



Research article

UDC 624

DOI: 10.34910/MCE.111.3

Detection and prediction of weak points of a frozen wall based on grey theory

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Keywords: frozen soils, longitudinal temperature monitoring, whole field monitoring theory, grey system theory, prediction

Abstract. The purpose of this article is to use theoretical analysis to determine the cause of an artificial frozen wall not closing. Due to the influence of the energy difference received by each freezing apparatus, groundwater flow, freezing hole spacing and other factors, their heat transfer with the surrounding soil layer has limited efficiency. Theoretical calculations and engineering examples are compared to verify the validity of the theoretical model. According to the longitudinal temperature measurement of the temperature measuring hole and the freezing apparatus, the whole field monitoring theory based on grey correlation method is proposed. Through the calculation of the hole-to-hole correlation degree to evaluate and predict the working effect of each freezing apparatus, it is determined that the freezing pipes No. 7~9 and No. 18~22 are the weak parts of the freezing wall. Therefore, the grey correlation analysis was compared and analyzed in the case of the "window" accident of the frozen wall of the shaft, which verified the practicability of the theory.

1. Introduction

Artificial ground freezing method (AGF) is a special construction method for solving the underground construction of soft water-bearing strata such as quicksand, silt, etc. By placing freezing pipes at intervals in the stratum and circulating low-temperature brine through them, the stratum becomes hard frozen soil [1, 2]. The artificial stratum freezing method has been developed for more than 130 years, and it has been vigorously promoted by various countries in coal mines, subways and foundation pit projects. The technology is also becoming more and more mature, but the groundwater flow often leads to the accident of the frozen wall not closing, causing project delays and economic losses. For the freezing construction under high water content and complex environmental conditions, the change of temperature field determines the thickness and average temperature change of the frozen wall. Therefore, the ways of determining the tightness and strength of the frozen wall are the focus of scholars from various countries. Therefore, the calculation of the freezing temperature field is related to the success or failure of the entire freezing project. The main research methods of the temperature field include analytical methods, numerical simulation methods, field data measurement, and in-situ test methods.

Professor Trupak first proposed the calculation method for the temperature field of the frozen wall [3]. He began to study the temperature field of a single-tube frozen cylinder, and derived the analytical solution formula of the temperature field in the frozen soil wall under the condition of a single-row linear

Wu, T., Zhou, X.M., He, X.N., Xu, Y., Zhang, L.G. Detection and prediction of weak points of a frozen wall based on grey theory. Magazine of Civil Engineering. 2022. 111(3). Article No. 11103. DOI: 10.34910/MCE.111.3

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arrangement of frozen tubes according to the geometric relationship between the frozen tube and the frozen soil column. Bakholdin uses the concept of ideal freezing tubes to interfere with each other to derive the theoretical formula of the temperature field distribution after the frozen wall of a single row of tubes under a linear arrangement [4]. Hu Xiangdong applied conformal transformation to transform the annular single-circle freezing model into a special linear arrangement model to complete the analytical solution of the unfrozen temperature field in the single-circle tube freezing circle [5]. In the literature [6–8], the calculation formula of the frozen wall thickness of the double-row tube and the 3-row tube and the analytical solution of the temperature field have been deduced successively, and the calculation results are relatively consistent with the measured data.

Literature [9] used the monitoring data of the construction site of the connected aisle freezing method to analyze the disturbance of each construction process on the freezing temperature field, grasped the influence law of each construction process on the connected aisle freezing temperature field, and provided a safety reference for similar projects in the future. Literature [10] based on the tunnel freezing project, studied the weakening effect of freezing waterproof curtain caused by construction thermal disturbance through model tests.

With the popularization of numerical software, the numerical simulation of the frozen wall temperature field has been valued by scholars [11–15].

However, freezing engineering usually sets up several temperature measuring holes to monitor the state of the frozen wall [16]. When calculating specific engineering examples, the temperature measurement data is substituted into the theoretical analytical formula or numerical software to obtain the thickness of the frozen wall, so the data of the temperature measuring holes can only represent the temperature characteristics at a specific location, and can not provide timely feedback on the overall state of the frozen wall.

The purpose of this article is to establish the research object of each freezing apparatus, and collect the longitudinal temperature data of freezing apparatus, and obtain the longitudinal temperature distribution in the freezing tube through comprehensive data analysis, in order to qualitatively judge and predict the overall state of the frozen wall. To achieve the above goals, the following problems need to be solved:

1. Establish a set of scientific mathematical analysis methods to form a complete basic theory.
2. In the engineering example, establish a model to determine the development status of the frozen wall, find the weak point of the frozen wall, and verify the theory.

2. Materials and Methods

2.1. Monitoring method and theoretical basis of frozen wall condition

The traditional method to study the temperature field of frozen wall is based on the single-hole freezing steady-state theory of Professor Trupak of the former Soviet Union [3, 17]. The temperature field model of frozen wall is established as shown in Fig. 1, and the temperature distribution function formula of frozen soil area is deduced as follows:

$$T_f(x, y) = T_p \operatorname{In} \frac{\sqrt{E^2 + L^2}}{2\sqrt{x^2 + y^2}} \bigg/ \operatorname{In} \frac{\sqrt{E^2 + L^2}}{d_p} \left(-\frac{L}{2} \leq x \leq \frac{L}{2}, -\frac{\sqrt{E^2 + L^2}}{2} \leq y \leq \frac{\sqrt{E^2 + L^2}}{2} \right), \quad (1)$$

where T_f is the temperature field function in the frozen soil area; T_p is the temperature of the freezing pipe wall; E is the thickness of the frozen soil wall; L is the hole spacing; d_p is the diameter of the freezing pipe.

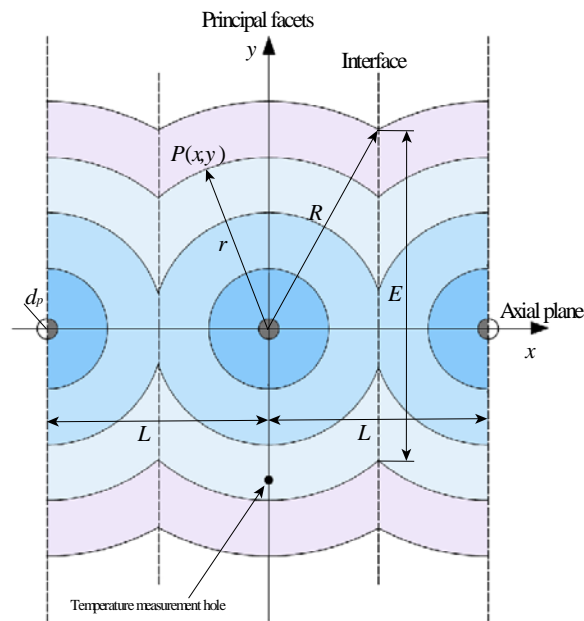


Figure 1. The frozen wall temperature field model.

A temperature measuring hole is provided near the freezing hole, and the frozen thickness of the frozen wall is calculated by the formula (1) through the detected temperature data in the temperature measuring hole, which is the traditional temperature field monitoring method. In practical engineering, the development of frozen wall is affected by many factors such as the energy regulation of each freezing apparatus, the flow of groundwater, the spacing of frozen holes, the heat conductivity of the stratum and so on. The interior of the stratum belongs to the concealed space, so it is difficult to fully grasp the development process of the frozen wall of the stratum.

In the engineering construction, a technical method of longitudinal temperature measurement in the freezing apparatus has been gradually developed to diagnose and deal with engineering accidents, as shown in Fig. 2. Literature [18] studied the relationship between the temperature change in the center of the freezing pipe and the thickness of the frozen wall after the freezing apparatus was temporarily shut down, and proposed a new method to use the longitudinal temperature measurement of the freezing apparatus to solve the internal situation of the frozen wall.

$$v = \left. \frac{dT}{dt} \right|_{r=0} = a \left(\frac{\alpha}{R} - \frac{\beta}{R^2} \right), \quad (2)$$

where v is the temperature rise rate; R is the radius of the frozen soil; a is the coefficient of thermal conductivity of the frozen soil; α and β are the correlation coefficients of the initial temperature field of the frozen wall before freezing.

Because there are a lot of random factors in the project, it is still difficult to apply formula (2) directly, but its significance lies in revealing the substantial value of the longitudinal temperature measurement data of the freezing apparatus, which is to make full use of the temperature measurement holes, hydrological holes and freezing apparatus to implement the whole field monitoring of the frozen wall temperature field thereby increasing the output of system information. The methods and requirements for the construction of freezing holes, temperature measuring holes and hydrological holes are basically the same, and the main difference lies in the different installation structures. The freezing apparatus is installed in the freezing hole, which is composed of a freezing pipe, a liquid supply pipe, and the head of the freezing apparatus, as shown in Fig. 3(a). The water pipe is arranged in the hydrological hole, which is composed of impervious steel pipe and permeable pipe. The permeable pipe is made of steel pipe with evenly distributed holes, as shown in Fig. 3(b). In the temperature measuring hole, the steel pipe with closed lower end is placed, and the inside of the pipe is generally in air state. Several temperature sensors are installed in the temperature measurement hole, as shown in Fig. 3(c). Then a large amount of information output of the system needs scientific information processing technology in order to extract more valuable parts for engineering construction services.

The main idea of the whole field monitoring theory is as follow. Firstly, we should vigorously develop new temperature measurement technology and increase the number of temperature measurement points to improve the amount of information in the temperature data of the shaft. Secondly, the longitudinal

temperature in the freezing apparatus is the main body, combining with the longitudinal temperature in the hydrological hole and the temperature measurement data of the temperature measuring hole to form a new temperature measurement engineering system. Finally, a data analysis system for predicting, forecasting and controlling the temperature field of the frozen wall is established to gradually transform the development of the frozen wall from a hidden state to a transparent state, thereby improving the safety of the quality of the frozen wall engineering and the technical level of freezing shaft construction.

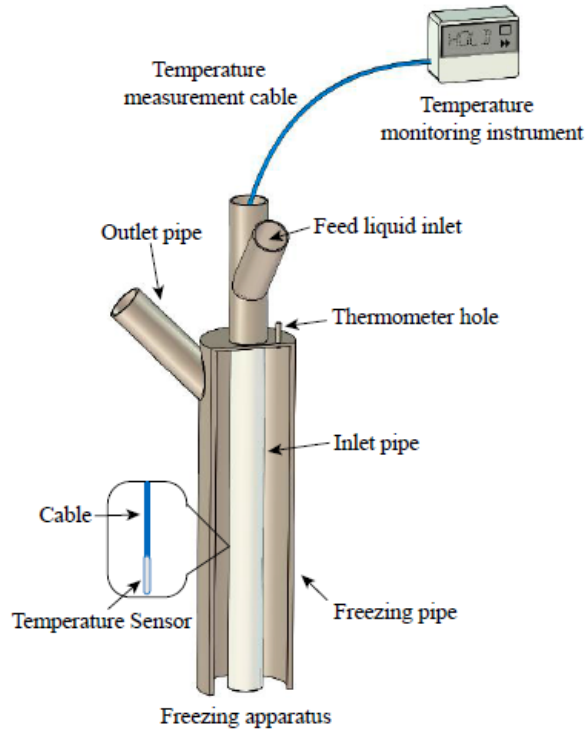


Figure 2. The principle of longitudinal temperature measurement in the freezing apparatus.

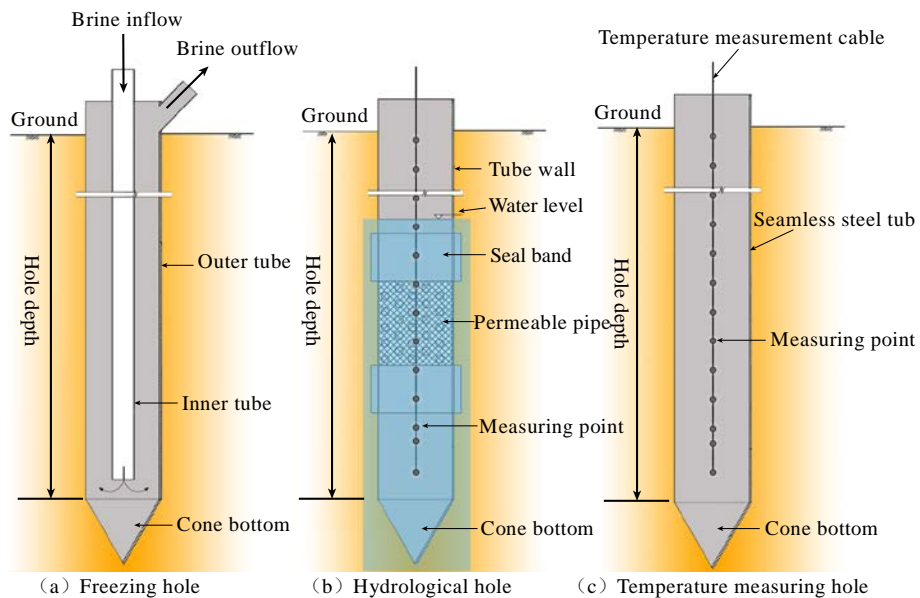


Figure 3. Different installation structures of freezing holes.

2.2. Principle of grey correlation analysis of longitudinal temperature measurement data

According to modern control theory, engineering system is a "black box". Ways to increase the amount of information in the system is an important technical direction of engineering monitoring. Grey theory [19–20] presents the output of the system state as a sequence, so the temperature data output along the depth direction is a data sequence. Here, the longitudinal temperature measurement data of the temperature measuring hole and the freezing apparatus are organized into the following sequence:

$$\begin{aligned}
 T_1(h) &= \{t_1(1), t_1(2), t_1(3), \dots, t_1(M)\}, \\
 T_2(h) &= \{t_2(1), t_2(2), t_2(3), \dots, t_2(M)\}, \\
 &\vdots \\
 T_i(h) &= \{t_i(1), t_i(2), t_i(3), \dots, t_i(M)\}, \\
 &\vdots \\
 T_N(h) &= \{t_N(1), t_N(2), t_N(3), \dots, t_N(M)\},
 \end{aligned}$$

where h is the depth or serial number of the longitudinal measuring point, $h = 1, 2, \dots, M$; i is the hole number of the frozen hole, $i = 1, 2, \dots, N$.

The definition of relevance in the grey theory borrows the concept of geometric similarity, assuming that these sequences are plotted on a two-dimensional rectangular coordinate system with time as the horizontal coordinate, as shown in Fig. 4. If the sequence $T_1(h)$ curve and the sequence $T_2(h)$ curve are relatively parallel or similar, it is said that the correlation between $T_2(h)$ and $T_1(h)$ is large, which is recorded as γ_{12} . If the similarity between curves $T_3(h)$ and $T_1(h)$ is small, and the similarity between curves $T_4(h)$ and $T_1(h)$ is the worst, then $\gamma_{12} > \gamma_{13} > \gamma_{14}$. Here, $T_1(h)$ is called the parent factor sequence, and $T_2(h)$, $T_3(h)$ and $T_4(h)$ are called sub factor sequences.

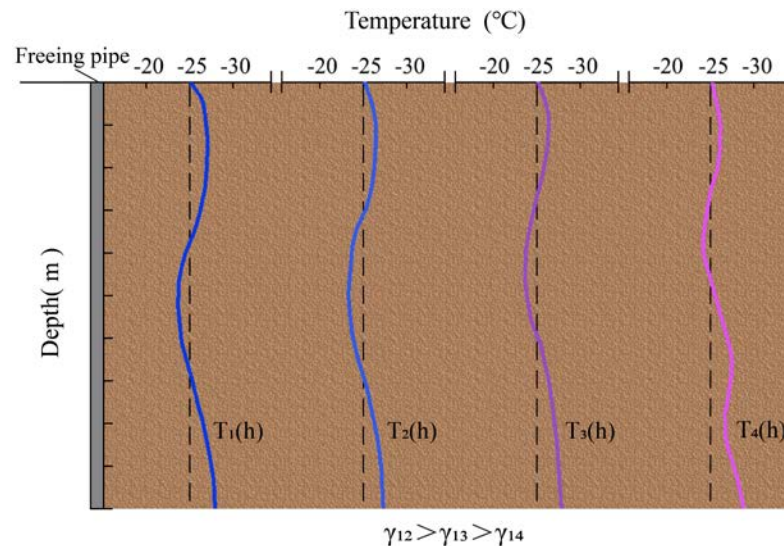


Figure 4. The geometric meaning of grey correlative degree.

In order to evaluate the overall condition of the frozen wall, it can be analyzed by the correlation of the temperature measurement data of each freezing apparatus. For example, in an ideal situation, the flow distribution of each freezing apparatus is uniform, and the conditions of the stratum are also the same. The temperature measurement curve between them should be exactly the same, the correlation degree should be very good. According to the temperature measurement data in the temperature field, the relevant concepts and definitions of the slope correlation degree are introduced.

Correlation coefficient $\xi(t)$: Let the sequence be $X(t)$ and $Y(t)$, $X, Y \in R$, $t \in T$, then

$$\xi(t) = \frac{1}{1 + \left| \frac{1}{\sigma_x} \frac{\Delta x(t)}{\Delta t} - \frac{1}{\sigma_y} \frac{\Delta y(t)}{\Delta t} \right|}, \tag{3}$$

where $\Delta x(t) = x(t+1) - x(t)$; $\Delta y(t) = y(t+1) - y(t)$; $\Delta t = (t+1) - t = 1$.

$$\sigma_{xi} = \sqrt{\frac{1}{N} \sum_{i=1}^N (x_{ik} - \bar{x}_i)^2}; \sigma_{yi} = \sqrt{\frac{1}{N} \sum_{i=1}^N (y_{ik} - \bar{y}_i)^2},$$

where σ_{xi} and σ_{yi} are the standard deviations of the $X(t)$ and $Y(t)$ sequences, respectively.

The nature of $\xi(t)$: $0 \leq \xi(t) \leq 1$, the closer the slope of $X(t)$ and $Y(t)$ at time t , the greater the $\xi(t)$, and the correlation coefficient is 1; $\xi(t)$ is related to the geometric shape, but not the starting position.

Definition of correlation degree:

$$\gamma_{xy} = \frac{1}{N-1} \sum_{t=1}^{N-1} \xi(t), \quad (t = \{1, 2, 3, \dots, N-1\}). \quad (4)$$

The main characteristics of slope correlation: (1) The original data does not need to select reference points and standardization, which is convenient for computer processing, and the restrictions on detection conditions are also small; (2) The calculation is not affected when the original data contains zero or negative values, so the adaptability of engineering data application is good; (3) The resolution of the correlation degree is high.

Each freezing apparatus can be regarded as either a parent factor or a sub factor, thus forming a correlation matrix; it is a symmetric matrix with a main diagonal element of 1, i.e.

$$\begin{array}{c} \text{Number} \\ 1 \\ 2 \\ \vdots \\ n-1 \\ n \end{array} \begin{array}{ccccc} 1 & 2 & \cdots & n-1 & n \\ \left[\begin{array}{ccccc} 1 & \gamma_{1,2} & \cdots & \gamma_{1,n-1} & \gamma_{1,n} \\ \gamma_{2,1} & 1 & \cdots & \gamma_{2,n-1} & \gamma_{2,n} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \gamma_{n-1,1} & \gamma_{n-1,2} & \cdots & 1 & \gamma_{n-1,n} \\ \gamma_{n,1} & \gamma_{n,2} & \cdots & \gamma_{n,n-1} & 1 \end{array} \right] \end{array}. \quad (5)$$

For the above matrix, you can use the analysis of variance to study the overall situation of the frozen wall. (1) The element $\gamma_{i,j}$ in the matrix represents the degree of correlation between holes i and j . Obviously, $\gamma_{i,j} = 1$, the closer $\gamma_{i,j}$ is to 1, the closer is the working condition between i and j holes. $\gamma_{i,j} < \gamma_{i,k}$, indicating that the freezing condition of hole k is closer to that of hole i than that of hole j ; (2)

The average correlation of hole i , $\bar{\gamma}_i = \frac{1}{n} \sum_{j=1}^n \gamma_{i,j}$, which reflects the comprehensive degree of the correlation between hole i and other holes, and reflects the state of hole i in the whole; (3) The overall

average correlation degree of the freezing apparatus $\bar{\gamma} = \frac{1}{n} \sum_{i=1}^n \left(\frac{1}{n} \sum_{j=1}^n \gamma_{i,j} \right)$, which reflects the average

effect of each freezing apparatus in the whole frozen wall. The closer $\bar{\gamma}$ is to 1, the more similar are the working conditions such as the distribution of cold energy and stratum conditions; (4) Parameters $\gamma_{i,j}$, $\bar{\gamma}_i$, $\bar{\gamma}$ are all functions of time (or depth), that is, when analyzed in different depth segments, the above parameters will change, which can directly reflect the changes of objective conditions, such as groundwater flow, lithology and other conditions.

2.3. Example calculation

The net diameter of Xipang shaft of Dongpang Mine in China is 4.5 m, the thickness of the topsoil layer is 136 m, the aquifer is mainly composed of a pebble formation with a vertical depth of 22 m, a sand layer and a pebble formation of 111~135.65 m, and the freezing depth is 175 m. The ring diameter of the freezing apparatus is 9.556 m, the hole spacing is 1.247 m, and there are 24 freezing holes. During the active freezing period, the brine temperature is $-26 \sim -28$ °C. BRR specific heat calorimeter and QTM-PD2

thermal conductivity measuring instrument in Fig. 5 are used to test the heat flux and thermal conductivity of soil layer respectively. The thermal physical properties of soil are shown in Table 1. When the active freezing time reaches the design period, and the temperature of the temperature measuring holes has reached the design temperature, an experimental excavation is performed on the shaft. When the fine sand layer of -41.7 m was excavated, water leaked in the shaft. After preliminary judgment, it was determined that the frozen wall of the shaft was not closed. In order to find the reason, the longitudinal temperature measurement of all freezing apparatuses of the shaft was carried out. Part of the data is shown in Table 2. Firstly, it was found that the longitudinal temperature of the C1 temperature measuring hole, No. 18, No. 19, No. 20, No. 21 and No. 22 freezing holes had an obvious inflection point at 50 m~60 m, and the temperature was 2~3 °C higher than other layers. Therefore, it was found the frozen wall did not close due to the abnormality of these freezing holes. However, it is necessary to specifically determine the location of the "window" of the frozen wall and take remedial measures.

Then, we used the grey correlation theory to conduct a whole field monitoring analysis and find the corresponding "window" position. We established the holes Nos. 20 and 21 had the largest longitudinal temperature fluctuation as the parent factor, and analyzed the longitudinal temperature measurement data of the freezing apparatus with a depth of 65m or more in the topsoil aquifer, using the slope correlation method. The freezing holes with a high degree of correlation with holes 20 and 21 may be weak holes in the frozen wall. After analyzing all the longitudinal temperature measurement data, it was finally confirmed that there was groundwater flow phenomenon at the vertical depth of about 57m on the frozen wall, and there were water-conducting channels between the holes No. 18~22 and the holes No. 7~9.

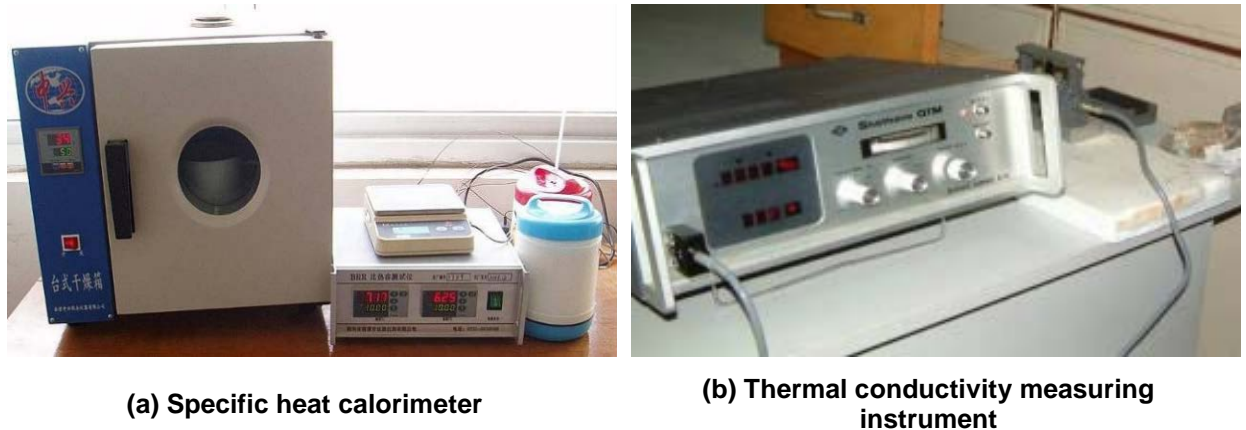


Figure 5. Measurement of soil thermophysical properties.

Table 1. Soil properties.

Soil property	Depth (m)	Moisture content (%)	Density (g/cm ³)	Dry density (g/cm ³)	Specific heat (J·g ⁻¹ ·K ⁻¹)	Thermal conductivity (kCal/m·h·°C)
Sandy clay	43.68~80.87	14.68	2.095	1.831	1.504	1.872
Clayey sand	25.30~72.20	24.76	1.898	1.521	1.485	1.787

Table 2. Lengthwise temperature survey along freeze pipe of main shaft in Dongpang mine, China.

Hole number	Depth							
	25 m	35 m	40 m	45 m	50 m	55 m	60 m	65 m
1	-19.8	-20.0	-20.1	-20.2	-20.5	-20.3	-20.4	-20.2
3	-21.6	-21.9	-21.8	-21.9	-22.1	-22.0	-22	-22.6
5	-22.1	-22.4	-22.5	-22.4	-22.3	-22.1	-22.1	-22.4
7	-21.5	-21.8	-22.1	-22.2	-22.2	-22.0	-22.1	-22.2
8	-21.5	-21.5	-21.6	-21.6	-21.4	-21.0	-21.3	-21.2
9	-21.8	-21.8	-21.8	-22.0	-21.8	-20.5	-21.7	-21.9
12	-21.4	-21.2	-21.1	-21.4	-21.4	-21.3	-21.3	-21.5
14	-19.7	-19.6	-19.7	-20.2	-20.3	-20.2	-20.2	-20.3
16	-20.1	-19.8	-19.9	-20.4	-20.4	-20.0	-20.2	-20.1

Hole number	Depth							
	25 m	35 m	40 m	45 m	50 m	55 m	60 m	65 m
18	-20.0	-20.0	-20.0	-20.1	-20.0	-20.0	-20.4	-20.6
19	-19.8	-20.2	-20.1	-19.7	-19.4	-18.1	-19.3	-19.8
20	-20.3	-20.9	-21.1	-21.1	-19.9	-19.0	-21.0	-21.3
21	-21.0	-21.2	-21.3	-21.2	-20.6	-19.3	-20.8	-21.1
22	-20.4	-20.4	-20.3	-20.3	-19.6	-19.1	-19.8	-20.3
23	-21.8	-21.8	-21.9	-21.9	-22.0	-21.9	-21.9	-22.1

3. Results and Discussions

3.1. Results

By using the slope correlation method to analyze the above longitudinal temperature measurement data, the results are shown in Fig. 6. It can be seen that the correlation between No. 7~9 and No. 18~22 is particularly high, indicating that the groundwater flow forms a channel along No. 18~22 and No. 7~9, so the correlation analysis between freezing holes can determine the location of the frozen wall weakness caused by abnormal freezing pipes.

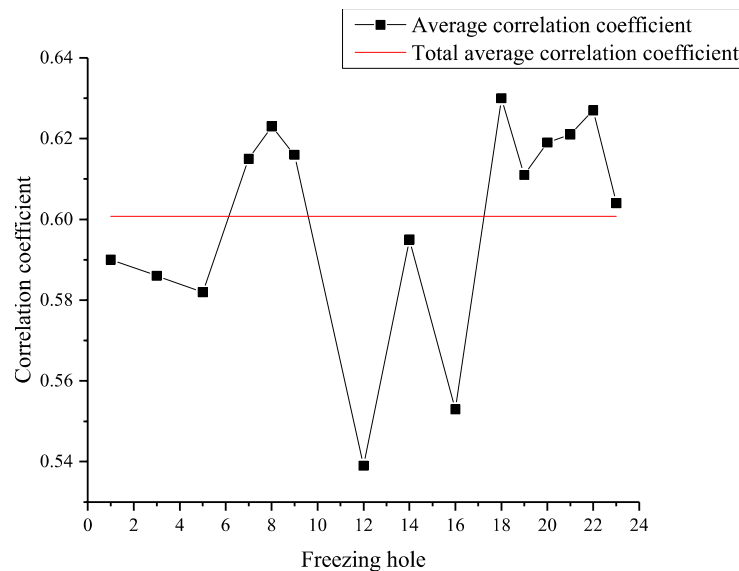


Figure 6. Correlation analysis of longitudinal temperature measurement data of freezing apparatus.

The on-site investigation and analysis showed that due to the accident stratum being a highly permeable gravel layer affected by spring irrigation pumping, the groundwater flow scoured the frozen wall from the northwest to the southeast taking away a large amount of cold energy and making it difficult to close the frozen wall. The water flow enters the shaft with the freezing holes No. 18, No. 19, No. 20, No. 21 and No. 22 as a main channel, one flow enters the shaft along the S2 hydrological hole, and the other flow flows from No. 7, No. 8 and No. 9 freezing holes taking away the cooling energy. This further confirms the accuracy of the grey correlation theory in accident analysis, as shown in Fig. 7. In order to speed up the progress of the freezing project, the brine temperature was lowered from $-28\text{ }^{\circ}\text{C}$ to $-33\text{ }^{\circ}\text{C}$. Pipeline pumps were added to the eight freezing holes of No. 18~22 and No. 7~9 of the annular groove of the shaft to increase the brine flow rate of the freezing hole. The C1 and C2 temperature measuring holes are changed into freezing holes to increase the cooling capacity. By taking the above measures, the healing of the frozen wall was ensured, and the accident was dealt with pertinently. During the excavation process, the location of the "window" of the frozen wall was verified. Holes 18~22 and 7~9 have traces of repair of the frozen wall window at a vertical depth of about 57 m, and there is ice formation in the gravel gap (size 50 mm \times 80 mm), as shown in Fig. 8, which is completely consistent with the correlation analysis.

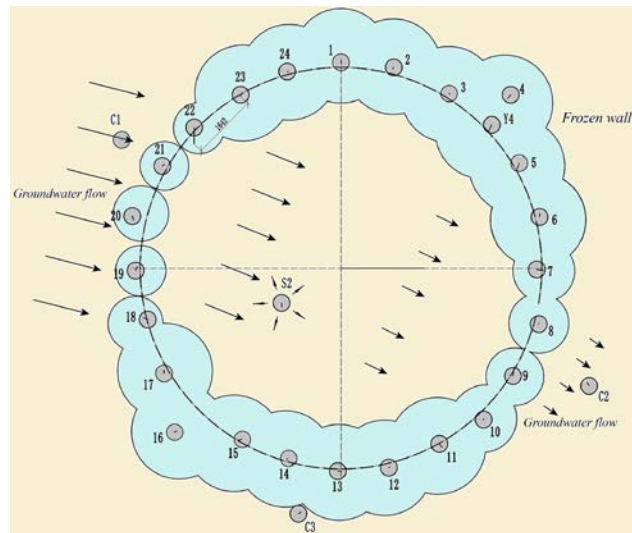


Figure 7. Schematic diagram of the 57 m frozen wall outlet of the shaft.



Figure 8. Freezing point on the west side of shaft at 57.3 m.

3.2. Discussions

In the process of studying the temperature field of the frozen wall, the researchers calculated the thickness, strength and closing time of the frozen wall using theoretical analysis methods and finite element numerical methods through the temperature measurement hole data. In this case, the calculation result obtained by the theoretical analysis method reflects the state of the local range of the frozen wall, and the solution is also approximate [3, 5, 21]. The modeling process of the finite element numerical calculation method is ideal, requiring the model to be homogeneous and continuous, and the obtained simulation calculation results have certain errors from the actual measurement [22, 23]. In addition, in the face of complex geological conditions and hydrological environments, the development of complex freezing temperature field under some special working conditions is no longer applicable to the above method.

Literature [24] applied the area correlation analysis method to analyze the longitudinal temperature measurement data of the frozen shaft of Jian Chang Ying Mine in China, and determined the weak position of the frozen wall. However, this method cannot be calculated when there are zero values in the original series.

In this study, the longitudinal temperature measurement data of freezing tube is used to comprehensively analyze the development of frozen wall, which overcomes the shortage of only relying on the local temperature measurement hole to understand the situation of frozen wall in the traditional technology; the slope correlation method also effectively solves the defect that zero value cannot be calculated in the data, and realizes the prediction and evaluation of the all-round development of frozen wall.

4. Conclusion

From the above results:

1. The longitudinal temperature measurement of the temperature measuring hole and the freezing apparatus is a technical means to understand the local and overall freezing conditions of the shaft. The proposal of whole field monitoring is the supplement and development of the longitudinal temperature measurement technology in theory.
2. The use of slope correlation analysis is a practical method of longitudinal temperature measurement data processing, and an application of grey theory in artificial ground freezing engineering.
3. The correlation analysis of the longitudinal temperature measurement data between freezing apparatuses reflects the relatively good or bad freezing effect of the freezing apparatus, and is the basis for conclusion on the weak position of the frozen wall.
4. The normal operation of each freezing apparatus is a necessary guarantee for the smooth freezing of the shaft. It is affected by many subjective and objective factors such as the cold supply of the freeze pipe, the flow of groundwater, and the thermal conductivity of the formation. It combines the longitudinal temperature detection data of the freezing pipe with the temperature measurement hole detection data to achieve the goal of comprehensively monitoring the formation effect of the frozen wall.

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Received 09.06.2020. Approved after reviewing 28.04.2021. Accepted 20.06.2021.