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Target reliability of alternative fundamental combinations in Eurocode EN1990

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Abstract. In Eurocode EN1990 action effects in persistent and transient design situations for ULS checks are derived according to three different alternative expressions for combinations of actions, to be chosen in the National Annex for use in a country. The three formulations, ([6.10], or [6.10a] and [6.10b], or [6.10a modified] and [6.10b]), which are substantially confirmed in the draft version of the new EN1990 (prEN1990:2019), are not completely equivalent in terms of structural reliability. In the present study, the reliability levels associated with each of them are compared in some relevant examples considering permanent and imposed loads for different buildings categories. In the analyses, the structural reliability indexes derived using level 2 and 3 methods are discussed considering the influences of different assumptions about statistical distributions and parameters of material resistances and action effects. The results of the sensitivity analyses confirm that the reliability level for ULS checks is also strongly dependent upon the statistical models adopted. The target reliability level recommended for use in EN 1990 (and in prEN1990:2019) is commonly reached using expression [6.10], while the adoption of expressions [6.10a] and [6.10b] can lead to lower values, especially when the coefficient of variation (COV) of the material resistance is high. Expressions [6.10a modified] and [6.10b] generally lead to very significant reductions of the reliability levels in all the investigated cases, especially when permanent loads dominate the structural design.

1. Introduction

EN1990 is the head code in the Eurocode suite and establishes the principles and requirements for safety and serviceability for all the Structural Eurocodes [1]. EN1990 provides the basis for structural design and verifications, introducing the limit state design concept to be used in conjunction with the partial factor method. In the limit state design, also known as load and resistance factor design [2], it should be verified that the design load effect does not exceed the design resistance of the structure in ultimate limit states. Design load effects are determined for each limit state considering a representative load combination defined to take into account the probability of the simultaneous application of the various load types. Load combination rules, with the associated partial and combination factors, should be defined with the final aim to guarantee a minimum target reliability level for the designed structural members for all construction materials. In this study, load combinations in the structural Eurocodes are discussed evaluating the resulting reliability levels for different building structures and considering the relevant influences of different assumptions about statistical distributions and parameters of material resistances and action effects.

Combination of actions, which are given in Section 6 of Eurocode EN 1990:2002 [3], are substantially confirmed in Section 8 of new draft of prEN1990:2019 [4]. The choice of the format to be adopted in a country is transferred to National authorities, according to clause 6.4.3.1(1) P of [1], which states that “for

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each critical load case, the design values of the effects of actions (E_d) shall be determined by combining the values of actions that are considered to occur simultaneously”.

The Ultimate Limit States (ULS) verifications in EN1990 regard the loss of static equilibrium (EQU), internal failure or excessive deformation of the structure (STR), failure or excessive deformation of the ground (GEO) and fatigue failure (FAT). The occurrence of any ULS is prevented by checking the fundamental inequality

$$E_d \leq R_d, \quad (1)$$

where E_d is the design value of action effects and R_d the design value of the resistance.

Following clause §6.4.3.2 of EN 1990, design values of the action effects in persistent and transient design situations can be derived adopting three alternative and mutually exclusive sets of fundamental combinations of actions. These sets, given below, are formally identified by expression [6.10] of EN1990 (Eq. (2)); or by the most adverse between expressions [6.10a] (Eq. (3.a)) and [6.10b] (Eq. (3.b)); or by the most adverse between expressions [6.10a modified] (Eq. (4)) and [6.10b] (Eq. (3.b)). Although in prEN1990:2019, ULS are no more formally distinct in EQU, STR and GEO, the sets of mutually exclusive combinations remain substantially unchanged: in fact, the “new” combination [8.12] corresponds to the “old” combination [6.10]; the “new” combination [8.13.a] to the “old” [6.10.a], the “new” combination [8.13.b] to the “old” [6.10.b] and, finally, the “new” combination [8.14] to the “old” combination [6.10.a modified]:

$$\sum_{j \geq 1} \gamma_{G,j} G_{k,j} + \gamma_P P + \gamma_{Q,1} Q_{k,1} + \sum_{i > 1} \gamma_{Q,i} \psi_{0,i} Q_{k,i}; \quad (2)$$

$$\sum_{j \geq 1} \gamma_{G,j} G_{k,j} + \gamma_P P + \gamma_{Q,1} \psi_{0,1} Q_{k,1} + \sum_{i > 1} \gamma_{Q,i} \psi_{0,i} Q_{k,i}; \quad (3a)$$

$$\sum_{j \geq 1} \xi \gamma_{G,j} G_{k,j} + \gamma_P P + \gamma_{Q,1} Q_{k,1} + \sum_{i > 1} \gamma_{Q,i} \psi_{0,i} Q_{k,i}; \quad (3b)$$

$$\sum_{j \geq 1} \gamma_{G,j} G_{k,j} + \gamma_P P. \quad (4)$$

In expressions (2), (3.a), (3.b) and (4), $G_{k,j}$ is the nominal or characteristic values of the j -th permanent actions, P is the relevant representative value of prestressing action, $Q_{k,1}$ is the characteristic value of the leading variable action, $\psi_{0,i} Q_{k,i}$ is the combination value of the i -th accompanying variable action, acting simultaneously with the leading one, $\gamma_{G,j}$, $\gamma_{Q,i}$ and γ_P are the partial factors for permanent, variable and prestressing actions, respectively, and ξ , considered as a combination factor, is a reduction factor for permanent actions in expression [6.10b], or [8.13.b], for which the recommended value is 0.85.

Considering that combinations in EN1990:2002 and in prEN1990:2019 coincide, in the following reference is made to the EN1990 nomenclature.

EN1990, the choice among the above formulations for use in a country is left to the National Annex, prepared by the National Standard Body.

The rationale of the alternative sets is evident, In fact, in expression [6.10a] the unfavourable part of permanent actions is assumed as leading action and all variable actions are considered as accompanying actions; in expression [6.10b] unfavourable permanent actions are also considered as accompanying ones; finally, in the third alternative, expression [6.10a modified] only includes permanent actions.

Typical argument in support of the alternative formulation [6.10a] and [6.10b] is that in this way it is possible to weaken the dependence of the reliability index β on the parameter χ , better described in the following, which is the ratio between the effects of variable actions and the total actions [1, 5], arriving to a more uniform distribution of β values over χ . Truly, very sound arguments can be also claimed endorsing adoption of the recommended formulation [6, 10]; among them, particularly relevant is the observation that the fraction of the design life, during which the minimum reliability should be assured, depends on the nature of the actions governing the design. In fact, the total duration of that fraction of the design life, which is very significant in case the structural design is governed by permanent actions, when the required minimum reliability is little varying over time, reduces as soon as the influence of variable actions increases.

Evidently, in case the variable actions predominate, the minimum reliability is required only occasionally, however, for a small amount of the life of the construction.

Finally, use of alternative [6.10a mod] and [6.10b] is supported only by merely economic arguments. In the Authors opinion, these arguments are very weak, since they lead, when permanent loads are governing the design, i.e. χ is small, to unacceptable reduction of β .

Load combinations in the Eurocodes were first discussed in [1] to formulate general recommendations for the BSI National Annex to EN1990. Simple examples for generic structural members showed that expression [6.10] leads to the most reliable structures, expressions [6.10a] and [6.10b] provide a lower but comparatively most uniform reliability level for all load ratios, while the use of modified expression [6.10a] and expression [6.10b] is not recommended since leads to rather low reliability level.

In any case, till now there are no logical reasons strong enough to clearly identify the most appropriate formulation, In fact, examining 24 National Annexes available to the Authors, it resulted that: 13 countries adopted expression [6.10]; 5 countries adopted expressions [6.10a] and [6.10b]; 5 countries allowed the designer to choose between [6.10], or [6.10a] and [6.10b]; and only one country adopted expressions [6.10a mod] and [6.10b].

An in deep evaluation about the Nationally Determined Parameter (NDPs) selected by each Member State and the reliability of structural members designed accordingly can be found in [5].

In the recent years, a discussion about reliability levels of structural members designed according to the partial factor method given in the Eurocodes has been initiated and it is currently ongoing also in view of the second generation of Eurocodes [6, 7] The discussion is mainly focused on target reliability levels [8–12] and the calibration of load factors to minimize variability of reliability levels [13, 14].

In this context, it is important to remind that the evaluation of reliability indexes and the calibration of partial factors is characterized by several uncertainties concerning for example, the assumptions on the distribution functions for loads and material resistances, the methods adopted to evaluate reliability [15–17], and not least the combination rule adopted for structural design.

Since the three formulations given in EN1990 are not equivalent in terms of structural reliability [1], aim of the present study is to compare the reliability levels associated with each expression in some significant case studies. In the examples, residential, commercial and storage buildings are considered, varying the representative parameters of the permanent and variable actions, as well as of the resistances of building materials, assuming different statistical distributions.

In the investigation, the reliability indexes β [15–17] were initially calculated according to level 2 approach; successively, a supplementary level 3 sensitivity analysis was carried out. The latter allowed to investigate the influence of different hypotheses, regarding statistical distributions of mechanical properties for structural materials and actions, on the structural reliability index β and to compare the results with the target reliability levels [9–12].

2. Methods

The study is articulated in three subsequent steps, increasing the level of deepening and complexity.

In a first phase (case 1) [18] the distribution of actions' effect (E_d) is derived, in turn, as the result of the combination of permanent and variable actions, according to one of the three alternative formulations, so focusing on their influence on the reliability. Briefly, that preliminary study aims to assess the sensitivity of the calculated reliability indexes on the adopted formulation.

In the second phase (case 2) [18], the actual distribution of effects of actions has been theoretically derived, taking also into account the influence of model uncertainties.

The outcomes of these first two phases confirmed, once again, that the reliability index β is a relative measure of the structural safety, strongly dependent on the starting assumptions.

Also aiming to clarify whether and how much level 2 simplifications influence the results, a more refined study (phase 3 – case 3) has been finally carried out, determining the actual probabilities of failure, by means of direct numerical integration of the relevant limit state functions.

The investigated case studies are referred to structural steel and reinforced concrete members in different building categories. Aiming to cover the most significant cases occurring in current design practice, they were considered residential buildings (categories A and B of EN 1991-1-1 [19]), commercial buildings

(categories C and D) and storage buildings (category E1). Besides the unfavourable permanent action G , only one unfavourable variable action Q , the imposed load, was considered, but the investigation could be easily extended to a greater number of actions.

The relative effects of permanent and variable actions were taken into account by means of the already cited parameter χ , defined as the ratio between the characteristic value of the variable action, Q_k , and the characteristic value of the global action, sum of the characteristic value of permanent and variable actions, $G_k + Q_k$:

$$\chi = \frac{Q_k}{G_k + Q_k}. \quad (5)$$

The χ values were assumed varying in the range $0 \leq \chi \leq 0.67$, being the upper limit approximately corresponding to a ratio $Q_k/G_k = 2.0$. Evidently, smaller values of χ correspond to heavy structures, where permanent load is governing the design, while $\chi \geq 0.5$ correspond to lighter structures, where imposed load is leading.

Failure in reinforced concrete structures was associated, in turn, to concrete crushing, like in compressed columns or in beams with high reinforcement ratio, or to yielding of reinforcing steel.

In each example, and for each given χ value, the structure was designed in the most economical way, thus equalizing design effects and design resistance in Eq. (1), i.e. setting $E_d = R_d = R_k/\gamma_M$, being γ_M the partial factor for resistance.

Design effects E_d were calculated adopting recommended values of partial and combination factors provided in EN1990: $\gamma_G = 1.35$; $\gamma_Q = 1.50$; $\xi = 0.85$; $\psi_0 = 0.70$ for residential and commercial buildings; $\psi_0 = 1.00$ for storage buildings. It must be remarked that, for storage buildings, the group [6.10a modified+6.10b] is the unique alternative to [6.10], since in that case the group [6.10a +6.10b] coincides with expression [6.10].

Of course, for a given value of χ , adoption of expression [6.10] leads to the maximum required design strength, $R_{d \max}(\chi)$, $R_{d \max}(\chi) = R_{d[6.10]}(\chi)$. Adopting alternative expressions [6.10a+6.10b] or [6.10a modified+6.10b], the required strength reduces to $R_{d[6.10a+b]}(\chi)$ or to $R_{d[6.10a \text{ mod}+b]}(\chi)$, where the index indicates the expressions used in designing the structures. With obvious symbolism [18, 19], the characteristic resistances required by the various alternative expressions can be easily obtained as a function of $R_{d[6.10]}(\chi)$, in the forms:

$$R_{k[6.10]}(\chi) = \gamma_M R_{d[6.10]}(\chi) = \gamma_M R_{d \max}(\chi); \quad (6.a)$$

$$R_{k[6.10a+b]}(\chi) = \gamma_M R_{d[6.10a+b]}(\chi) = \gamma_M R_{d[6.10]}(\chi) \frac{\max\left(1 + \gamma_Q \gamma_G^{-1} \psi_0 \frac{\chi}{1-\chi}; \xi + \gamma_Q \gamma_G^{-1} \frac{\chi}{1-\chi}\right)}{1 + \gamma_Q \gamma_G^{-1} \frac{\chi}{1-\chi}}; \quad (6.b)$$

$$R_{k[6.10a \text{ mod}+b]}(\chi) = \gamma_M R_{d[6.10a \text{ mod}+b]}(\chi) = \gamma_M R_{d[6.10]}(\chi) \frac{\max\left(1; \xi + \gamma_Q \gamma_G^{-1} \frac{\chi}{1-\chi}\right)}{1 + \gamma_Q \gamma_G^{-1} \frac{\chi}{1-\chi}}. \quad (6.c)$$

In the first two phases of the study, for the resistance and actions effects they were considered the statistical parameters, mean, μ , and coefficient of variation (COV), V , summarized in Table 1, assuming normal or log-normal probability density functions.

Table 1. Statistical properties of relevant actions and strengths.

Strength/Action	Symbol	Mean μ	COV (V)	γ_M
Steel (yield stress)	R	$R_k + 1.64 \sigma_R$	0.07	1.00
Rebars (yield stress)	R	$R_k + 1.64 \sigma_R$	0.07	1.15
Concrete compressive strength	R	$R_k + 1.64 \sigma_R$	0.15	1.50
Concrete compressive strength ⁽¹⁾	R	$R_k + 1.64 \sigma_R$	0.20	1.50
Permanent action (case 1)	G	$G_k - 1.64 \sigma_G$	0.10	
Permanent action (case 2)	G	G_k	0.10	
Imposed load (residential etc.)	Q	$\sim 0.3 Q_k$	1.42	
Imposed load (shopping etc.)	Q	$\sim 0.6 Q_k$	0.35	
Imposed load (storage etc.)	Q	$\sim 0.8 Q_k$	0.15	

⁽¹⁾ Cast in situ, no special control

In the following sections, the reliability levels will be assessed for different categories of imposed loads (residential, shopping and storage). Climatic actions can be easily added in the analysis; however, the obtained reliability levels will be influenced also by the considered locations. For example, for snow loads different climatic zones can be characterized by significantly different values of the coefficient of variation [20]. The reliability of roof structures designed according to the Eurocodes and subjected to snow loads have been already discussed in [21, 22] and by the authors in [23, 24] and [25] considering also climate change influence. The results highlighted lower values than the target ones for lightweight roof structures; however, it must be recalled that often additional safety margins are intentionally introduced in codes, increasing the characteristic ground snow loads resulting from the statistical analysis of data in the adopted snow load maps, as observed by the authors in [26].

Concerning wind loads, the evaluation of structural reliability levels involves a complete probabilistic description of all the relevant parameters in the wind load chain (shape, roughness and gust factor, together with the basic wind velocity). An example of reliability assessment is carried out with reference to the benchmark two-storey steel frame defined in the example applications of the JCSS model code [27]. The benchmark structure is subjected to permanent (G), imposed (Q) and wind load (W) and designed adopting the expression [6.10] in EN1990 [3]. Adopting the probabilistic models given by the JCSS [27], the variation of reliability levels with load ratio $\chi = W_k / (G_k + Q_k)$ is evaluated considering different probability density function (pdf) for the wind velocity, Gumbel, Weibull and Generalized Pareto Distribution (GPD). The results are shown in Fig. 1 and highlight the sensitivity of the reliability measure to the adopted pdf for the wind velocity. The heavy tail of the GPD leads to significantly lower values of reliability than those obtained adopting Gumbel and Weibull pdf. Beside the high sensitivity of the outcomes on the adopted extreme value distribution, we can notice the relevant decrease of reliability when the COV of the wind action is increased: in fact, looking at the Fig. 1, it clearly emerges that reliability indexes obtained assuming COV = 0.1 (solid lines), are significantly higher than those pertaining to COV = 0.2 (dashed lines).

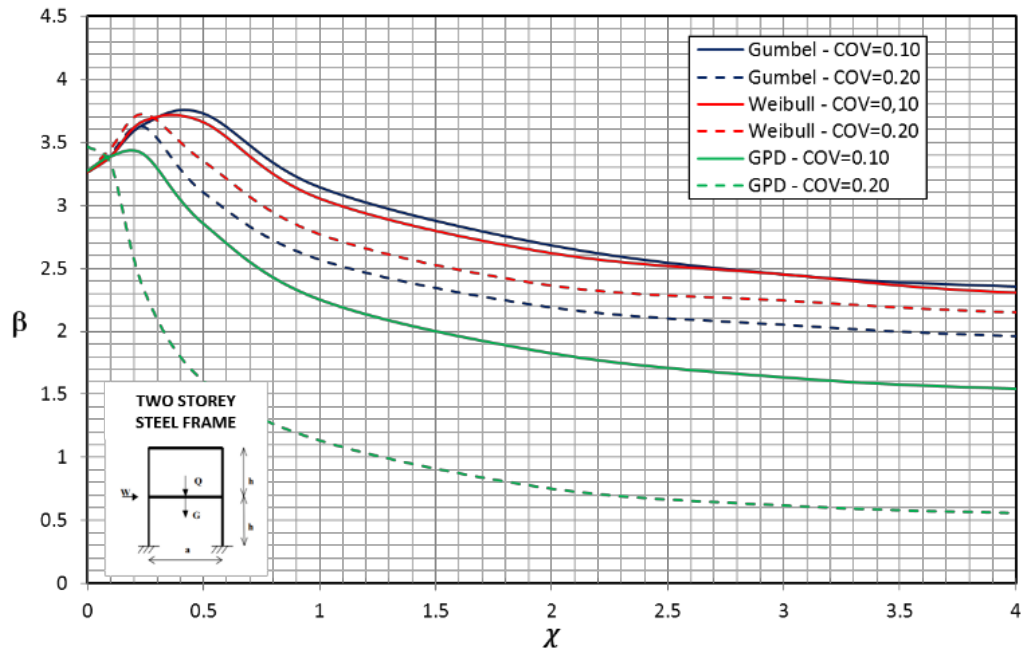


Figure 1. $\beta - \chi$ reliability curves for the benchmark two-storey steel frame described in [27].

Values in table 1 are consistent with those recommended by the *Joint Committee on Structural Safety* (JCSS) [27] and currently adopted in literature [5, 28–33]. As better described in §3.1, in the first two phases of the research, the statistical parameters of the imposed load were obtained fitting the upper tail of the extreme values distribution with the upper tail of a normal distribution, according to the JCSS recommendations. Furthermore, in order to adequately cover the whole range of current design applications, the coefficient of variation of imposed loads was selected close to its upper bound, in the case of residential buildings, and close to its lower bound, in the other cases.

The resistance R was modelled as the product of three independent log-normally distributed variables,

$$R = Azf, \quad (7)$$

where A and z are suitable geometrical properties, for instance an area and an inner lever arm, and f an appropriate strength, namely yield stress f_y for steel, and compressive strength f_c for concrete.

The COVs for the geometrical properties (V_A and V_z) were taken equal to 6 % for r.c. sections and to 3 % for steel ones. Regarding the strength of building materials, the COVs were adopted according to Table 1. More into detail, for concrete, beside the usual value $V = 0.20$, it was also explored the reduced value $V = 0.15$, which is consistent with a more stringent quality control during execution. Whence, recalling eq. (7), the COV of R results 17.3 % for high quality control concrete ($V = 0.15$), 21.8 % for concrete cast in situ with normal quality control ($V = 0.20$), and 8.19 % for structural steel and reinforcing steel ($V = 0.07$).

It must be underlined that results pertaining to different materials or to different failure modes cannot be directly compared, since the assessment formulae currently provided in Eurocodes, or in other structural standards, frequently contain additional safety, although not explicitly declared.

2.1. Evaluation of statistical parameters for imposed loads

Live loads on building floors are induced by the weight of furniture, equipment, stored items and occupants; they depend on building's category and vary in time and space. In a first approximation, spatial variations of live loads can be assumed to be homogeneous, while time variations can be represented by two components, the sustained load and the intermittent load [27, 34, 35].

The sustained load includes the weight of furniture and heavy equipment; its short-term variations are generally included in uncertainties. The intermittent load, characterized by usually short relative

duration, represents all other kinds of live loads not covered by the sustained load, like gathering of people, or stacking of furniture during maintenance, as well as, when load-structure interaction is negligible, the dynamic magnification.

The equivalent uniformly distributed load q of the sustained load part is characterized by its overall mean intensity, μ_q , and by its standard deviation σ_q , expressed by [27].

$$\sigma_q = \sqrt{\sigma_y^2 + k\sigma_u^2 \min\left(1; \frac{A_0}{A}\right)}; \quad (8)$$

being σ_y is the standard deviation of the effects of a zero mean normal variable Y ; σ_u the standard deviation of the effects of a zero mean random field $U(x, y)$ with a specific skewness to the right; A the tributary area of the considered element, A_0 the reference area and k an adjustment factor depending on the shape of the influence surface. Generally, k varies in the range 1.0 – 2.4 [27, 35]. In the present study it has been assumed $k = 2.0$, which is a typical value for bending moments on beams or axial forces in columns.

If the time between load changes is exponentially distributed, the number of load changes is Poisson distributed and the process is a Poisson pulse process. The cumulative distribution function (CDF) of the arbitrary point in time value q of a rectangular pulse process is expressed by

$$F_q(x) = (1-d)H(x) + dF_{q'}(x), \quad (9)$$

where $H(x)$ is the Heaviside unit step function, $F_{q'}(x)$ is the CDF of q' and d is the fraction of time the process is non-zero. For the sustained load, obviously, it is $d = 1.0$.

From equation (9), the CDF of the life-time maximum value q_{\max} results

$$F_{q_{\max}}(x) = \left[(1-d)H(x) + dF_{q'}(x) \right] \exp\left[-\lambda T (1 - F_{q'}(x)) \right], \quad (10)$$

where T is a suitable time interval linked with the expected life of the building and λ the rate of the process, i.e. the number of pulses in the time unit. For high values of x it is $F_{q'}(x) \approx 1.0$, therefore (10) reduces to

$$F_{q_{\max}}(x) \approx \left[(1-d)H(x) + d \right] \exp\left[-\lambda T (1 - F_{q'}(x)) \right]. \quad (11)$$

The equivalent uniformly distributed load (EUDL) p of the intermittent load part can be represented by the same stochastic field as the sustained load, whose representative parameters, mean intensity, μ_p , and standard deviation σ_p , depend on the building's category. The intermittent load maxima occur as a Poisson rectangular pulse process characterized by a mean occurrence rate ν , and by an average duration of each pulse, d_p , depending on the imposed load classification.

The extreme values of EUDL, p , in the reference time interval T are again expressed by equations like (10) and (11).

Adopting input data parameters for live load distributions consistent with table 2.2.1 of JCSS Code [27] and assuming that q and p are stochastically independent and described by appropriate gamma distributions, the following parameters of the extreme value distributions of the maxima of imposed loads can be found: $\mu_{q,\max} \approx 0.39 \text{ kN/m}^2$, $\text{COV} \approx 1.30$, $Q_k = 2.00 \text{ kN/m}^2$ in residential buildings; $\mu_{q,\max} \approx 2.19 \text{ kN/m}^2$, $\text{COV} \approx 0.50$, $Q_k = 5.00 \text{ kN/m}^2$ in commercial buildings and $\mu_{q,\max} \approx 4.15 \text{ kN/m}^2$, $\text{COV} \approx 0.30$, $Q_k = 7.50 \text{ kN/m}^2$ in storage buildings. Obviously, in storage buildings intermittent loads are not significant.

Since characteristic values of imposed loads are extremely sensitive to input data variations, the parameters in Table 1 were adopted for the analyses, in order to appropriately cover the possible range of variations.

3. Results and Discussion

In the following sub-sections, the results in terms of reliability curves are presented and discussed for the three case studies described in the Introduction:

- Case 1: effects of actions represented by alternative expressions of load combinations given in EN1990;
- Case 2: effects of actions represented by the theoretical combination, taking into account also the influence of model uncertainties;
- Case 3: Level 3 sensitivity analysis determining the actual probabilities of failure, by means of direct numerical integration of the relevant limit state functions.

3.1. Case 1: distribution of effects represented by alternative load combinations

In the first phase of the analysis (case 1), effects of the actions were evaluated considering all the three EN1990 alternative formulations, already recalled in §1, setting the characteristic value of permanent actions as the 95 % fractile.

To make easier the analytical treatment, two limit state functions were analyzed: the safety margin, S , in case of normal variables,

$$S = R - E \quad (12)$$

and the safety factor, Z , for log-normal variables,

$$Z = \frac{R}{E}. \quad (13)$$

The probability of failure is then given by

$$P_f = P[(R - E) < 0] = \Phi(-\beta) \quad (a) \quad \text{or by} \quad P_f = P\left[\left(\frac{R}{E}\right) < 1\right] = \Phi(-\beta) \quad (b), \quad (14)$$

being Φ the normal cumulative distribution function and β the associated reliability index. To estimate (14.b), the right tail of the sum of two independent log-normal variables G and Q was fitted with a log-normal tail using the Fenton and Wilkinson approximation [36, 37].

It must be underlined that, at the level 2, expressions (14.a) and (14.b) lead to different β values, despite from the theoretical point of view they are absolutely equivalent. However, these different outcomes can be easily explained, recalling that, in the aforementioned cases, the tails of the actual distributions of R and E have been fitted with different *pdfs*: normal distributions referring to safety margin, Eq. (14.a), log-normal distributions referring to safety factor, Eq. (14.a). That is a further reason to emphasize that β values cannot be directly compared, as their significance is largely conventional [38].

It is worth to note that in EN 1990 characteristic values of variable actions are defined as those characterized by 2 % of exceedance on annual basis (roughly corresponding to 50 years return period), and the target value of β for 50 years reference period is fixed to 3.8 for Consequence Class 2 structures (i.e. "normal" consequences), corresponding to an accepted failure probability $P_f \approx 7.23 \cdot 10^{-5}$ in 50 years.

3.1.1. Evaluation of β reliability index for normal variables

If R and E are normally distributed, the reliability index β is given by [1, 39, 40–42]:

$$\beta = \frac{\mu_R - \mu_E}{\sqrt{\sigma_R^2 - 2\rho_{RE}\sigma_R\sigma_E + \sigma_E^2}}; \quad (15)$$

being μ_R and μ_E the mean values of variables, σ_R and σ_E their standard deviations, and ρ_{RE} the correlation coefficient, which is zero for uncorrelated variables, like assumed in the present investigation.

As remarked in the introduction, the Eurocode does not give a unique expression to evaluate actions' effects E_d , leaving the choice among the three alternatives (2–4) to the National Annex. Clearly, since the intent of the fundamental combination is to represent, as close as possible, the design values of the actual actions' effects, none of the three alternatives is unanimously recognized as the one able to represent in the most effective way the design load combination.

As better explained in the following §3.3, it is possible to directly compare alternative load combination rules, in terms of structural reliability, only when the reliability is calculated with reference to the effective joint probability distribution of actions, to which the structure will be subjected during its design life.

Evidently, being at least $R_d = E_d$ (see Eqs. (6)), the design depends on the choice of the expression for the fundamental combination. Once designed the structure, its reliability will be a function of the effective distribution of actions' effects. In the spirit of the sensitivity analysis, to investigate how different choices on combination rules influence the structural reliability, each structural element was designed according to one of the given expressions (e.g. [6.10]; or [6.10a] and [6.10b]; or [6.10a mod] and [6.10b]) and the β values, associated to each set of the alternative load combinations, were calculated. Each combination rule is assumed to be, in turn, the one best representing the actual situation, which is denoted with the prefix "act" in the following: in that way, nine $\beta - \chi$ reliability curves are obtained, three for each set of expressions. Each curve represents the reliability curve associated to a structural element designed using one of the three alternative sets of load combinations, depending on the load combination best representing the actual situation.

Let Ξ the load combination used for the design, $\Xi = ([6.10]; [6.10a+b]; [6.10a \text{ mod}+b])$. Manifestly, for each value of χ , the reliability index β satisfies the following inequalities:

$$\beta(\chi, \Xi, [\text{act } 6.10]) \leq \beta(\chi, \Xi, [\text{act } 6.10a + b]) \leq \beta(\chi, \Xi, [\text{act } 6.10a \text{ mod} + b]), \quad (16)$$

where, as said, terms in square brackets indicate the combination of actions assumed to be the best representation of the real situation.

Significant outcomes of the analysis are shown in Fig. 2 and 3. To make easier their interpretation, and according to what previously said, in the legends the nine $\beta - \chi$ reliability curves are labelled in such a way that the term in brackets indicates the expression better representing the actual situation and the first term indicates the expression adopted to design the element.

The case illustrated in Fig. 2 refers to steel members' failure in a commercial building. Reliability curves are characterized by different trends:

- Let consider structures designed according to [6.10]:
 - i. if the actual combination is represented by [6.10], reliability increases in the interval $\chi = 0 - 0.3$, and decreases for $\chi > 0.3$, when the quota of variable imposed loads becomes more relevant;
 - ii. if the actual combination is represented by [6.10a+b] or by [6.10a mod+b], the calculated reliability further increases, since designing with [6.10] introduces overstrength. Again, an increasing trend is shown when permanent action predominates and χ is up to 0.3.
- Let consider structures designed according to [6.10a+b]:
 - iii. if the actual combination is represented by [6.10a+b], the trend of the $\beta - \chi$ curve is similar to that described in the previous case I;
 - iv. if the actual combination is represented by [6.10] a decreasing reliability trend is observed when [6.10a] governs the design, the trend is inverted when [6.10b] dominates;

- v. if the actual combination is represented by [6.10a mod+b] the structure results oversized and the behavior of the $\beta - \chi$ curve is similar to that described in previous case ii.
- Let, finally, consider structures designed according to [6.10a mod+b]:
 - vi. if the actual combination is represented by [6.10a mod+b] an almost constant reliability is observed as soon as [6.10a mod] governs;
 - vii. if the actual combination is represented by [6.10] the $\beta - \chi$ curve is significantly decreasing in the range where [6.10a mod] governs. A similar trend is observed if the actual combination is represented by [6.10a+b].

Similar trends characterize curves in the following Fig. 3, 4 and 5.

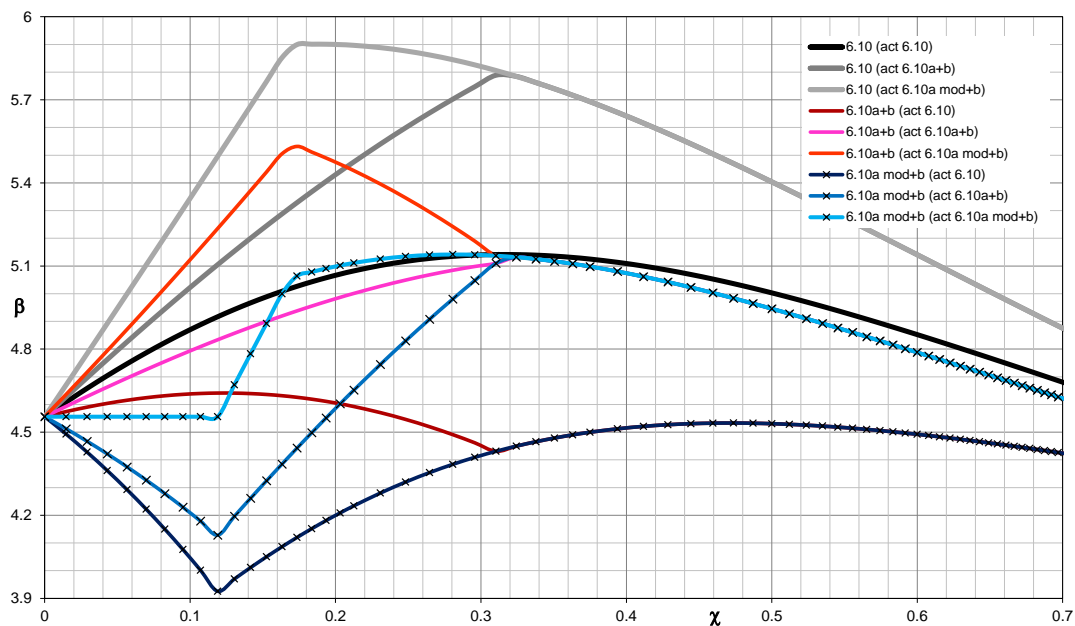


Figure 2. $\beta - \chi$ reliability curves for steel member in commercial buildings – normal variables.

For the resistance of reinforced concrete structures, they are considered two relevant cases, according as yielding of steel rebars or crushing of concrete governs. In the following cases indicated as “rebars” correspond to yielding of steel reinforcement, cases indicated as “concrete” correspond to crushing of concrete.

The diagrams in Fig. 3 refer to compressed concrete failure of reinforced concrete members in a residential building.

The two complete sets of curves refer to different coefficient of variations of concrete strength: $V = 0.20$, corresponding to “normal” quality control level, and $V = 0.15$, corresponding to improved quality control level. The results are of particular interest, as they refer to an extreme case, where small variations of input data are associated to relevant variations of the reliability index. The phenomenon is so evident that the two sets of curves do not overlap.

In a previous work [18], similar results were reported more extensively, considering also the effects of material over-strength, drawing the following conclusions:

- although, as explained later, reliability indexes calculated here are overestimated, since the incidence of model uncertainties is disregarded, β values seem often smaller than the target value, 3.8, especially in r.c. residential buildings, when concrete crushing governs the failure and no special quality control measures are adopted for the material itself ($V = 0.2$);
- in case of over-strength [18], the reliability indexes increase; the increment is more relevant for steel members than for reinforced concrete members;

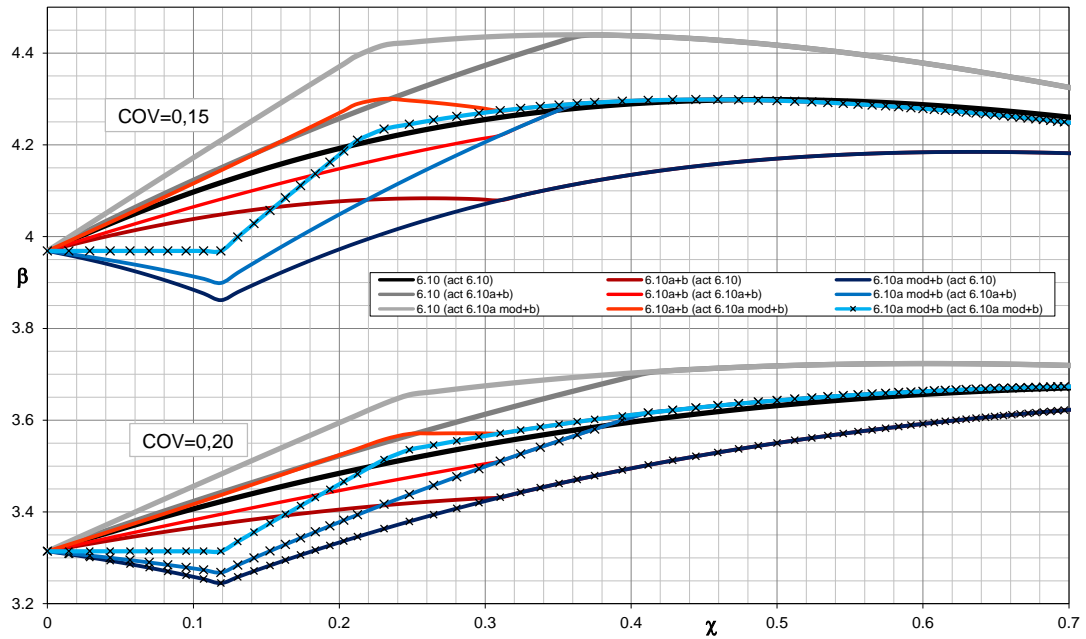


Figure 3. β – χ reliability curves for concrete failure of r.c. members in residential buildings (different quality control level for concrete) – normal variables.

- in all the investigated cases, structures designed using [6.10] exhibit higher β values than those designed using alternative expressions, with peaks in the range $0.1 < \chi < 0.35$; the gain is particularly relevant when the actual situation is best fitted by alternative expressions;
- values associated with [6.10a] and [6.10b] can be significantly smaller than those associated with [6.10], mainly in vicinity of $\chi = 0.35$, which is quite often the case in current design; moreover, in r.c. residential buildings β values are often below the target;
- expressions [6.10 a modified] and [6.10b] lead in all considered cases to significant reduction of β , especially in the range $0 < \chi < 0.25$, where local minima of β – χ curves are very low; since this range corresponds to heavy structures, like r.c. massive ones, this behaviour seems to be in contrast with the inspiring criterion of such an expression, which is based on permanent loads primacy;
- the lowest β curve associated to [6.10] practically always envelopes alternative curves.

Since the influence of the model uncertainty factors is disregarded, β values evaluated up to now appear generally overestimated. Actually, the design value of the capacity demand E_d [1],

$$E_d = \gamma_f \gamma_{Sd} E_k = \gamma_F E_k, \quad (17)$$

is higher than that strictly required by reliability analysis. In equation (17) E_k is the characteristic value of the effect of the load combination, γ_f the partial factor for uncertainty in representative values of the effect, γ_{Sd} the model uncertainty factor and γ_F the partial factor for actions.

Hence, despite of the relative meaning of β and regardless of the disturbance caused by the model uncertainty factors, an accurate calculation requires to consider, instead of γ_F , only its quota γ_f . To check the sensitivity of β on model uncertainty factors, the above-mentioned commercial building were re-examined considering the contribution of the γ_{Sd} factors on the action side, and updating the β – χ reliability curves for steel members' failure. The uncertainty factor γ_{Sd} has been hypothesized only depending on the nature of the action, and, according to recommendations in EN1990, it has been assumed 1.06 for permanent loads and 1.15 for variable loads.

Since the study aimed to compare different $\beta-\chi$ reliability curves, derived under homogeneous hypotheses, model uncertainty factors were disregarded on the resistance side, leaving unchanged the partial factors γ_M , since they do not affect the comparison.

Diagrams in in Fig. 4, compared to the corresponding ones in Fig. 2, show the influence of γ_{Sd} on the evaluation of the reliability index β .

3.1.2. Evaluation of β reliability index for log-normal variables

In case of log-normal variables, the expression for the reliability index β is [39–41]

$$\beta = \frac{\ln(m_R) - \ln(m_E)}{\sqrt{\ln(1+V_R^2) + \ln(1+V_E^2) - 2\rho_{RE}\sqrt{\ln(1+V_R^2)\ln(1+V_E^2)}}}; \quad (18)$$

where m_R and m_E are the medians of R and E , V_R and V_E are the corresponding coefficients of variation and ρ_{RE} is the correlation coefficient, which is again assumed to be zero under the hypothesis of uncorrelated variables.

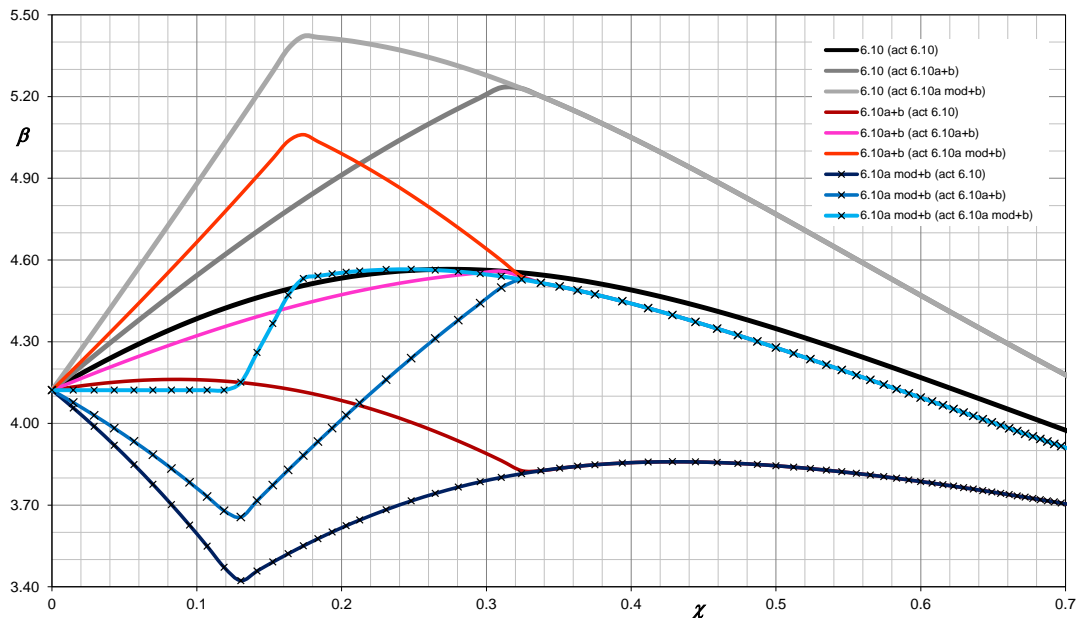


Figure 4. $\beta-\chi$ reliability curves for steel members in commercial buildings – normal variables, model uncertainty factors on actions considered.

Fig. 5 shows the outcomes of the analysis developed for the same case as in Fig. 2. Comparing diagrams in Fig. 2 and 5, it is evident that the shape of the curves and their trends are similar, so that the analysis of the diagram in Fig. 5 leads to analogous conclusions to those expressed in §3.1. At the same time, it must be highlighted that β values evaluated considering safety factor Z and log-normal variables are different, and in some cases rather significantly, than those calculated considering safety margin S and normal variables. These discrepancies, due to the inaccuracies of level 2 approach and to the errors of the Fenton and Wilkinson approximation, confirm once again that β is a relative measure of the reliability. In other words, the reliability index β fully represents structural reliability only if it is calculated exactly matching the assumptions adopted to evaluate the target values.

3.2. Case 2: effects of actions represented by the theoretical combination

When G and Q are both normal variables, their joint *pdf* can be derived theoretically. In this case, the appropriateness of the three normative sets of load combinations can be directly checked, comparing the design capacity associated with each of them with the effective design capacity derived by the

theoretical joint *pdf*. This is the aim of the second phase of the present study (case 2) [18], where permanent actions have been assumed not to significantly vary during the design working life of the structure.

To emphasize the comparison, the characteristic value of permanent actions has been set to $G_K = \mu_G$ (see table 1), assuming again $COV = 0.1$, which is upper limit justifying the assumption $G_K = \mu_G$.

The design value of the effect E of a single normally distributed action, considered alone, is (17)

$$E_d = \gamma_{Sd} (\mu_E + \alpha_E \beta \sigma_E) = \gamma_{Sd} (\mu_E + 2.66 \beta \sigma_E) = \gamma_f \gamma_{Sd} E_k = \gamma_F E_k; \quad (19)$$

where μ_E and σ_E are the mean and the standard deviation of E , respectively, $\beta = 3.8$ is the target reliability index for a 50-year reference period and a CC2 structure [3], and $\alpha_E = 0.7$ is the FORM [16] [42] sensitivity factor. Hence, from (19) it follows

$$\gamma_{Sd} = \frac{\gamma_F E_k}{\mu_E + 2.66 \beta \sigma_E}, \quad (20)$$

which, applied separately on G and Q , gives partial factors γ_g and γ_q to be introduced in formulae (2), (3.a), (3.b) and (4) to cut off the model uncertainties.

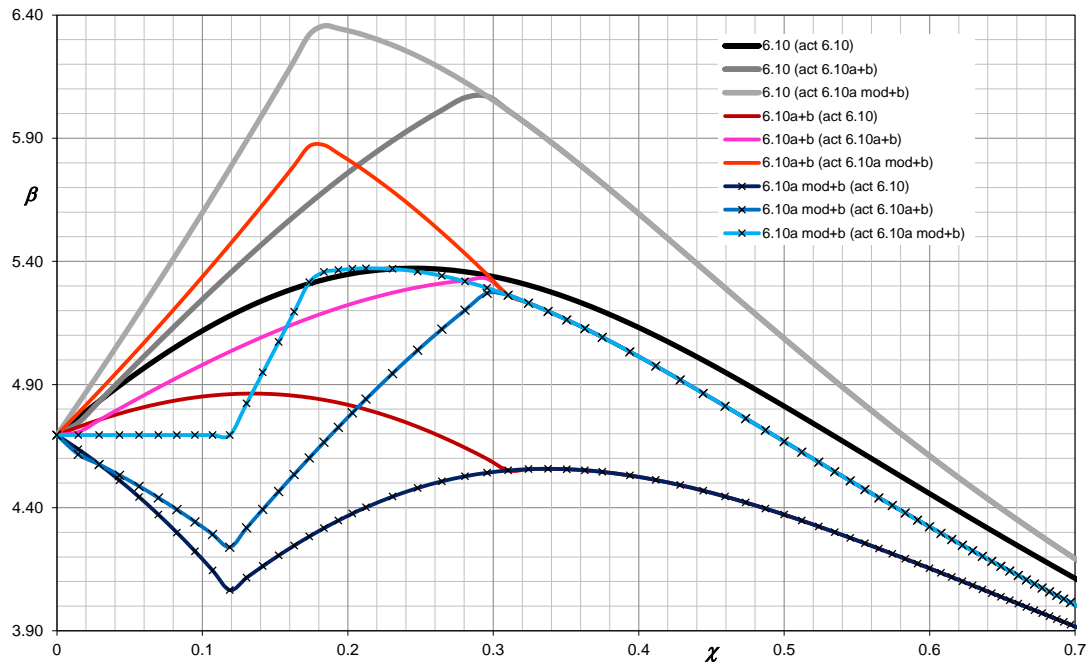


Figure 5. $\beta - \chi$ reliability curves for steel members in commercial buildings – log-normal variables.

Since G and Q are assumed to be normally distributed, and $G_K = \mu_G$, the theoretical design value $(G+Q)_d$ results

$$(G+Q)_d = \mu_G \left[1 + \frac{\chi}{1-\chi} (1+1.645V_Q)^{-1} + 2.66 \sqrt{V_G^2 + \left(\frac{\chi}{1-\chi} \right)^2 V_Q^2 (1+1.645V_Q)^{-2}} \right], \quad (20)$$

where V_G and V_Q are the coefficients of variation of G and Q , respectively, and χ is defined by (5).

3.2.1. Evaluation of β reliability index

The influence of the model uncertainties γ_{Sd} can be easily evaluated adopting, in eqs. (2), (3) and (4), γ_g and γ_q instead of γ_G and γ_Q . For each case study, assuming G and Q normally distributed, it is possible to evaluate the reliability index β by means of (15), excluding the effects of model uncertainty factors γ_{Sd} . In accordance with §3.1.1, weighted values of γ_{Sd} depend on the ratio χ .

Fig. 6 shows the results obtained for the case of commercial buildings, with reference to the alternative sets of load combinations in case of structural steel, re-bars, and concrete with different levels of quality control ($V = 0.15$ and $V = 0.20$).

Results for residential or storage building categories look similar, both in terms of curves' shapes and values, to those illustrated in Fig. 6, and the following remarks can be drawn:

- Eq. [6.10] guarantees almost constant reliability, independently on structural material and on χ , while alternative sets lead to considerable reduction of β ; particularly when $0 < \chi < 0.25$ or failure is governed by steel (either re-bars or structural steel);
- r.c. structures are particularly sensitive to the level of quality control measures adopted during concrete preparation and casting; beside that, when yielding of steel re-bars govern the failure, they are more reliable than the other structures examined in the present study;
- β values seem to be above the target value only for steel reinforcement; nevertheless this conclusion should be carefully considered: since β increases considerably whenever the material's strength increases or its coefficient of variation decreases, this phenomenon is consequence of the adoption of γ_M values for re-bars ($\gamma_M = 1.15$) sensibly higher than for structural steel ($\gamma_M = 1.00$).

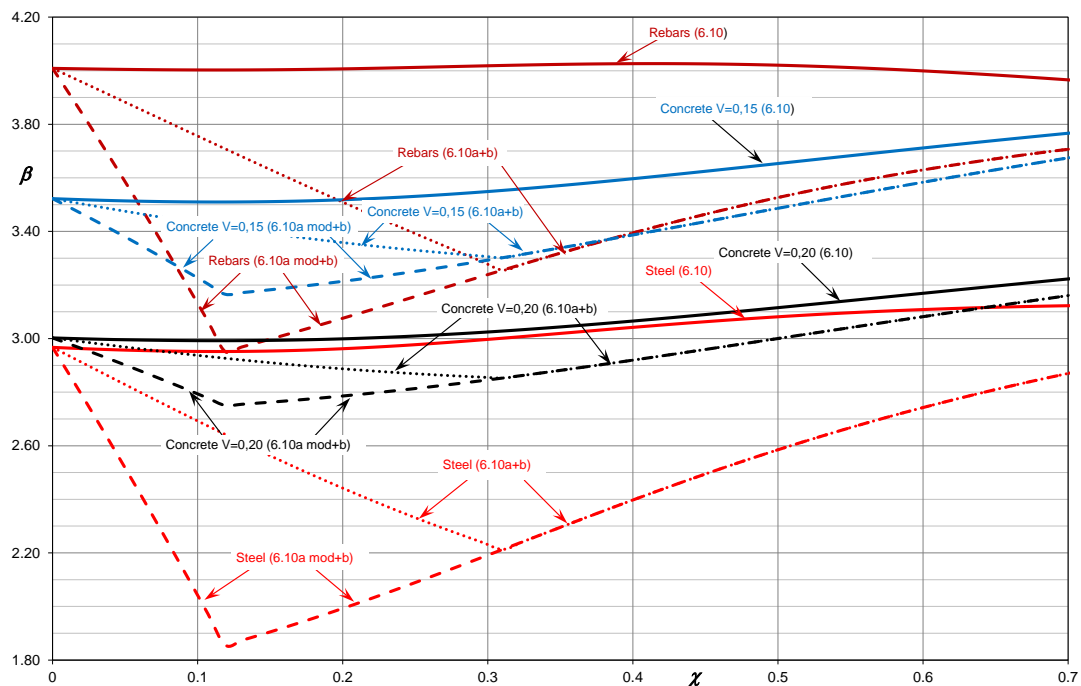


Figure 6. $\beta - \chi$ reliability curves for commercial buildings – $G_K = \mu_G$.

In addition, the influence of the replacement of $G_K = \mu_G$, with $G_K = \mu_G + 1.645 \sigma_G$ has been investigated. An example of these analyses is illustrated in Fig. 7, which refers as well as Fig. 6, to building categories C and D.

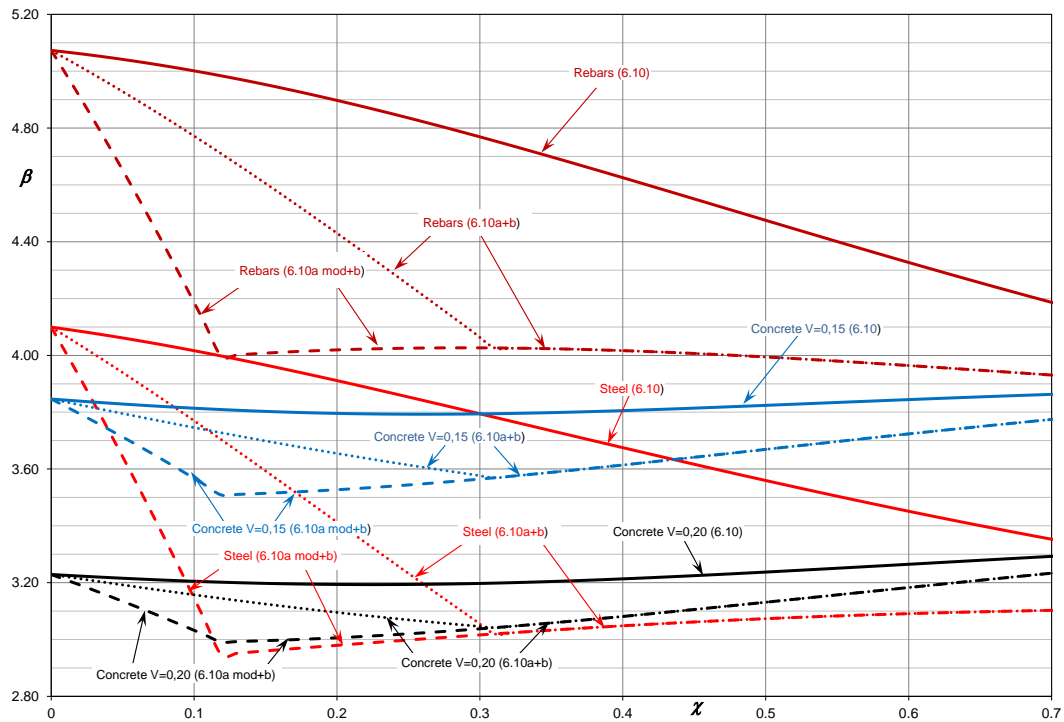


Figure 7. β – χ reliability curves for commercial buildings – $G_K = \mu_G + 1.645 \sigma_G$

The curves in Fig. 7 demonstrate that β values strongly depend on the definition of G_K , in effect, if compared with the previous ones (Fig. 6) all the curves are considerably up-shifted, in particular those pertaining to structural steel and to concrete failure of r.c. elements subjected to strict quality controls.

3.3. Case 3: Level 3 sensitivity analysis

The results discussed above, and particularly the circumstance that β values depend on the initial assumptions, suggested the need of more refined investigation to separate the intrinsic variations of β , effectively influenced by the assumptions, from those pertinent to level 2 approximations. For that reason, a more advanced sensitivity analysis (case 3), aiming to calculate the exact probability of failure, P_f , by means of a level 3 reliability approach has been undertaken.

The present subsection illustrates the comparison between reliability levels reached by applying expression [6.10] (Eq. (2)) and the reliability levels obtained with expressions [6.10a] and [6.10b] (Eqs. (3.a) and (3.b)).

Owing the fact that, as remarked before, expression [6.10a modified] (Eq. (4)) leads to probably excessive, reduction of the reliability index in a particularly relevant interval of the parameter χ , it will be not further considered.

To investigate the influence on structural reliability of the variable action models, the extreme values of imposed loads have been hypothesized described, in turn, by a normal, a log-normal, or a gamma distribution, often adopted in current practice. Obviously, the parameters describing these three distributions have been calibrated to attain the same characteristic values and similar upper tails.

Varying load combinations and building materials, the following cases were investigated:

- variable (imposed) loads described by normal, log-normal or gamma distributions;
- characteristic value of permanent load expressed by $G_K = \mu_G$ or by $G_K = \mu_G + 1.645 \sigma_G$;
- partial factors for permanent and variable actions γ_G and γ_Q or γ_g and γ_q , according if the model uncertainty factors γ_{Sd} is considered or not, as previously discussed in §3.1.2.

Assuming resistances R log-normally distributed, and permanent loads G normally distributed with $V = 0.1$ (see §2), once assigned the actual *pdf* of each random variable, the P_f associated to each case

study was calculated, by means of a full probabilistic procedure, via numerical integration of the limit state function. To make easier the interpretation of results, the probability of failure P_f was converted into the corresponding β value, using the inverse of the Φ function.

Fig. 8, 9, 1 and 11 summarize some particularly relevant results.

In Fig. 8 the plotted reliability curves refer to the concrete failure of r.c. members in storage buildings.

The curves are grouped into families, derived according to expression [6.10], consisting of three curves each. The curves of each family are associated to different *pdfs* (normal, log-normal or gamma) for imposed loads, as indicated in the figure.

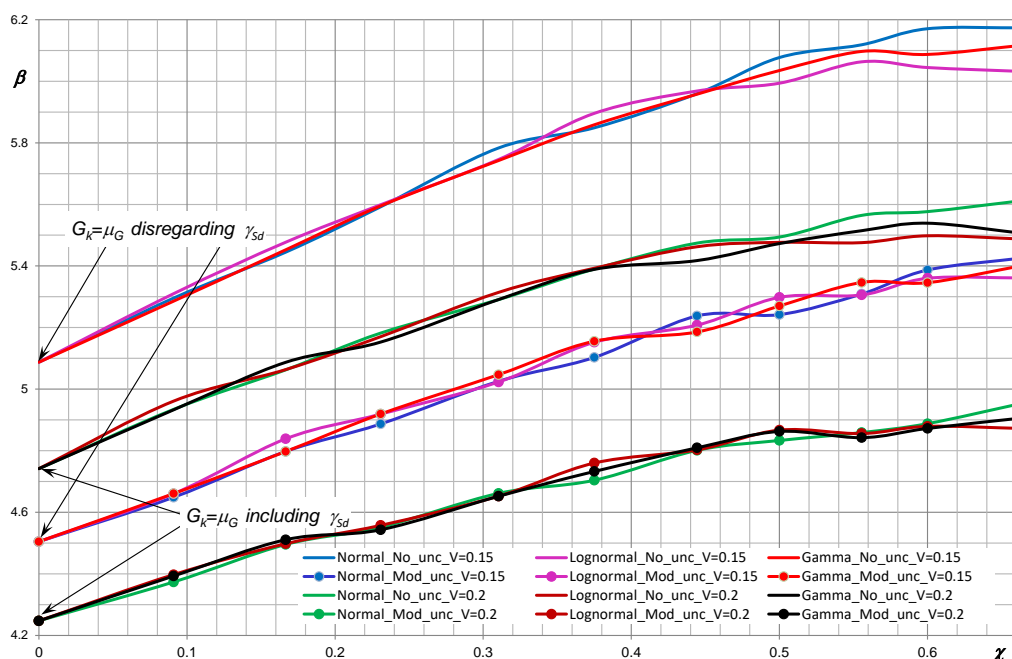
Concerning concrete strength, in Fig. 8 three different families of curves are plotted for each examined case, $V = 0.15$ and $V = 0.20$. These families correspond to the cases: $G_K = \mu_G$ and partial factors γ_g and γ_q , i.e. disregarding the model uncertainty factors γ_{Sd} ; $G_K = \mu_G$ and partial factors γ_G and γ_Q , i.e. including the model uncertainty factors γ_{Sd} ; and $G_K = \mu_G + 1.645 \sigma_G$; and partial factors γ_G and γ_Q , respectively, characterized by increasing values of reliability index.

Investigating diagrams in Fig. 8, we can observe that:

- the level of quality controls influences considerably the reliability; in fact, the reduction of scattering of concrete strength, associated with more stringent controls leads to significant increases of β values, especially for lightweight structures ($\chi > 0.5$);
- in the present example, the reliability is slightly influenced by the *pdf* of variable actions. That is not surprising since the COVs of live loads in storage buildings are comparable with the COVs of concrete strength and partial factor for concrete is relatively high ($\gamma_C = 1.50$);
- the values of β are always well above the target value for all the considered cases; therefore, a sufficient safety margin exists to cover also the model uncertainty on the resistance side.

Fig. 9, built up according to the same criteria followed for Fig. 8, refers to concrete failure of a r.c. member in a residential building, which is the case study previously considered in Fig. 3.

For the sake of clarity, in the figure curves referred to expressions [6.10a) and [6.10b), which lead in some cases to considerable reductions of the reliability index, are omitted.



(a)

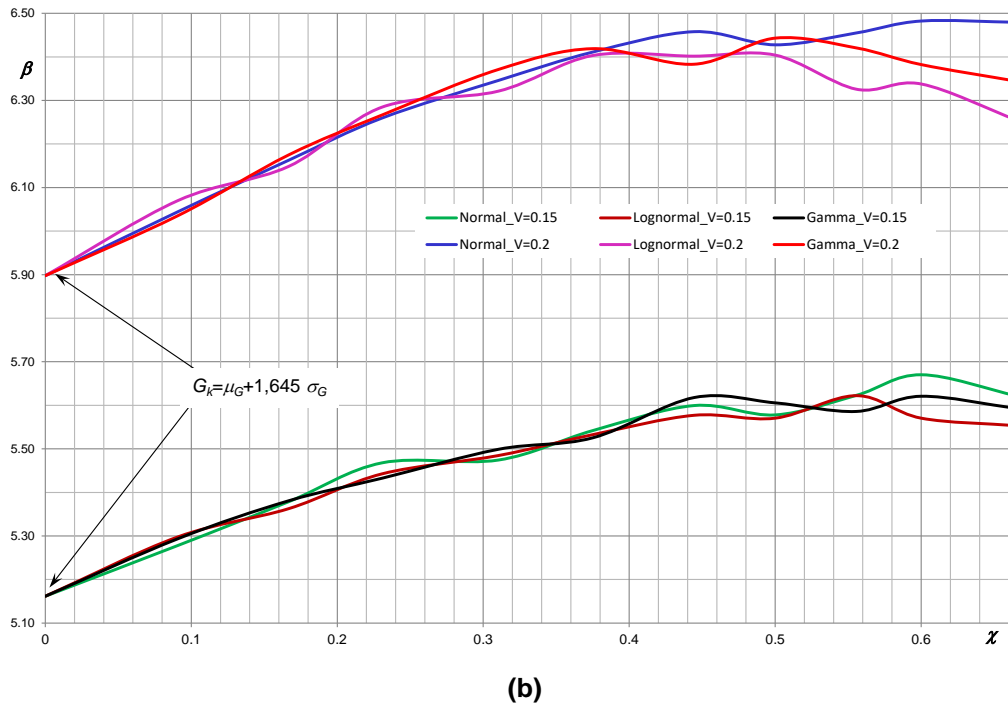
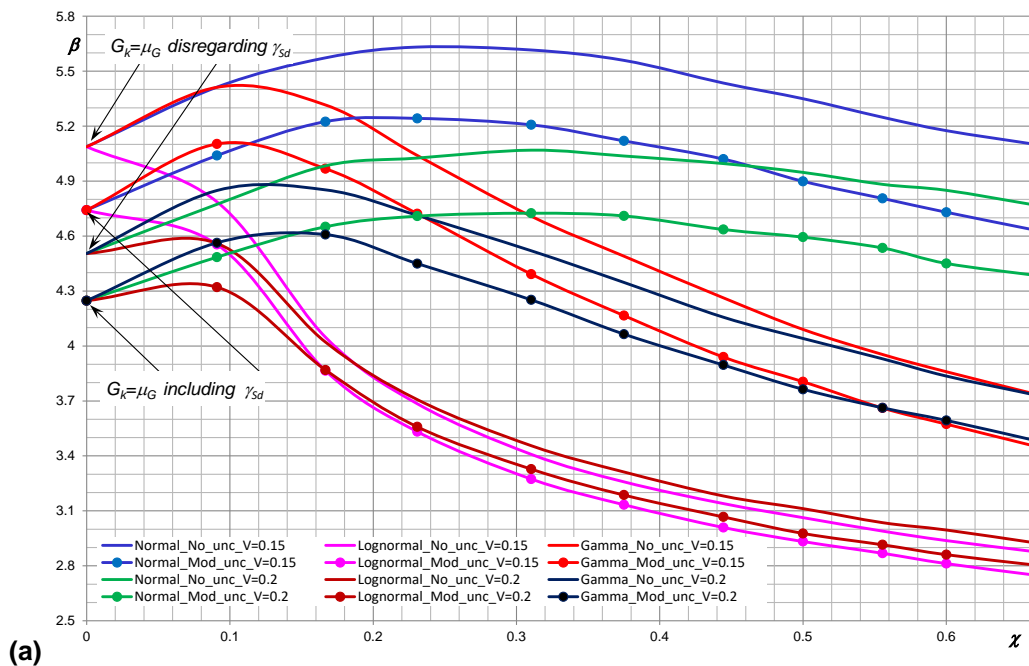
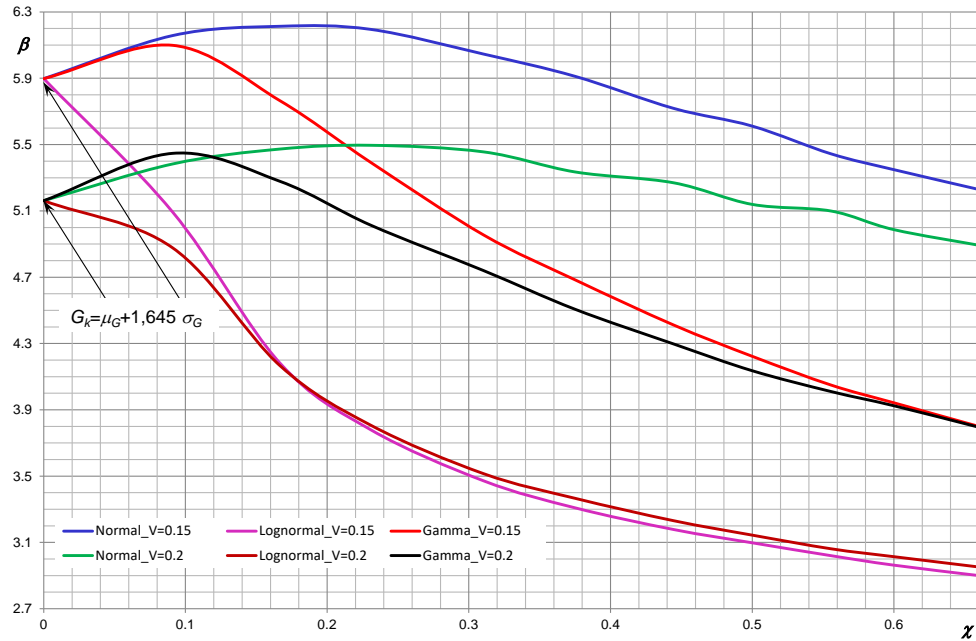


Figure 8. Sensitivity analysis of $\beta-\chi$ curves considering normal, lognormal, and gamma distributions for imposed loads, expression [6.10]: concrete failure of r.c. members in storage buildings (50 years ref. period): a) Case $G_K = \mu_G$, including or disregarding the model uncertainty factors γ_{Sd} ; b) Case $G_K = \mu_G + 1.645 \sigma_G$, disregarding the model uncertainty factor γ_{Sd} .

Examining curves in Fig. 9, we can highlight that:

- influence of concrete quality controls is still relevant even if it decreases for lightweight structures ($\chi > 0.45$);
- the reliability significantly depends on the statistical distribution of actions. When gamma and log-normal distributions are adopted, β value decreases as χ increases: this effect is more pronounced in log-normal case, where, for $\chi > 0.3$, β values are well below the target value.



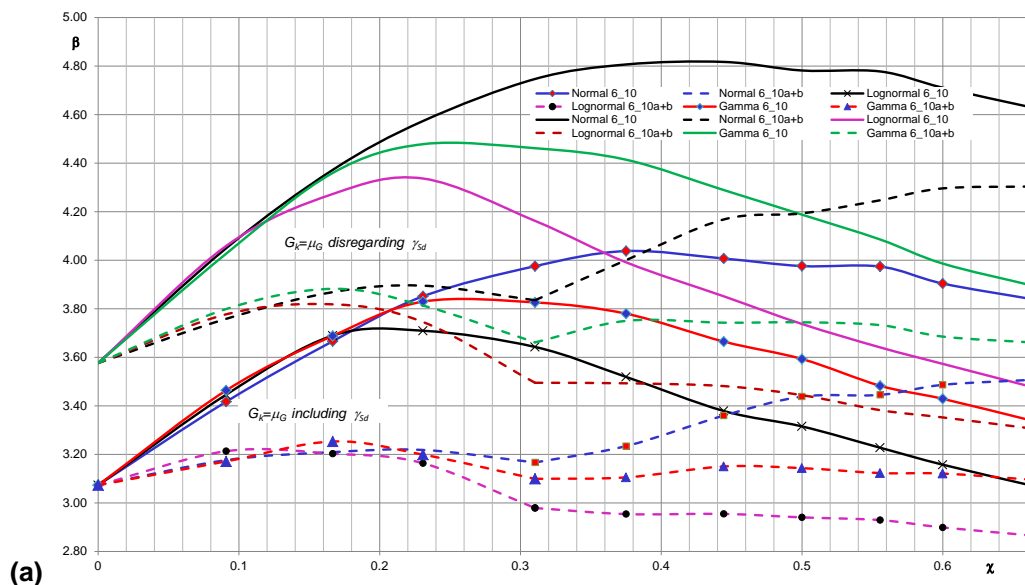


(b)
Figure 9. Sensitivity analysis of $\beta-\chi$ curves considering normal, lognormal, and gamma distributions for imposed loads, expression [6.10]: concrete failure of r.c. members in residential buildings (50 years ref. period): a) Case $G_K = \mu_G$, including or disregarding the model uncertainty factors γ_{Sd} . b) Case $G_K = \mu_G + 1.645 \sigma_G$, disregarding model uncertainty factors γ_{Sd} .

Since the imposed loads are highly scattered and the results are extremely sensitive on the *pdfs*, in residential buildings level 2 approximations seems to be unacceptable, leading to questionable results.

Fig. 10, correlated with previous Fig. 2 and 5, concerns steel members in a commercial building. Inspecting the curves pertaining to this example, the following remarks can be formulated:

- β values depend on the statistical distribution, and they do not always reach the target value;
- as the partial factor for steel resistance is small, $\gamma_M = 1.0$, β depends on model uncertainty factors as well as on the definition of characteristic value of permanent loads;
- expressions [6.10a+b] can lead, as already observed, to severe reductions of the reliability;
- comparison with Fig. 2 and 5 shows that, adopting combination [6.10], level 2 analysis gives acceptable estimates of the reliability indexes.



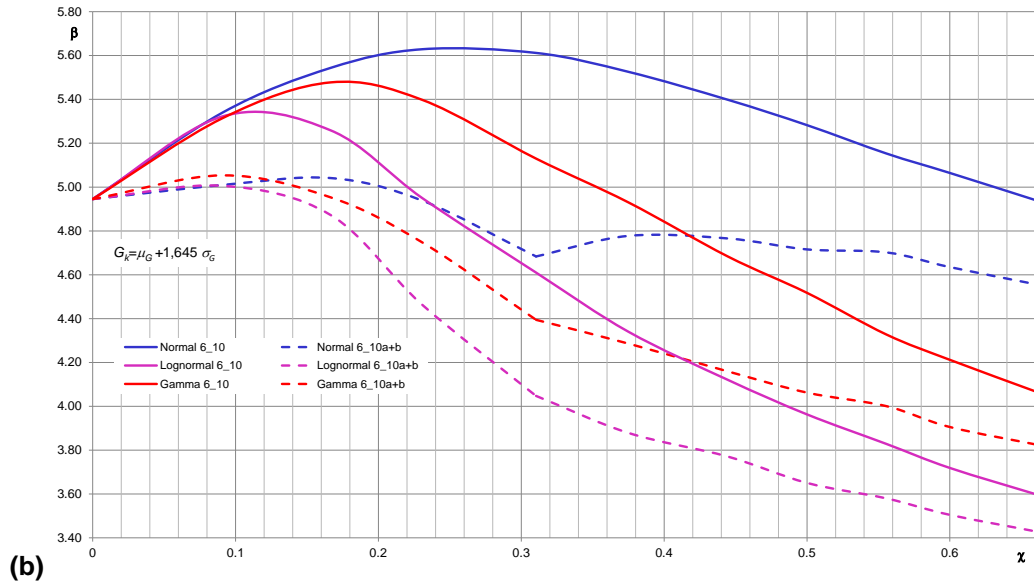
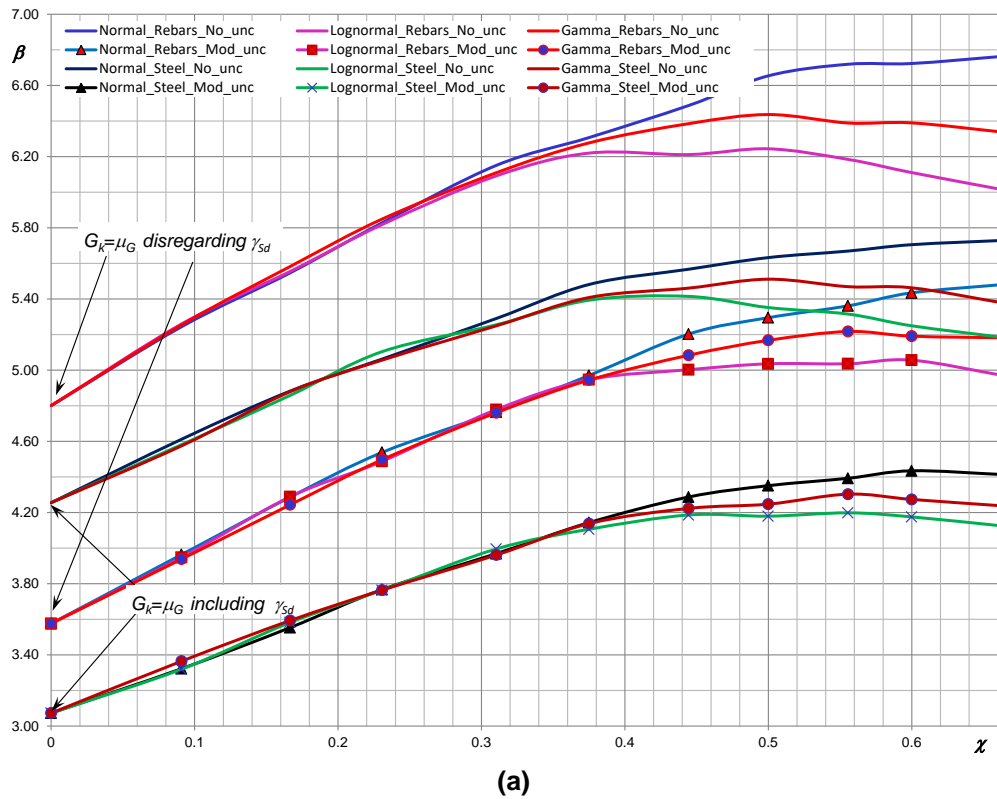


Figure 10. Sensitivity analysis of $\beta - \chi$ curves considering normal, lognormal, and gamma distributions for imposed loads, expressions [6.10] and [6.10a+b]: steel members in commercial buildings (50 years ref. period): a) Case $G_K = \mu_G$, including or disregarding the model uncertainty factors γ_{Sd} ; b) Case $G_K = \mu_G + 1.645 \sigma_G$, disregarding model uncertainty factors γ_{Sd} .

Examining Fig. 11, which refers to steel members and rebar's failure in r.c. members of storage buildings, we can finally remark that:



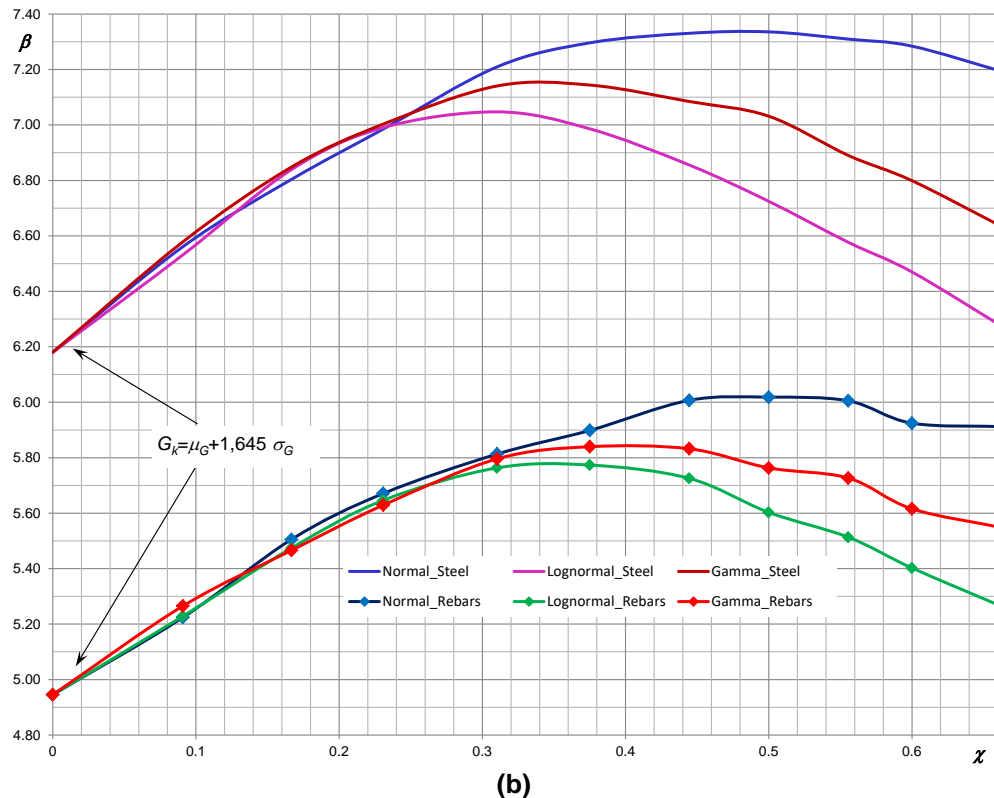


Figure 11. Sensitivity analysis of β - χ curves considering normal, lognormal, and gamma distributions for imposed loads, expression [6.10]: steel members and reinforcement failure of r.c. members in storage buildings (50 years ref. period): a) Case $G_K = \mu_G$, including or disregarding the model uncertainty factors γ_{Sd} ; b) Case $G_K = \mu_G + 1.645 \sigma_G$, disregarding model uncertainty factors γ_{Sd} .

- since the strength distributions of structural steel and reinforcing steel are described by the same statistical parameters, reliability index raises when the partial factor γ_M increases from $\gamma_M = 1.0$, as for structural steel, to $\gamma_M = 1.15$, as for reinforcing steel.
- heavier structures reveal smaller reliability indexes, while *pdf* of imposed loads influences the results mainly for lightweight structures, where imposed loads predominate;
- independently on χ -value, structural steel members, affected by a smaller partial factor γ_M , exhibit lower reliability than reinforcing steel members.

4. Conclusions

The present study discusses the reliability levels associated with the three mutually exclusive expressions provided in EN1990:2002 [3] and in prEN1990:2019 [4] for ULS combination of actions, taking into account the effect of the ratio χ between imposed loads and the total acting loads (permanent and imposed loads), in some significant case studies.

In each example, resorting both to level 2 and level 3 approaches, the reliability index β pertaining to each set of load combinations has been derived, assuming that extreme values of imposed loads are described, in turn, by a normal, a log-normal, or a gamma distribution.

The following general conclusions can be drawn:

- the reliability index β is extremely sensitive to the coefficient of variation of the strength's distribution, to partial factors and to the assumptions on the type and the relevant parameters of the *pdf* adopted to model extreme values;
- in principle, adoption of partial factors for actions and strengths adequately calibrated according to the actual statistical distribution could allow to reduce the above variability, but the adoption of the

appropriate distribution is a still open question, since the upper tails of the distributions of variable actions are largely unknown, lacking sound enough data. Moreover, despite of its effectiveness, this approach would significantly complicate the application of the code, since level 3 analyses would be systematically required;

- the study confirms that the reliability index is not an univocal property of the structure (or the structural element), on the contrary, it is a function of the method adopted to evaluate the reliability itself. For that reason, as already pointed out, the reliability index β is to be regarded as a relative measure of safety, which strongly depends on the assumptions, and its use should be limited to compare, adopting consistent approaches, the reliability of structures designed under the same hypotheses, and built with the same material;
- since a quota of the reliability index β is spent to cover model uncertainties, actual values of β could result sensibly smaller than the calculated ones, in particular when the tails of the actual distributions of actions deviate, or are not adequately fitted, by the assumed distributions;
- the target reliability level indicated in the Eurocodes (EN1990) for a 50-year reference period and for structures belonging to Consequence Class 2, i.e. “normal” consequences in case of failure, is generally achieved using expression [6.10];
- adoption of alternative expressions [6.10 a] and [6.10b] leads in some cases to reliability indexes smaller than the target, while use of [6.10.a modified] plus [6.10.b] leads to reliability indexes often unacceptably below the target for massive structures, whose design is governed by the self-weight.

Although not a direct consequence of the study, it should be also considered that, to cover failure modes occurring without significant warnings, like in case of buckling in steel members or shear failure in concrete, verification formulae provided in structural codes can intentionally include often not explicitly declared additional safety. That aspect should be duly taken into account comparing reliability levels pertaining to different limit state functions.

Further investigations are in progress, based on level 3 analyses, mainly focusing on the refinement of the partial factors, avoiding the, often controversial, introduction of sensitivity factors α , required by FORM or SORM [16][43][44], as well as and a more precise estimate of uncertainty factors for actions and resistance.

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