



Research article

UDC 624.046

DOI: 10.34910/MCE.111.1

## Limit states design theory based on critical energy levels criterion in force method form

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**Keywords:** structural analysis, limit states design, force method, critical levels of energy, pin-jointed structures

**Abstract.** The article deals with the development of the theory of limit states of structures. There are various approaches to the formulation of limiting states of structures. They are related to issues of strength, structural stability (ULS) or safe operational requirements (SLS). All known theories are oriented to a certain hypothesis of the limiting state and exist separately from each other. Therefore, there is an opinion that there is no systematic view of the limit state theory. The article suggests a general approach to creating the theory of limit states based on the criterion of critical levels of internal energy proposed by one of the authors. The problem of determination of the structure limit state is formulated as finding critical energy levels by varying system stresses. The change of energy level is accompanied by removal of linkages in the structure. Self-stressing states of the system make it possible to find out the most loaded elements. Mathematical model of general approach is the eigenvalue problem. It is formulated in the force method form and the resolving equations are derived. The physical meaning of the obtained relations and the links between the variables are explained. The proposed technique allows making a forecast that shows the element and the load from which destruction of the system will begin and trace the progressive failure of structural elements. Depending on the hypothesis of the ultimate limit state of the structure or serviceability limit state, we can analyze the behavior of the structure under load and predict violation of the conditions LSD. The problem solving method of finding the limit states is shown in the example of the pin-jointed structure.

### 1. Introduction

Issues that related to the structures bearing capacity loss are still relevant from the moment the doctrine of the strength deformable systems arose. The conceptual framework of the limit state design (LSD) of the structure is the main in the construction field.

The limit state method came into designers practice of building codes in the Soviet Union in 1955 (N.S. Streletsky and others) and became the main throughout the world [1, 2].

In the regulatory documents two main groups are identified: ultimate limit state (ULS) that associated with the appearance of the greatest stresses or critical forces in the structure, and serviceability limit state (SLS) associated with safe operational requirements. The first includes hypotheses of design criteria: strength, stability, etc., related to the behavior of materials and structures, based on the continuum models of the deformable body [3–11].

Stupishin, L.Yu., Moshkevich, M.L. Limit states design theory based on critical energy levels criterion in force method form. Magazine of Civil Engineering. 2022. 111(3). Article No. 11101. DOI: 10.34910/MCE.111.1

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The hypotheses of the second group do not take into account the causes of structural failure associated with the defects of the material of construction and the structure listed above. The SLS in the practice of designing building structures means a state that does not meet the operational requirements for building structures. Therefore, the designer takes into account the requirements of safe operation, that do not directly lead to bearing capacity loss, but make the operation processes dangerous or complicate normal operation [12–14].

There are researches connecting the physicochemical processes occurring in the material with its strength characteristics [15]. Particularly intensive studies of bearing capacity loss occasion are conducted for materials at the nano-scale [16–18] that allow a deeper understanding of the processes occurring in deformable bodies.

For combining existing theories of limit states into one it seemed the most productive to use probabilistic approaches. Research in this direction is ongoing, but work is not quite finished [19–21].

The optimal design of structures is another way to combine criteria of different nature into one statement of the problem [22–25]. However, too many hypotheses of the limiting state make it difficult to formulate the objective function, and its large number does not allow obtaining a solution to the optimization problem.

The above approaches try to combine many hypotheses that cannot be limited. The attempt to combine existing LSD theories in a single mathematical model causes great difficulties. Therefore, there is an opinion that “there is still no theory of LSD as an exposition of a certain systematic view of the subject” [26].

This paper is devoted to the formulation of a general theory and criterion of the LSD for the deformable structures. The created methodology for structural analysis of deformable systems is illustrated with a simple example.

## 2. Methods

### 2.1. Backgrounds of the method

One of the goals of structural mechanics is to determine the maximum displacements and stresses of structure element. For this structure element must be written down the corresponding condition of LSD. At the same time, the formulation of LSD criteria for the structure elements remains outside the structural mechanics boundaries. In this regard, the question about the ultimate goal of structural mechanics as a scientific discipline, which, obviously, consists in determining the parameters of LSD for structure, remains open. Accordingly, there must be a phenomenological criterion that allows, from a single point of view, to find a structural element in LSD, or to determine the LSD criterion of a complex physical model of a building (structure). In turn, indication of the parameters of the LSD of the structure will provide the basis for the further application of hypotheses about the causes of destruction or the appearance of defects in the material of the structure.

In structural mechanics, there are criteria for the LSD based on the phenomenological approach of structural loss of stability, or the occurrence of resonance, which can be figuratively formulated as “at some point, everything went not as it used to be”. Such a general approach it can be possible to construct a mathematical model of physical phenomena related to the concept of LSD on the basis of eigenvalue problem.

LSD hypothesis can be formulated as a statement that the structure response different than before the moment of the limiting state. The next fact is that in formulating limit state of structures, as a rule, it is not the deformation process is the main, but the structure parameters at the time of a critical condition. Finally, it should be recognized that the design takes only such deformed forms that depend on the conditions of support, mechanical and geometric characteristics and the physical law of deformation. The type of load and the law of its distribution over the structure have little effect on its shape in the ultimate state.

It is possible to determine the limiting states of the structure in the sequence:

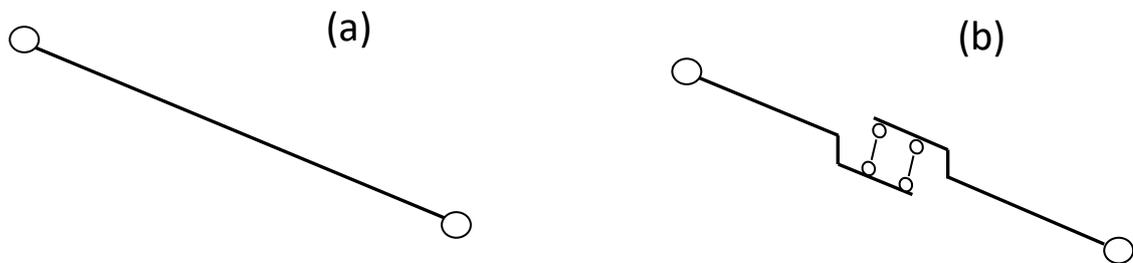
1. determination of the element with the highest stresses and strains based on the criterion of critical levels of the internal potential energy of the system [25];
2. to compare this state of possible types of loading and the corresponding level.

That is, to make a map of the external loads for the system under LSD condition. Then you can select the hypothesis for the possible types of loss of structural bearing capacity.

## 2.2. The loss of the bearing capacity as the removal of linkages

The fact that the failure of the system to bear the load is associated with the loss of geometric stability, forces us to turn to the structure instability analysis (kinematic analysis of structures). There are only three objects of kinematic analysis: a rigid body, constraints (linkages, in rare cases, deformable), and a hinge, which allows us to conclude that the appearance of geometric instability depends on the removal of linkages.

In this case, the removal of linkages leads to a decrease of the degree of static indeterminability and, ultimately, to a geometrically unstable system. That is means the loss of bearing capacity by the entire structure. The Fig. 1 shows a rod with a remote longitudinal linkage, illustrating the developed theory using the example of a system of pin trusses.



**Figure 1. (a) Bar element, (b) element with longitudinal bond removed.**

Thus, in order to formalize the concept of structural failure to satisfy operational requirements from the point of view of structural mechanics, we can accept it as equivalent to removing the system connection [27].

The reason for removing the connection is determined by the requirements for structural elements. This may be a loss of bearing capacity during tension (or compression) of the bar due to the appearance of yield strains. This can be loss of stability or brittle fracture due to the appearance and development of cracks, etc. Theoretically, any criterion corresponding to the concept of LSD can be used.

## 2.3. Self-stressing state of the structure

The concept of the self-stressed structure was introduced almost a century ago, but is used by various authors to explain the physical phenomena that occur in a solid deformable body, depending on the problem being solved. So A.R. Rzhancyn [28] stated that the state of self-stress can occur only in statically indeterminate systems at zero loads. In this case, the cause of such stresses is the discontinuity of elements or the effect of temperature, i.e. external influences. A.V. Perel'muter and V.I. Slivker [29] talk about the state of self-stress in the main system of the force method, which is also a statically determinate system. Dzh. Robinson, G.V. Haggemajer, R. Kontini [30] said about the equivalent concept of a self-balanced system is introduced and the concept of amplitudes of self-balanced forces is used.

The criterion of critical energy levels requires an understanding of the state of self-stress as an objectively existing state of the entire deformable structure as a whole at any stage of its loading. This state is determined by the geometry of the structure, the geometric and mechanical characteristics of its elements and the conditions of support. The distribution of forces in the static indeterminate system is determined by the system of equations written in the form of the force method, displacement method, etc. The solution of the problem will be the principal values of the deflections of the entire system and eigenvectors present internal forces distributions in the system. The principal stiffness values and the corresponding displacement distributions are determined similarly.

A state of self-stress without a load on the system is detected by varying displacements or forces in the bars, while the magnitude of the variations is not specified, since the energy levels are a discrete set of critical values. The state of self-stress, in the sense of the distribution of internal forces in the bars or their deformations, exists both in the absence of loading and at any non-destructive level of loading. It can be argued that the state of self-stress in the system does not change during loading process until any linkage will remove. Failure will occur in the most rigid (flexible member) element of the system, that is, the element with the maximum (minimum, depending on the statement of the problem) eigenvalue.

We obtain a new design scheme without one connection, to which the above reasoning applies. The new system has a different level of internal potential energy of deformation, as well as a system of self-balanced internal forces (stresses), which may differ from the previous one.

The most important property of self-balanced efforts or movements is that they are described by an orthonormal system of functions.

## 2.4. Critical strain levels in form the force method

The criterion of critical energy levels can be used as the basis for formulating the limiting state, which allows one to describe the limiting states of the system.

Generalized internal effort  $\Phi_i^{\text{in}}$  in  $i$ -th displacement connection  $m$  times of a statically indeterminable system arising from a generalized external force  $\Phi^{\text{ex}}$ , can be represented as the sum of internal efforts

$$\Phi_i^{\text{in}} = \Phi_{i\Phi^{\text{ex}}} + \sum_{k=1}^m \Phi_k^{\text{in}} \Phi_{ki}^{\text{in}}. \quad (1)$$

Here is  $\Phi_{i\Phi^{\text{ex}}}$  the effort in the  $i$ -th element of the main system, devoid of all unknown redundant constraints arising from the applied external load  $\Phi^{\text{ex}}$ ,

$\Phi_k^{\text{in}}$  is the internal efforts in the redundant structure links ( $k = 1, 2, \dots, m$ );

$\Phi_{ki}^{\text{in}}$  is the unit efforts in  $i$ -th element of the simple system from the effort  $\Phi_k^{\text{in}} = 1$ , exerted on  $k$ -th element.

Note that the presented expression for efforts does not depend on the choice of the method for static indeterminacy solving. Moreover, the initial statically indeterminate structure can be taken as the primary system. The principle of deriving equations in the form of the force method and the displacement method, as you know, is the same.

We will apply the methodology of the force method in the following considerations, which will not change the results for the case of using other methods for revealing static indeterminacy.

Define generalized deflections  $\delta_k$  in the direction redundant unknowns. We write down the possible work of the redundant unknowns  $\Phi_{kk}^{\text{in}}$  and the internal efforts caused by them  $\Phi_{ik}^{\text{in}}$  in the  $n$  elements of the simple system at the corresponding generalized displacements  $\xi_i$

$$\Phi_{kk}^{\text{in}} \delta_k - \sum_{i=1}^n \Phi_{ik}^{\text{in}} \xi_i = 0, \quad k = 1, 2, \dots, n. \quad (2)$$

Here, the summation is carried out over all elements of the system  $n$ .

Putting force in  $k$ -th element equal to one, i.e.  $\Phi_{kk}^{\text{in}} = 1$ , getting for  $k$ -th state equation of displacements in this rod

$$\delta_k = \sum_{i=1}^n \Phi_{ik}^{\text{in}} \xi_i, \quad k = 1, 2, \dots, n. \quad (3)$$

Considering that the variation of the self-stress system will always occur near the equilibrium state, we introduce the stiffness coefficient of the system element  $K_i$ . In our case denoted  $K_i = E_i A_i$ , where elastic module of rods is  $E_i$ , area of its cross section is  $A_i$ . Then we can determine the generalized displacements through the generalized efforts in the form:

$$\delta_{k\Phi^{\text{ex}}} = \sum_{i=1}^n \frac{\Phi_{ik}^{\text{in}} \Phi_{i\Phi^{\text{ex}}}}{K_i}; \quad (4)$$

for deflections dependent on external load, and

$$\delta_{kl} = \sum_{i=1}^n \frac{\Phi_{ik}^{\text{in}} \Phi_{il}^{\text{in}}}{K_i}, \quad (5)$$



$$[L] = \begin{bmatrix} \delta_{11} & \delta_{12} & \dots & \delta_{1n} \\ \delta_{21} & \delta_{22} & \dots & \delta_{2n} \\ \dots & \dots & \dots & \dots \\ \delta_{n1} & \delta_{n2} & \dots & \delta_{nn} \end{bmatrix}, [\lambda_0] = \begin{bmatrix} \lambda_{01}^G & 0 & \dots & 0 \\ 0 & \lambda_{02}^G & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 0 & \dots & \lambda_{0n}^G \end{bmatrix}.$$

Eigenvalues  $\lambda_{0i}^G$  is the main values of deflections in the structure bars, and the eigenvectors  $\{\delta\Phi_k^{\text{in}}\}$  is amplitude values of the distribution of self-stress. For small variations, the relation for the eigenvalues of the flexibility and stiffness matrices is valid

$$[\lambda_{0i}^K] = [(\lambda_{0i}^L)]^{-1}.$$

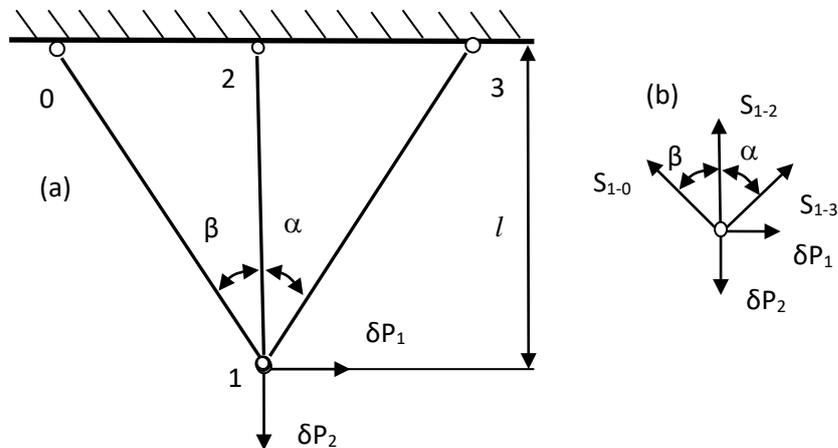
Here are  $\lambda_{0i}^K$  is the eigenvalues of the system stiffness matrix.

The physical meaning of the derived equations (12), (13) is the state of self-stress in a multiply connected system at critical energy levels. Solving the eigenvalue problem gives the main node displacements of the core system and the corresponding vectors of the amplitude values of the structure nodal reactions.

### 3. Results and Discussion

#### 3.1. Example of calculating a redundant construction

Let us take as an example the three-bar pin – jointed system shown in Fig. 2. The cross-sectional areas of the structural members  $A_i, i=1,2,3$  and the elastic modulus  $E_i, i=1,2,3$  are designated for members 0-1, 1-2, and 1-3, respectively. It is required to indicate the possible limiting states of the structure, as well as the values of ultimate loads.



**Figure 2. Three-bar system: (a) design scheme, (b) node 1.**

To simplify the calculations, we will carry out the calculation in the matrix form of the force method in accordance with (13). Equilibrium equations in Node 1

$$\begin{aligned} -S_{0-1} \sin \beta + S_{1-3} \sin \alpha + \delta P_1 &= 0, \\ -S_{0-1} \cos \beta - S_{1-2} - S_{1-3} \cos \alpha + \delta P_2 &= 0, \end{aligned} \quad (14)$$

allow you to compose a static matrix of the three-rod system

$$[A] = \begin{bmatrix} -\sin \beta & 0 & \sin \alpha \\ -\cos \beta & -1 & -\cos \alpha \end{bmatrix}. \quad (15)$$

The matrix of internal deflections of the three-bar system, taking into account the notation  $\eta_1 = EA/E_1A_1$ ,  $\eta_2 = EA/E_2A_2$ ,  $\eta_3 = EA/E_1A_1$ , has the form

$$[\mathbf{B}] = 1/EA \begin{vmatrix} \eta_1/\cos\beta & 0 & 0 \\ 0 & \eta_2 & 0 \\ 0 & 0 & \eta_3/\cos\alpha \end{vmatrix}. \quad (16)$$

Expressions for calculating next matrixes, the eigenvalues and eigenvectors will not be written out due to the large volume.

The flexibility matrix of the three-rod system is written as

$$[\mathbf{L}] = \left( [\mathbf{A}]^T [\mathbf{B}]^{-1} [\mathbf{A}] \right)^{-1}. \quad (17)$$

The internal forces in the structure rods are defined as

$$[\mathbf{N}] = [\mathbf{B}]^{-1} [\mathbf{A}]^T [\mathbf{L}] \{\mathbf{P}\}. \quad (18)$$

The strain caused by internal forces is

$$[\boldsymbol{\varepsilon}] = [\mathbf{A}] [\mathbf{L}] [\mathbf{A}]^T \{\mathbf{N}\}. \quad (19)$$

The deflections of node 1 in the direction of the initial degrees of freedom

$$[\mathbf{u}] = [\mathbf{L}] \{\mathbf{P}\}. \quad (20)$$

**Example 1.** Let the stiffness values of the rods be the same ( $\eta_1 = \eta_2 = \eta_3 = 1$ ), and the rods are tilted at the same angles ( $\alpha = \beta = \pi/4$ ).

Stage 1.

The system is considered at the first critical level (when all connections are in place). Due to the complete symmetry of the system, the flexibility matrix is diagonal.

$$[\mathbf{L}] = \frac{1}{EA} \begin{vmatrix} 1.4143 & 0 \\ 0 & 0.5858 \end{vmatrix}. \quad (21)$$

We consider the unit impacts in the node directed along the degrees of freedom of node 1 shown in Figure 1.

$$[\boldsymbol{\Psi}] = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}. \quad (22)$$

We consider the load to be unit and directed along the degrees of freedom of node 1 shown in Fig. 1.

If we take only  $\delta P_2$  force to load the system, will have the same internal forces, which A.R. Rzhanicyn yield in "Theory of plasticity"

$$\bar{S}_{1-2} = 0.5858, \bar{S}_{1-0} = \bar{S}_{1-3} = 0.2929. \quad (23)$$

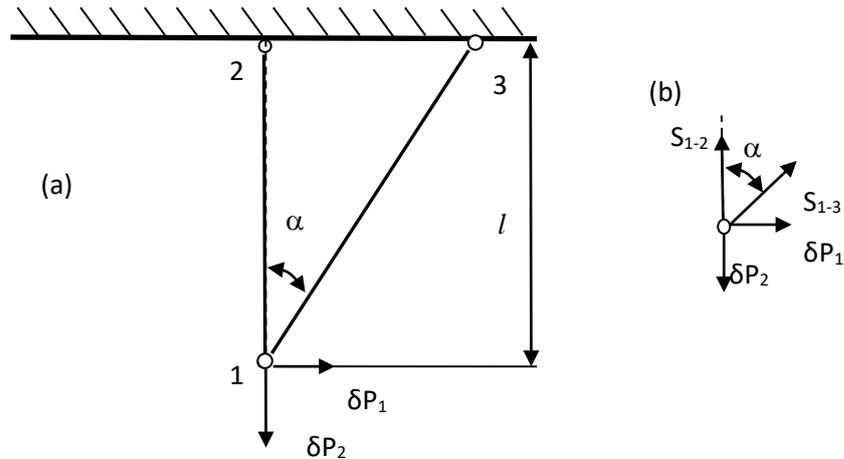
Internal forces from two directed unit external forces are

$$\bar{S}_{1-0} = 1, \bar{S}_{1-2} = -0.5858, \bar{S}_{1-3} = 0.2929. \quad (24)$$

At the first stage, the ultimate limit state occurs in the extended rod 0-1.

Stage 2

Further investigation is conducted for the three-bar system without linkage 0-1. Fig. 3 shows a new design scheme of the system, which continues to be loaded in the directions of the degrees of freedom  $\delta P_1, \delta P_2$ . Loads continue to increase.



**Figure 3. (a) Three-bar system without one bar, (b) node 1 (a).**

Equilibrium equations in Node 1

$$\begin{aligned} S_{1-3} \sin \alpha + \delta P_1 &= 0, \\ -S_{1-2} - S_{1-3} \cos \alpha + \delta P_2 &= 0. \end{aligned} \quad (24)$$

allow you to compose a static matrix of the two-rod system

$$[A] = \begin{bmatrix} 0 & \sin \alpha \\ -1 & -\cos \alpha \end{bmatrix}. \quad (25)$$

Easy to see that  $\text{Det}[A] \neq 0$ , and therefore, the system is stable.

The matrix of the external deflections of the two-rod system is written as

$$[L] = \begin{bmatrix} 3.829 & 1 \\ 1 & 1 \end{bmatrix}. \quad (26)$$

Eigenvalues of the flexibility matrix is  $\lambda_1 = 4.146$ ,  $\lambda_2 = 0.682$ .

The eigenvectors of the external deflection matrix

$$[\Phi] = \begin{bmatrix} 0.9530 & -0.3029 \\ 0.3029 & 0.9530 \end{bmatrix}. \quad (27)$$

Longitudinal forces in the system

$$S_{1-2}^{\text{cr}} = 5.207, S_{1-3}^{\text{cr}} = -5.588. \quad (28)$$

The ultimate state from compressive strength will occur in inclined rod, the inclined one should be checked for stability.

The full ultimate load taking into account the both stage of deformation will be

$$P_{\text{total}}^{\text{cr}} = 6.588P^{\text{cr}}. \quad (29)$$

The critical force acting on the structure is taken in accordance with the concept of the type of loss of bearing capacity by the rod.

If we assume that the bars lose strength in the plastic stage of deformation  $\sigma_{\max} = \sigma_t$ , and the load acting in the direction of degrees of freedom  $P_2 = 1, P_1 = 0$ , we get a complete match with the result given A.R. Rzhanicyn.

$$P_{\text{total}}^{\text{cr}} = 2.414P^{\text{cr}}. \quad (30)$$

where  $P^{\text{cr}} = A\sigma_t$ .

**Example 2.** Let the stiffness values of the rods be the same ( $\eta_1 = \eta_2 = \eta_3 = 1$ ), and the rods are tilted at the same angles ( $\alpha = \pi/4, \beta = \pi/3$ ).

Stage 1.

The system is considered at the first critical level (when all connections are in place).

Due to the system nonsymmetrical, the flexibility matrix is

$$[L] = \frac{1}{EA} \begin{vmatrix} 1.397 & 0.1295 \\ 0.1295 & 0.6883 \end{vmatrix}. \quad (31)$$

Eigenvalues of the flexibility matrix is

$$[\lambda] = \frac{1}{EA} \begin{vmatrix} 1.4199 & 0 \\ 0 & 0.6654 \end{vmatrix}. \quad (32)$$

Vector-matrix of the eigenvectors is

$$[\Phi] = \begin{vmatrix} 0.9847 & -0.1743 \\ 0.1743 & 0.9847 \end{vmatrix}. \quad (33)$$

For the main vector with the maximum eigenvalue, we obtain the longitudinal self-stress forces

$$\bar{S}_{1-0} = 0.9475, \bar{S}_{1-2} = 0.3514, \bar{S}_{1-3} = -0.8169.$$

The greatest tensile force arises in the left bond, inclined at an angle of 60 degrees to the vertical.

The full maximum load at the first stage for removing one connection will be

$$P_1^{\text{cr}} = 0.9475P^{\text{cr}}. \quad (34)$$

Stage 2.

Further investigation is conducted for the three bar system without linkage 0-1. Fig. 3 shows a new design scheme of the system, which continues to be loaded in the directions of the degrees of freedom  $\delta P_1, \delta P_2$ . Loads continue to increase.

Since the solution for such a scheme was already carried out in the previous problem, we write out the value of the critical force that turns the system into an unstable one

$$\bar{S}_{1-3}^{\text{cr}} = -5.5882P^{\text{cr}}. \quad (35)$$

The total limit load in two stages will be

$$P_{\text{total}}^{\text{cr}} = 6.5357P^{\text{cr}}. \quad (36)$$

## 4. Conclusion

1. The limiting states of the structure correspond to the extreme values of stresses arising in the deformable body or the corresponding deformations.

2. Critical strain energy levels of a deformable body are detected by changing self-stress state. Due to this fact, it is possible to identify for the designed structure under what conditions the ultimate state and values of ultimate loads come.

3. It is possible to predict from which element and at what load the destruction of the building will begin and trace the sequence of exclusion of structural elements from work on the load.

4. Depending on the hypothesis of the ultimate limit state of the structure or serviceability limit state, one can analyze the behavior of the structure under load and predict violation of the conditions of the LSD at the one mathematical model.

5. The proposed technique opens up new possibilities for the analysis and synthesis of structures:

- 1) analysis of the sequence of rods output from work on the load;
- 2) selection of the structural element from which its progressive destruction will begin;
- 3) reinforcement of structural elements in order to regulate the forces in the structure;
- 4) design of smart structures.

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*Received 25.03.2020. Approved after reviewing 20.07.2021. Accepted 21.07.2021.*