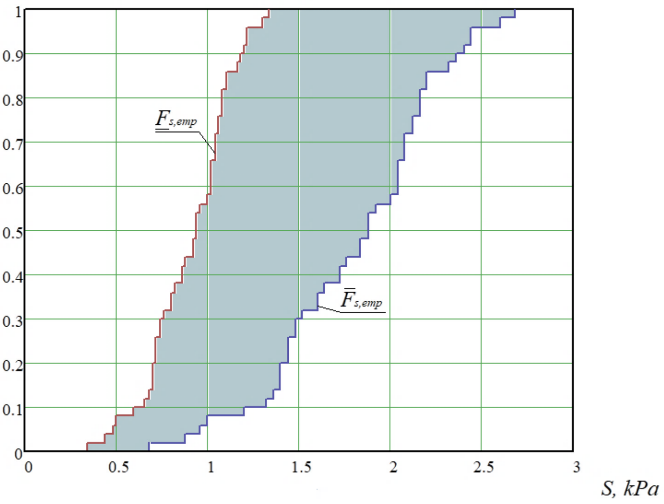

Figure 4. Boundary empirical cumulative distribution functions $\underline{F}_{s,emp}$ and $\overline{F}_{s,emp}$ for snow weight (kPa) in Vologda, Russia.

If to take the distance between the trusses 6 m for considered example, then the load area per node will be 18 m^2. The level part of (13) inequality can be written as:

$$\tilde{N}_{4-7}\left(\tilde{F}_{snow}\right) = \tilde{X} = 18 \cdot 4 \cdot \tilde{S} = 72 \cdot \tilde{S}. \tag{14}$$

The right part of (13) is described by the fuzzy probability distribution functions (6)-(7). There is subtraction of the fuzzy variables $\tilde{N}_{4-7,ult} = A \cdot \tilde{\sigma}_{s,ult}$ and $\tilde{N}_{4-7} = 4 \cdot \tilde{F}_{sw}$. The design parameters a_Y and b_Y can be calculated as: $a_Y = A \cdot a_{\sigma,ult} - 4 \cdot a_{F,sw}$, $b_Y = A \cdot b_{\sigma,ult} + 4 \cdot b_{F,sw}$, where $a_{\sigma,ult} = 0.5\left(\sigma_{s,ult,\max} + \sigma_{s,ult,\min}\right)$, $b_{\sigma,ult} = 0.5\left(\sigma_{s,ult,\max} - \sigma_{s,ult,\min}\right) / \sqrt{-\ln\alpha}$, and $a_{F,sw} = 0.5\left(F_{sw,\max} + F_{sw,\min}\right)$, $b_{F,sw} = 0.5\left(F_{sw,\max} - F_{sw,\min}\right) / \sqrt{-\ln\alpha}$. $\sigma_{s,ult,\max}$, $\sigma_{s,ult,\min}$ and $F_{sw,\max}$, $F_{sw,\min}$ are maximum and minimum values of fuzzy variables $\tilde{\sigma}_{s,ult}$ and $\tilde{F}_{sw}$ obtained by tests and estimations.

For considered example, let the truss member 4-6 have a cross-section of a rectangular tube 100×5 mm with a cross-sectional area $A_{4-6} = 18.57 \cdot 10^{-4}$ m^2. Control samples of steel were tested for tensile strength. The minimum and maximum ultimate stress values during the tests are $\sigma_{s,ult,\max} = 310$ MPa and $\sigma_{s,ult,\min} = 290$ MPa. By collecting loads from the self-weight of the truss and the self-weight of the structural cover, the maximum and minimum values of loads in the truss node are $F_{sw,\max} = 86$ kN and $F_{sw,\min} = 74$ kN.

The parameters of the fuzzy probability distribution functions are $a_{\sigma,ult}=300$ MPa, $b_{\sigma,ult}=5.78$ MPa, $a_{F,sw}=80$ kN, $b_{F,sw}=3.47$ kN. Then $a_Y=237.1$ kN; $b_Y=24.61$ kN.

P-boxes of the random variables X and Y show at the Fig. 5.

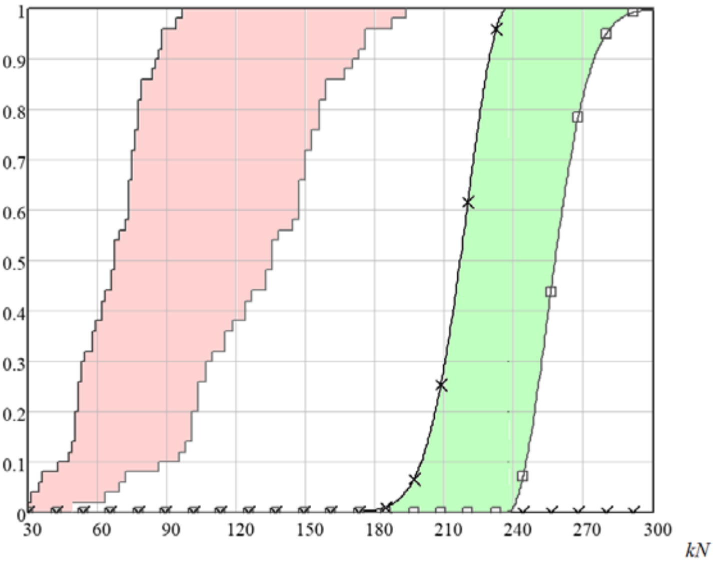

Figure 5. P-boxes of random variables X and Y.

The failure probabilities (5) are:

$$\underline{Q}=n^{-1}\cdot\sum_{i=1}^{n}\overline{F}_Y(\underline{x}_i)\rightarrow\frac{1}{50}\cdot(50\cdot 0)=0,$$

$$\overline{Q}=n^{-1}\cdot\sum_{i=1}^{n}\underline{F}_Y(\overline{x}_i)\rightarrow\frac{0.064}{50}=0.00128.$$

These failure probability values are obtained with full confidence to experts and the complete set of interval data. To adjust the reliability value, we will introduce the uncertainty measure χ, since the number of intervals was small. We will consider the parameter χ as the confidence degree to expert estimates of a statistical subset or an assessment of the accuracy of measuring instruments or methods.

If to take s = 0.3, then $\chi=(1+s/N)^{-1}\rightarrow(1+0.3/50)^{-1}=0.9940.$

The probability of non-failure is $P'=\chi\underline{P}=\chi\left(1-\overline{Q}\right)=0.99872\cdot 0.99400=0.99272$ and $\overline{P}'=1-\chi\underline{Q}=1-0=1.$

The reliability of the steel truss bar according to the steel strength [24] criterion is characterized by the interval [0.99272; 1].

The theoretical reliability value can be calculated using the FOSM (First Order Second Moment) approach by equation [16]:

$$P=\int_{0}^{+\infty}f_X(x)\cdot F_Y(x)\,dx, \tag{15}$$

where $f_X(x)$ is a probability density function (PDF) for the random variable $\tilde{X}$; $F_Y(x)$ is a cumulative distribution function (CDF) for the random variable $\tilde{Y}$ in the mathematical model (3).

The random variable $\tilde{X}$ can be described by Gumbel distribution (or Generalized Extreme Value distribution Type-I) [22]. The random variable $\tilde{Y}$ can be described by normal distribution in accordance with Eurocode 0 "Basis of structural design". Using following PDF and CDF [16] for the random variables $\tilde{X}$ and $\tilde{Y}$, the equation (15) can be presented as:

$$P = \frac{1}{S_Y\sqrt{2\pi}} \cdot \int_0^{+\infty} \exp\left[-\exp\left(\frac{\alpha - x}{\beta}\right)\right] \cdot \exp\left[\frac{-(x-m_Y)^2}{2S_Y^2}\right] dx, \tag{16}$$

or:

$$P = 1 - \frac{1}{2\beta} \cdot \int_0^{+\infty} \exp\left[\frac{\alpha - x}{\beta} - \exp\left(\frac{\alpha - x}{\beta}\right)\right] \cdot \left[1 + erf\left(\frac{x - m_Y}{\sqrt{2S_Y^2}}\right)\right] dx, \tag{17}$$

where α and β are Gumbel's distribution parameters; $erf()$ is the error function.

Let $\alpha = 1.08\ kPa \cdot 72\ m^2$ and $\beta = 0.41\ kPa \cdot 72\ m^2$ by the [22] recommendations; $m_Y = 237$ kN and $S_Y = 25$ kN similarly with the a_Y and b_Y parameters above.

The result of the numerical experiment in the form of reliability 0.99354 also fell into the interval obtained within the framework of inaccurate interval and fuzzy estimates [0.99272; 1]. Therefore, the proposed method of reliability analysis can be used in engineering practice of structural reliability analysis.

Reliability intervals for other truss member can be obtained by a similar approach. The structural reliability interval for the strength of the whole truss as the structural system can be calculated [15] as:

$$\begin{cases} \underline{P} = \max\left(0, \sum_{i=1}^{n} \underline{P}_i - (n-1)\right), \\ \overline{P} = \min\left(\overline{P}_i\right) \end{cases}$$

where $\underline{P}_i$ and $\overline{P}_i$ are lower and upper bounds of non-failure probability for i-th member of the truss; n is a number members in the truss.

The table with reliability intervals for truss bars can be formed for a plane truss. Table 2 contains the example of such table.

Table 2. Reliability of planar truss elements.

Planar truss element (Fig. 1)	$\underline{P}$	$\overline{P}$
1–2	0.99420	1.00000
1–3	0.99920	1.00000
2–3	0.99420	1.00000
2–4	0.99670	1.00000
3–4	0.99750	1.00000
3–5	0.99284	0.99994
4–5	0.99750	1.00000
4–6	0.99272	1.00000
5–6	0.99750	1.00000
5–7	0.99284	0.99993
6–7	0.99750	1.00000
6–8	0.99670	1.00000
7–8	0.99420	1.00000
7–9	0.99920	1.00000
8–9	0.99420	1.00000

For statistical data in the Table 2, the following bounds can be obtained:

$$\underline{P} = \max\left(0, \quad \sum_{i=1}^{n} \underline{P}_i - (n-1)\right) = 14.9370 - (15-1) = 0.9370,$$

$$\overline{P} = \min\left(\overline{P}_i\right) = 0.99993.$$

The reliability of planar truss is [0.93700; 0.99993]. If this interval is too wide to make a decision about the level of truss safety, then it needs to improve the quality of statistical information: refine probability functions of distributions, increase the number of control samples, etc.

Target values for the reliability index β for various design situations, and for reference periods of 1 year and 50 years, are indicated in Appendix C, Eurocode 0 “Basis of structural design”. For example, reliability index for serviceability limit state (for reference periods of 1 year) is $\beta = 2.9$. Joint Committee on Structural Safety (JCSS) Probabilistic Model Code sets target values for the reliability index β in dependence with a comparative cost of safety measures and failure consequences. However, the reliability index should be calculated individually for each structure (and structural element) based on a value of an acceptable risk [25, 26].

4. Conclusions

1. The article proposes the method for structural reliability analysis with limited statistical information about random variables: some random variables are described by a subset of interval data and others by some fuzzy probability distribution.

2. The reliability analysis examples were considered for the steel truss by the steel strength criterion based on various approaches to reliability analysis;

3. The reliability interval [0.99272; 1] by proposed method covers the reliability values by the FOSM approach 0.99354. Thus, the proposed approach allows obtaining a more cautious reliability interval for an incomplete statistical information case.

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