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Analytical estimation of the first natural frequency and analysis of a planar regular truss oscillation spectrum

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Abstract. A statically determinate truss of a beam type with a triple lattice with short descending and long ascending braces is considered. The mass of a truss is modeled by concentrated loads at its nodes. For the first natural frequency, the Dunkerley's method derives a formula for the dependence of its lower boundary on the number of panels. The calculation of the efforts in the truss required to obtain the stiffness value according to the Maxwell-Mohr formula is performed in the Maple computer mathematics system by cutting out the nodes. It is shown that for a certain number of panels the proposed scheme of the truss has the property of kinematic variability. For admissible numbers of panels, by induction, the sequence of solutions for trusses with different numbers of panels is generalized to an arbitrary case. The coefficients of the required dependence are obtained as solutions of linear homogeneous recurrent equations. To obtain and solve recurrent equations, the operators of the Maple system are used. The found solution is compared with the minimum frequency of the spectrum obtained numerically. It is shown that the accuracy of the analytical assessment monotonically increases with the increase in the number of panels. Multiple frequencies and the independence of several higher frequencies from the number of panels were found in the frequency spectrum.

1. Introduction

Although one of the most important characteristics of a structure for practice is its lowest natural frequency, for many engineering problems it is also necessary to know the entire frequency spectrum of the structure. Such tasks, in particular, include the tasks of seismic resistance of structures [1–3]. In most cases, the frequency spectrum is calculated numerically in a linear [4, 5] or nonlinear [6–10] setting. In rare cases, the first frequency is determined analytically [11–16]. Particularly effective are formulas for the vibration frequencies of regular structures, depending on their order (the number of periodicity elements, for example, panels). General questions of the existence of statically determinate regular rod systems were studied in [17–19]. Formulas for the deflection of planar regular trusses for various types of lattices and supports are obtained in handbooks [20, 21], and [22–24]. The formulas for deflection make it possible to calculate the stiffness of the truss and calculate the natural frequencies based on it. Methods for calculating natural frequencies are reviewed in [25, 26].

In this paper, we derive the analytical dependence of the lower boundary of the first natural vibration frequency of a regular truss and analyze its spectrum. In contrast to the known similar problems, the

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proposed truss scheme has a special feature. For some values of the number of panels, the design allows kinematic variability. This complicates the derivation of the formula for the desired dependency. In addition, this study takes into account both degrees of freedom of each mass in the truss node. The solution is compared with the numerical one obtained for a refined (non-uniform) distribution of masses over nodes.

2. Methods

2.1. Calculation of forces. Kinematic analysis

The symmetric truss of a beam type with a triple diagonal lattice is considered (Fig. 1). Ascending and descending braces have different angles of inclination. The inertial properties of the structure are modeled by the same masses concentrated in all hinges. In this case, the rods themselves are assumed to be devoid of masses. The truss has 2n panels, length a and 2h high. The truss consists of $n_e = 8(n+1)$ elements, including three rods, simulating a movable left and fixed right hinge support.

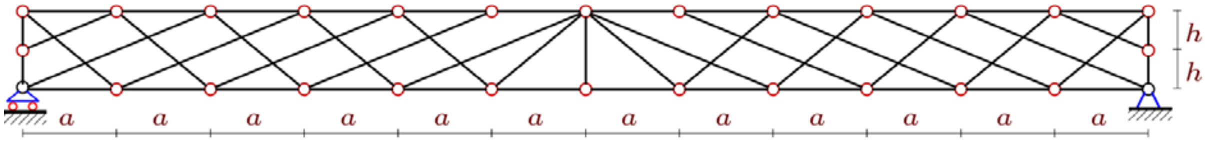

Figure 1. Truss, *n* = 6.

To calculate the forces in the structural members, the program [27], written in the language of symbolic mathematics Maple [28], is used. The algorithm used in this program also allows you to use other similar systems: Mathematica, Maxima, Reduce, etc. The coordinates of the hinges are entered into the program, assuming that the origin is located in the left support:

$$x_i = x_{i+2n+2} = a(i-1),\ y_i = 0,\ y_{i+2n+2} = 2h,\ i = 1,..,2n+1,$$
$$x_{2(n+1)} = 0,\ x_{4(n+2)} = 2na,\ y_{2(n+1)} = y_{4(n+1)} = h.$$

The lattice structure is defined by ordered lists of the vertices of the rods $T_i,\ i = 1,...,n_e$. For the rods of the lower belt, for example, we have:

$$T_i = [i, i+1],\ i = 1,...,2n.$$

The equilibrium equations of all nodes of the truss constitute a system, the solution of which gives the values of the forces in symbolic form, depending on the given number of panels. The matrix of the system of equations consists of the direction cosines of the forces, determined by the values of the coordinates of the nodes of the rods. The matrix is filled in a cycle along all the bars of the truss $i = 1,...,n_e$

$$L_1 = x_{T[i,2]} - x_{T[i,1]},\ L_2 = y_{T[i,2]} - y_{T[i,1]},$$
$$l_i = \sqrt{L_1^2 + L_2^2},$$
$$G_{\alpha j} = -L_j / l_i,\ \alpha = 2T[i,2] + j - 2 \le n_e,$$
$$G_{\alpha j} = L_j / l_i,\ \alpha = 2T[i,1] + j - 2 \le n_e,\ j = 1,2.$$

The system of equilibrium equations is written in matrix form $\mathbf{GS} = \mathbf{P}$, where $\mathbf{S}$ is the vector of unknown forces in the rods and supports, $\mathbf{P}$ is the vector of loads.

Trial calculations of the forces in the rods from the action of arbitrary loads have shown that for some numbers of panels, for example $n = 2,5,8,11,...$, or $n = 3k-1,\ k = 1,2,3...$, the determinant of the system of equilibrium equations vanishes. This is possible in the case of instantaneous change of the design, which, of course, is completely unacceptable here. This fact is confirmed by the diagram of possible velocities (Fig. 2). This is not the only scheme of possible velocities. Its asymmetric variants are also admissible when all nodes in one of the truss halves are stationary or the directions of the velocities are asymmetric. There is a connection between the values of the velocities $v/h = u/a$.

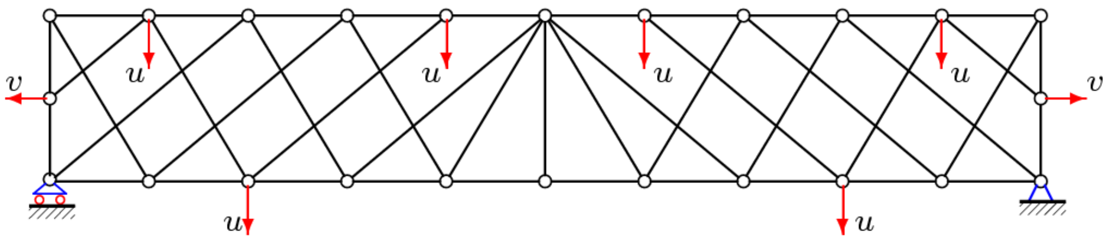

Figure 2. Scheme of possible velocities of truss nodes, *n* = 5.

Eliminating the unacceptable values of n from the calculations, we set the following sequence of panel numbers used to display the dependence of the vibration frequency on the number of panels:

$$n=\left(6k+5-(-1)^{k}\right)/4,\ k=1,2,\ldots. \tag{1}$$

In what follows, by the order of the truss, we mean the number k. The dependence of the deflection of this truss on the number of panels at various loads is given in the handbook [21].

2.2. First frequency. The analytical solution according to the Dunkerley's method

Since each node endowed with mass has two degrees of freedom, the number of degrees of freedom of the considered truss is equal to $N_s=2N$, where $N=4n+2$ is the number of masses. The formula for determining the lower (first) frequency by the Dunkerley's method [29, 30], which gives an estimate of this value from below, has the form

$$\omega_D=\left(\sum_{j=1}^{N}1/\omega_{h,j}^2+1/\omega_{v,j}^2\right)^{-1/2}, \tag{2}$$

where $\omega_{h,j},\ j=1,..,N$ is partial frequencies of horizontal vibrations, and $\omega_{v,j}$ is partial frequencies of vertical ones. Partial frequencies are calculated from the analysis of the differential equation of motion of one mass fixed at node *j* of the truss

$$m\ddot{u}_j+D_ju_j=0,$$

where u_j is the vertical displacement of the mass, $\ddot{u}_j$ is the acceleration, D_k is the stiffness coefficient (*j* is the mass number). The frequency of vibration of the load is $\omega_j=\sqrt{D_j/m}$. The stiffness coefficient is calculated using the Maxwell – Mohr formula under the assumption that the stiffness of all the rods are the same

$$\delta_j=1/D_j=\sum_{\alpha=1}^{n_e-3}\left(S_\alpha^{(j)}\right)^2 l_\alpha\Big/(EF).$$

Here it is indicated $S_\alpha^{(j)}$ is the forces in the element number α from the action of a unit vertical or horizontal force applied to the node where the mass numbered k is located. According to (2) we have

$$\omega_D^{-2}=m\sum_{j=1}^{N_S}\delta_j=m(\Delta_h+\Delta_v). \tag{3}$$

Sequential calculation of trusses with an increasing number of panels shows that the expression for the coefficient Δ_v has a constant form

$$\Delta_v=\left(C_1a^3+C_2c^3+C_3d^3+C_4h^3\right)/\left(h^2nEF\right), \tag{4}$$

where $c=\sqrt{a^2+h^2}$, $d=\sqrt{a^2+4h^2}$ are the lengths of the braces.

The coefficients $C_1,...,C_4$, as functions of the number k, related to the number of panels n in half the span by formula (1), are found by the induction method with the involvement of the Maple system operators.

Using the operator *rgf_findrecur*, we make sure that the sequence of coefficients C_1 satisfies the linear homogeneous recurrent equation of the ninth order

$$C_{1,k}=C_{1,k-1}+4C_{1,k-2}-4C_{1,k-3}-6C_{1,k-4}+\\+6C_{1,k-5}+4C_{1,k-6}-4C_{1,k-7}-C_{1,k-8}+C_{1,k-9}.$$

The *rsolve* operator solves this equation

$$C_1=(k+1)\Big(54k^4+9\big(19-5(-1)^k\big)k^3+3\big(123-35(-1)^k\big)k^2+\\+3\big(92-45(-1)^k\big)k-40(-1)^k+50\Big)\Big/40.$$

The remaining coefficients are found as solutions to similar equations of the sixth and seventh orders:

$$C_2=(k+1)\Big(78k^2+3\big(27-11(-1)^k\big)k-14(-1)^k+14\Big)\Big/12,$$

$$C_3=\Big(78k^3+3\big(77-17(-1)^k\big)k^2+\big(233-101(-1)^k\big)k-53(-1)^k+77\Big)\Big/96,$$

$$C_4=\Big(9\big(5-(-1)^k\big)k^2+12\big(11-3(-1)^k\big)k-35(-1)^k+83\Big)\Big/6.$$

Similarly, we obtain the sum related to the partial frequencies of horizontal vibrations:

$$\Delta_h=\left(C_5a^3+C_6c^3+C_7d^3+C_8h^3\right)\Big/\left(a^2n^2EF\right), \tag{5}$$

where

$$C_5=\Big(648k^4+54\big(47-8(-1)^k\big)k^3+9\big(425-157(-1)^k\big)k^2+\\+3\big(893-505(-1)^k\big)k-547(-1)^k+731\Big)\Big/32,$$

$$C_6=\Big(36k^3+6\big(19-2(-1)^k\big)k^2+4\big(29-10(-1)^k\big)k-29(-1)^k+37\Big)\Big/4,$$

$$C_7=(k+1)\Big(18k^2-\big(15(-1)^k-57\big)k-8(-1)^k+52\Big)\Big/16,$$

$$C_8=\Big(18k^2+12\big(5-3(-1)^k\big)k-47(-1)^k+55\Big)\Big/4.$$

As a result, we obtain the desired lower estimate of the first frequency in analytical form

$$\omega_D=\sqrt{\frac{nEF}{m\left(\left(C_1a^3+C_2c^3+C_3d^3+C_4h^3\right)\big/h^2+\left(C_5a^3+C_6c^3+C_7d^3+C_8h^3\right)\big/\left(na^2\right)\right)}}. \tag{6}$$

If we do not take into account the horizontal fluctuations of loads, then the solution turns out to be much simpler

$$\tilde{\omega}_D=h\sqrt{\frac{nEF}{m\left(C_1a^3+C_2c^3+C_3d^3+C_4h^3\right)}}. \tag{7}$$

3. Results and Discussion

3.1. Numerical calculation of the spectrum of natural frequencies

Let us estimate the accuracy of the obtained dependence by comparing it with the result of the numerical solution. To calculate the forces in the rods, which are used to calculate the stiffness of the structure, you can use the same program used to calculate the estimate (6).

The system of differential equations for the movement of loads has the form

$$\mathbf{M}_N \ddot{\mathbf{U}} + \mathbf{D}_N \mathbf{U} = 0, \quad (8)$$

where $\mathbf{U} = [x_1, ..., x_N,\ y_1, ..., y_N]^T$ is the displacement of the masses, $\mathbf{D}_N$ is the stiffness matrix, $\mathbf{M}_N$ is the diagonal inertia matrix of size $N \times N$, and $\ddot{\mathbf{U}}$ is the acceleration vector. Multiplying (8) by the compliance matrix $\mathbf{B}_N$, we reduce the problem of finding natural frequencies to the problem of the eigenvalues of the matrix $\mathbf{\Phi}_N = \mathbf{B}_N \mathbf{M}_N$. Indeed, taking into account the ratio $\ddot{\mathbf{U}} = -\omega^2 \mathbf{U}$, where ω is the natural frequency of oscillations, we obtain $\mathbf{\Phi}_N \mathbf{U} = \lambda \mathbf{U}$, where $\lambda = 1/\omega^2$ is the eigenvalue of the matrix $\mathbf{\Phi}_N$. Maple has an *Eigenvalues* operator from the *LinearAlgebra* package for calculating eigenvalues and vectors of matrices.

Example. The elastic modulus of steel $E = 2 \cdot 10^5$ MPa. The rods have a cross-section of $F = 40.5\,\text{sm}^2$ and a linear mass of $\rho = 31.8\,\text{kg/m}$ (channel-shaped cross-section No. 30). Dimensions $a = 3\,\text{m}$, $h = 1\,\text{m}$ are accepted. The mass of each node in the analytical solution (6) is calculated by the formula $m = \rho L_0 / N$ where $L_0 = 4an + 4(n-1)c + (2n-1)d + 6h$ is total length of all truss rods. In the numerical solution, the masses of the nodes depend on the lengths of the rods connected to the nodes. The mass of each rod is divided in half between the nodes at its ends. For the assumed cross-section of the rods and the dimensions of the truss at $n = 3$, the corresponding mass distribution is given in Fig. 3. In an analytical solution designed for equal masses, each node has a mass in this case $m = 203\,\text{kg}$.

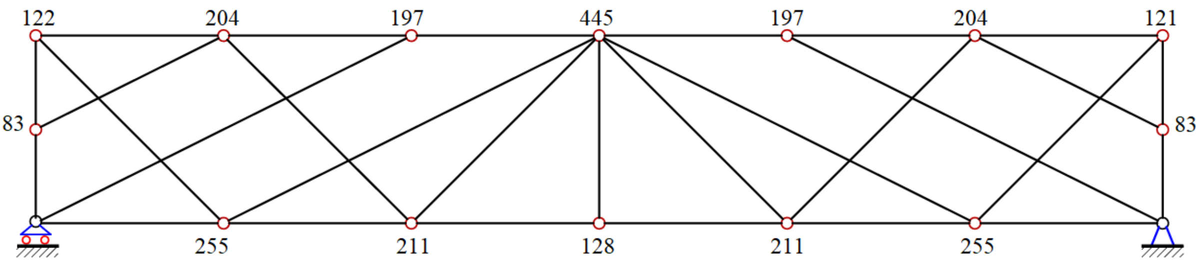

Figure 3. Mass distribution over the truss nodes in the numerical solution of the problem (kg).

Graph 4 compares the analytical estimate (6) and the lowest frequency of the spectrum of the truss, obtained numerically.

With an increase in the number of panels, the vibration frequency decreases, and the degree of approximation of the analytical estimate also increases. More precisely, you can trace the dependence of the error on the number of panels, if you enter the value $\varepsilon = (\omega_1 - \omega_D)/\omega_1$ of the relative discrepancy. For $k > 12$, the discrepancy of solution (5) decreases to a quite acceptable value of several percent (Fig. 4). An increase in the accuracy of the solution with a decrease in the height *h* of the structure is also noticeable.

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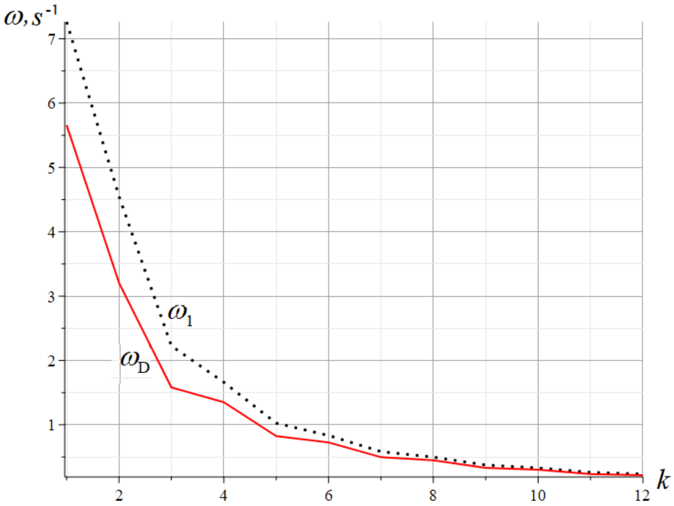

Figure 4. Dependence of the lower Dunkerley's estimate ω_D and the first frequency ω_1 obtained numerically on the number of panels.

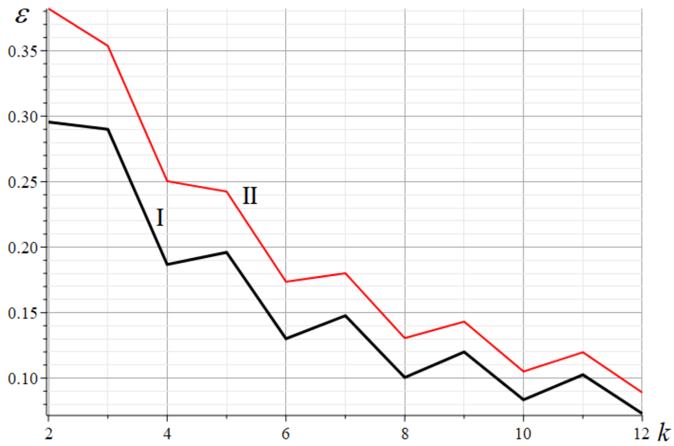

Figure 5. Solution (6) discrepancy depending on the number of panels and the height of the truss; I – h = 1 m; II – h = 2 m.

Increasing analytical accuracy with the increasing number of panels is of fundamental importance to practice. The "curse of dimension" is known, according to which with an increase in the complexity of the design, associated, for example, with an increase in the number of panels, the accuracy of solving a system of algebraic equations inevitably decreases, and the calculation time increases. Of course, the high bit depth and power of modern computing technology overcome this difficulty to some extent, but there can always be a design of such complexity that the power of the computing system and the amount of memory may not be enough. An analytical solution gives a simple solution and an increase in the order of the calculated system only goes to increase its accuracy. Note, however, that the effect of increasing the accuracy of the lower Dunkerley's estimate is not valid for all structures. For spatial systems, for example, the error grows with an increase in the number of panels, asymptotically approaching a value of the order of 30–40 %.

Note that without taking into account the horizontal fluctuations in the Dunkerley's formula (3), the result does not change much.

The corresponding frequency ratio according to the formula (6) and (7) decreases with an increase in the number of panels and a decrease in the height of the truss (Fig. 6).

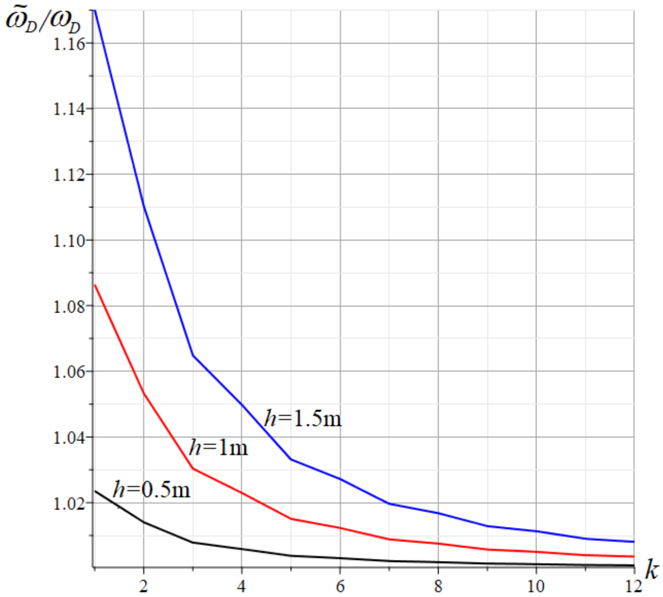

Figure 6. Comparison of frequencies obtained by formulas (6) and (7).

3.2. Natural frequency spectrum

An analysis of the entire frequency spectrum of the truss revealed interesting features. In Fig. 7, with the same data as for Fig. 3, the spectra of trusses with k = 1,2, ..., 8 are plotted. Horizontal mass fluctuations are not taken into account here. Each broken curve corresponds to a truss of order k, the abscissa shows the numbers of frequencies in the spectrum. The characteristic independence of the last five higher frequencies in each spectrum from the order of the system is noticeable. Here, the equality of frequencies is taken within the framework of a certain accuracy. Let us denote $\omega_j^{(k)}$ is frequencies with number j in the spectrum of a truss of the order of k, ordered in ascending order. According to (1), the number of degrees of freedom of a truss of the order of k is equal to $N = 6k + 7 - (-1)^k$, $k = 1, 2, \ldots$. The higher frequencies in the spectra of the truss under consideration are multiples of pairs and do not depend on accuracy δ_ω on the order of k. For example, for h = 1 m, we have higher frequencies $\omega_j^{(8)} = 119.138\ \text{c}^{-1}$, $j = N = 6k + 7 - (-1)^k$, that are multiple and equal to each other for different k with a relative accuracy of $\delta_\omega = \left(\omega_j^{(8)} - \omega_j^{(1)}\right) \Big/ \omega_j^{(8)} = 1.26 \cdot 10^{-5}$, and for h = 2 m the frequency and error of equality of conditionally multiple frequencies are less: $\omega_j^{(8)} = 88.885\ \text{s}^{-1}$, $\delta_\omega = 0.643 \cdot 10^{-5}$.

The same pattern also applies to the second and third frequencies from the end of the spectrum: $j = N - 2$, $j = N - 3$. The lowest frequencies in the spectra are located more chaotically.

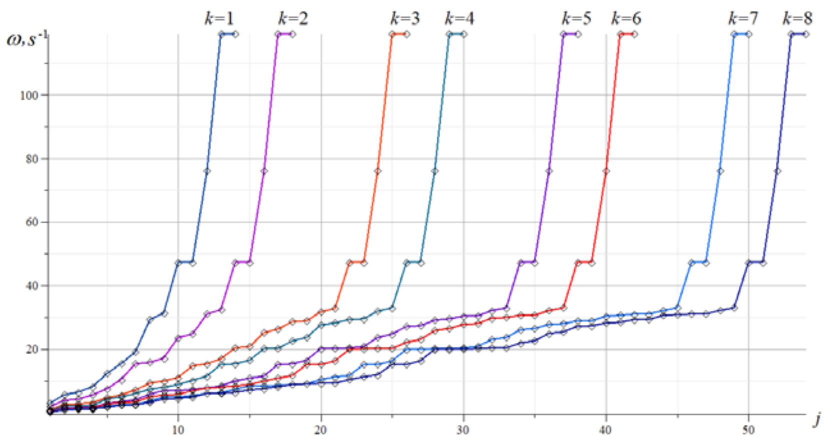

Figure 7. Spectra of natural frequencies of trusses of orders 1-8.

4. Conclusion

1. For the considered planar truss, an analytical lower estimate of the first frequency is obtained for an arbitrary number of panels. The coefficients of the resulting formula have the form of polynomials no higher than the fifth order in the number of panels.

2. It is noted that for a certain number of panels, the truss allows instantaneous kinematic variability. This is confirmed by the distribution pattern of possible node velocities.

3. A comparison of the analytical estimate with the numerical result shows that with an increase in the number of panels, the accuracy of the estimate increases.

4. It is shown that the highest frequency in the truss spectrum and several higher frequencies do not depend on the order of the truss. Multiple frequencies are found.

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References

1. Resatalab, S., Ahmadi, M.T., Alembagheri, M. Seismic response sensitivity analysis of intake towers interacting with dam, reservoir, and foundation. Magazine of Civil Engineering. 2020. 98(7). Pp. 9901–9901. DOI: 10.18720/MCE.99.1.
2. Pekcan, G., Itani, A.M., Linke, C. Enhancing seismic resilience using truss girder frame systems with supplemental devices. Journal of Constructional Steel Research. 2014. 94. Pp. 23–32. DOI: 10.1016/j.jcsr.2013.10.016.
3. Vatin, N., Ivanov, Ay., Rutman, Y., Chernogorskiy, S., Shvetsov, K. Earthquake engineering optimization of structures by economic criterion. Magazine of Civil Engineering. 2017. 8(76). Pp. 67–83. DOI: 10.18720/MCE.76.7
4. Siriguleng, B., Zhang, W., Liu, T., Liu, Y.Z. Vibration modal experiments and modal interactions of a large space deployable antenna with carbon fiber material and ring-truss structure. Engineering Structures. 2020. 207. Pp. 109932. DOI: 10.1016/j.engstruct.2019.109932
5. Liang, L., Li, X., Yin, J., Wang, D., Gao, W., Guo, Z. Vibration characteristics of damping pad floating slab on the long-span steel truss cable-stayed bridge in urban rail transit. Engineering Structures. 2019. 191. Pp. 92–103. DOI: 10.1016/j.engstruct.2019.04.032
6. Khodzhaev, D., Abdikarimov, R., Vatin, N. Nonlinear oscillations of a viscoelastic cylindrical panel with concentrated masses. MATEC Web of Conferences. 2018. 245. DOI: 10.1051/matecconf/201824501001
7. Santana, M.V.B., Gonçalves, P.B., Silveira, R.A.M. Closed-form solutions for the symmetric nonlinear free oscillations of pyramidal trusses. Physica D: Nonlinear Phenomena. 2021. 417. Pp. 132814. DOI: 10.1016/j.physd.2020.132814
8. Liu, M., Cao, D., Zhang, X., Wei, J., Zhu, D. Nonlinear dynamic responses of beamlike truss based on the equivalent nonlinear beam model. International Journal of Mechanical Sciences. 2021. 194. Pp. 106197. DOI: 10.1016/j.ijmecsci.2020.106197
9. Ufimtcev, E. Dynamic Calculation of Nonlinear Oscillations of Flat Trusses Part 2: Examples of Calculations. Procedia Engineering. 2017. 206. Pp. 850–856. DOI: 10.1016/j.proeng.2017.10.562.
10. Ufimtsev, E., Voronina, M. Research of Total Mechanical Energy of Steel Roof Truss during Structurally Nonlinear Oscillations. Procedia Engineering. 2016. 150. Pp. 1891–1897. DOI: 10.1016/j.proeng.2016.07.188.
11. Kirsanov, M.N., Petrichenko, E.A., Vorobev, O.V. The formula for the lower estimate of the fundamental frequency of natural vibrations of a truss with an arbitrary number of panels. Construction of Unique Buildings and Structures. 2021. 94(1). Pp. 9403–9403. DOI:10.4123/CUBS.94.3. URL: https://unistroy.spbstu.ru/article/2021.94.3 (date of application: 6.03.2021)
12. Vorobyev, O. About methods of obtaining analytical solution for eigenfrequencies problem of trusses. Structural mechanics and structures. 2020. 1(24). Pp. 25–38. URL: http://vuz.exponenta.ru/PDF/NAUKA/elibrary_42591122_21834695.pdf
13. Vorobev, O.V. Bilateral Analytical Estimation of the First Frequency of a Plane Truss. Construction of Unique Buildings and Structures. 2020. 92(7). Pp. 9204–9204. DOI: 10.18720/CUBS.92.4.
14. Kirsanov, M.N., T.D.V. Analysis of the natural frequencies of oscillations of a planar truss with an arbitrary number of panels. Vestnik MGSU. 2019. 14(4). Pp. 284–292. DOI: 10.22227/1997-0935.2019.3.284-292. URL: http://vuz.exponenta.ru/1/kt.pdf (date of application: 11.03.2021)
15. Kirsanov, M.N., Vorobyev, O.V. Calculating of a spatial cantilever truss natural vibration frequency with an arbitrary number of panels: analytical solution. Construction of Unique Buildings and Structures. 2021. 94. Pp. 9402. DOI: 10.4123/CUBS.94.2
16. Petrichenko, E.A. Lower bound of the natural oscillation frequency of the Fink truss. Structural Mechanics and Structures. 2020. 26(3). Pp. 21–29.
17. Hutchinson, R.G., Fleck, N.A. Microarchitectured cellular solids - The hunt for statically determinate periodic trusses. ZAMM Zeitschrift fur Angewandte Mathematik und Mechanik. 2005. 85(9). Pp. 607–617. DOI: 10.1002/zamm.200410208
18. Hutchinson, R.G., Fleck, N.A. The structural performance of the periodic truss. Journal of the Mechanics and Physics of Solids. 2006. 54(4). Pp. 756–782. DOI: 10.1016/j.jmps.2005.10.008
19. Zok, F.W., Latture, R.M., Begley, M.R. Periodic truss structures. Journal of the Mechanics and Physics of Solids. 2016. 96. Pp. 184–203. DOI: 10.1016/j.jmps.2016.07.007.
20. Kirsanov, M.N. Planar trusses: Schemes and Formulas. Cambridge Scholars Publishing Lady Stephenson Library. Newcastle upon Tyne, GB, 2019.
21. Kirsanov, M. Trussed Frames and Arches: Schemes and Formulas. Cambridge Scholars Publishing Lady Stephenson Library. Newcastle upon Tyne, GB, 2020.

22. Ovsyannikova, V.M. Dependence of deformations of a trapezous truss beam on the number of panels. Structural Mechanics and Structures. 2020. 26(3). Pp. 13–20.
23. Ilyushin, A.S. The formula for calculating the deflection of a compound externally statically indeterminate frame. Structural mechanics and structures. 2019. 22(3). Pp. 29–38.
24. Rakhmatulina, A.R., Smirnova, A.A. Analytical calculation and analysis of planar springel truss. Structural mechanics and structures. 2018. 17(2). Pp. 72–79. URL: http://vuz.exponenta.ru/PDF/NAUKA/Rahm-Smirn2018-2.pdf (date of application: 27.02.2021)
25. Kreith, K. Oscillation Theory. 324. Springer Berlin Heidelberg. Berlin, Heidelberg, 1973.
26. Liu, X., Li, Y., Lin, Y., Banerjee, J.R. Spectral dynamic stiffness theory for free vibration analysis of plate structures stiffened by beams with arbitrary cross-sections. Thin-Walled Structures. 2021. 160. Pp. 107391. DOI: 10.1016/J.TWS.2020.107391
27. Buka-Vaivade, K., Kirsanov, M.N., Serdjuks, D.O. Calculation of deformations of a cantilever-frame planar truss model with an arbitrary number of panels. Vestnik MGSU. 2020. (4). Pp. 510–517. DOI: 10.22227/1997-0935.2020.4.510-517
28. Rapp, B.E. Introduction to Maple. Microfluidics: Modelling, Mechanics and Mathematics. Elsevier, 2017. Pp. 9–20.
29. Trainor, P.G.S., Shah, A.H., Popplewell, N. Estimating the fundamental natural frequency of towers by Dunkerley's's method. Journal of Sound and Vibration. 1986. 109(2). Pp. 285–292. DOI: 10.1016/S0022-460X(86)80009-8
30. Low, K.H. Modified Dunkerley's formula for eigenfrequencies of beams carrying concentrated masses. International Journal of Mechanical Sciences. 2000. 42(7). Pp. 1287–1305. DOI: 10.1016/S0020-7403(99)00049-1

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