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Model of pile-frozen soil interaction in a closed form

O. V. Tretiakova ✉

Perm National Research Polytechnic University, Perm, Russia

Perm State Agro-Technological University named after Academician D.N. Pryanishnikov, Perm, Russia

✉ *olga_wsw@mail.ru*

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Abstract. The object of the research is pile-soil interaction under freezing and frost heaving. Modeling of pile-soil interaction on the basis of joint solution of static equilibrium equations, physical equations for stresses and equation of thermal conductivity is considered. No such solutions have been found in existing publications. In this study, a model of pile-soil interaction in the form of a closed analytical solution with respect to the pile geometry was developed. Methods of continuum mechanics and elasticity theory were used. The model was a mathematical record of the equilibrium of forces acting on the pile. It was reduced to second-order algebraic equations with respect to its geometric parameters. The static equations of force equilibrium written in quadratic form related the geometric parameters of the pile to the stress-strain state and the thermal characteristics of the soil. Physical equations for heaving stresses and the thermal conductivity equation closed the problem. The model was developed as applied to a pile with an upper reverse taper. It reflected the performance of the pile during freezing and frost heaving and allowed determining its required geometric parameters under the given soil and climatic conditions.

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1. Introduction

The geometric parameters of foundations, especially of piles under freezing and soil frost heaving, are governed by the equilibrium condition of acting forces. They include the external load and the stress-strain components of soil. The external load is specified in each design case. The components of the stress-strain state of soil are the stresses and forces arising at the boundary of the soil-pile system, including frost heaving stresses and forces. Such a stress-strain state can be described by Hooke's law in the framework of elasticity theory. The stress-strain state is conditioned by thermal-physical characteristics of soil and the position of frost boundary.

It can be seen that to determine geometric parameters of piles in freezing and frost heaving conditions, it is necessary to jointly solve static equations of force equilibrium with physical equations in the form of Hooke's law and thermal conductivity equation.

The thermal conductivity equation makes it possible to determine the position of the frost boundary. Bonacina and Comini [1] gave an analytical solution to the problem of phase transitions in time in a range of temperatures. The thermal effect of the phase transition was approximated by the heat capacity of the soil. This method can be applied to determining the position of the frost boundary in time. The thermal conductivity equation, which takes into account the heat capacity, as a function of the temperature field in three dimensions, was studied by Dauzhenko and Gishkelyuk [2]. The method can be successfully applied to solve Stefan problem for the phase transition and to determine the position of the frost boundary. Vlasov and Volkov [3] considered the thermal state of a two-phase system and presented a mathematical model of the quasi-stationary temperature field of the system. The system was represented as an isotropic half-space with a moving phase boundary. Such a model reflects the state of freezing soil with moving frost boundary, but requires a special solution for this case. Alekseyev in his paper [4] analyzed the methods for numerical simulation of heat transfer in freezing soil based on Stefan problem solution. The thermal impact of structures located in the soil was taken into account. He presented the results of determining the position of the frost boundary in different software packages. However, the above studies did not relate the thermal-physical characteristics of soil and the position of the frost boundary to the stress-strain state of soil and the parameters of foundations.

The physical equations, in the form of Hooke's law, make it possible to calculate frost heaving stresses around foundations. Yin, X. and others [5] developed a system of equations describing the state of soil during freezing and frost heaving. Their system included equations for modeling the migration of water, steam, and heat transfer in freezing soil. They also gave the equations for stresses in soil in the framework of elasticity theory. The author of the current research believes that it would be promising to extend this system of equations to the case of a rigid inclusion in soil, i.e. a pile. The next important research was carried out by Korshunov, Doroshenko and Nevzorov [6]. They used a numerical model of frozen soil to obtain the coefficients of elastic and elastic-plastic compressibility of soil, and also the rate of change in the Young's modulus with temperature. However, they did not introduce a rigid inclusion in the form of a pile into the model. If they had done that, it would make it possible to obtain static equilibrium equations within the framework of elasticity theory, which would connect the law of motion of the frost boundary, frost heave stresses, and geometric parameters of the piles. Volokhov in his paper [7] predicted values of the tangential frost heaving forces on the side surface of the foundation in a range of temperatures. He also showed the dependence of these forces on the properties of the contact zone of the frozen soil and the foundation, i.e., the foundation material and the roughness of its surface. It would also be interesting to consider the geometric parameters of the foundation, as well as its configuration. Ladanyi and Foriero [8] proposed a closed solution for calculating the tangential frost heaving forces acting on a pile. They combined the effects of frost freezing and frost heaving rate, pile size and soil temperature at any depth along the pile in time. Domashchuk [9] calculated the tangential frost heaving forces on the basis of the rate and value of soil surface frost heaving, and also the temperature of the frozen soil. He also investigated frost heaving forces acting on vertical, horizontal, and inclined elements embedded in frozen soil. Kim and his colleagues [10] established the dependence of tangential frost stresses on the foundation material, soil type and temperature, and the position of the frost boundary. The data they obtained could be used to make static equations of force equilibrium with respect to geometric parameters of foundations under these conditions. Nazarov and Istomin [11] analyzed the stress-strain state of a pile foundation under the influence of the stress state and the thermal expansion coefficient of the material in a range of temperatures. Nazarov and Poselsky [12] assessed the stress-strain state of the pile foundation under the influence of the moisture content and the temperature of the soil for different materials of the pile. But they did not take into account the geometric parameters of the foundation and therefore did not make up the static equations of force equilibrium with respect to these parameters. Alekseyev [13] presented the cases of the stress-strain state of the freezing soil around foundations of different types and made up the equilibrium equations. These cases were described using generalized Hooke's law and the theory of thermoelasticity without considering geometric parameters of the foundation. Normal stresses of frost heaving were considered in [14] as a function of soil deformation and stiffness of structures experiencing frost heaving pressure. However, the stiffness of concrete foundations is relatively constant in many cases. Therefore, models of normal frost heaving stresses relative to the geometric parameters of foundations are of greater interest. Liu, Wang [15] obtained a model of the interaction of frozen soil with a single pile on the basis of experimental data. The model took into account the effects of the changing normal pressure of frozen soil, negative temperature and soil moisture over time. Unfortunately, we did not see the development of the model in the form of an equilibrium force equation with respect to the geometric parameters of the pile. Alekseev [16] conducted extensive studies of normal horizontal frost heaving forces. A more complete use of the results would give the dependence of these forces on the geometric parameters of the foundation and its configuration. It is known that normal frost heaving forces reach significant values. If the direction of normal heaving forces was changed, positive factors for the performance of the foundation could be obtained.

Such factors were found by Huang and Sheng [17]. They illustrated the effect of belled pile geometry on the stress-strain state of freezing soil. It was shown that around the pile with an extension in the lower part, the restraining forces were formed. Those forces counteracted the tangential frost heaving forces causing the pile to rise. This fact complied with the results obtained by the author [18-19] of the current research. However, Huang and Sheng did not show the dependence of forces acting on the pile on their geometry although the results of their research allowed them to obtain such a dependence and on its basis to make static equations of equilibrium forces with respect to the geometric parameters of the piles. Chae, Cho and others [20] used the numerical simulation results in order to find out the influence of the shape and size of the pile bell on the belled piles displacement behaviors under pullout load in thawed soil conditions. Dickin and Leung [21] reported on the effect of the angle of the enlarged pile base on its uplift capacity. However, their studies were not related to the frost heaving forces of the soil. Li and Xu [22] considered a set of force factors around a composite pile with carved grooves in seasonally frozen and permafrost soil layers. The force factors included frost heaving forces, forces from negative friction during soil thawing, which arise in the course of several autumn-winter seasons. Li and Xu showed the influence of the pile geometry on the permafrost layer and its importance for keeping the layer frozen. The obtained components of the stress-strain state of the soil, together with the results of thermophysical studies, could be used to make the static equilibrium equations of forces with respect to the pile geometric characteristics. Abbasov [23] developed recommendations for the use of flat profiled piles in freezing soil. The self-anchoring effect of the pile due to its geometry was shown in his work. However, static equations of force equilibrium with respect to the geometric parameters of the pile were not given.

Yushkov and Repetsky [24] determined the stresses and heaving forces acting on the double-tapered pile and the displacements of the pile. The anchoring effect of normal heaving forces acting on the top cone of the pile during its lifting by tangential frost heaving forces was noted. The equation for pile bearing capacity, based on the static equilibrium of acting forces, was derived. However, the solution of this equation with respect to the geometric parameters of the pile was not given. Oswell and Nixon [25] analyzed the problem of reducing the bearing capacity of foundations under changing thermal characteristics of permafrost. The adaptation of foundations to such conditions can be achieved through their geometry. Alekseev [26] studied the pile-frozen soil system. The system was represented as a cylindrical element, which was shifted under the effect of soil frost heaving. The problem of determining the dimensions and components of the stress-strain state of this element was solved. But the results were not written down in the form of static equilibrium equations of forces in the considered system with respect to geometry of the pile. Kurbatsky's method [27] provided a non-standard solution of the static problem with respect to these parameters. Linell and Lobacz in their report [28] presented technical recommendations on the design and construction of foundations in the areas of deep seasonal frost and permafrost. The report took into account the geological and climatic conditions of the construction site. It would be useful to combine these data into a general scheme of finding the geometric parameters of foundations. Ladanyi and Foriero [29] considered the calculation of a vertically and horizontally loaded pile in permafrost based on the balance between axial and bending stresses. They defined a general procedure for calculating these parameters with respect to the pile geometry. Their calculation could be used for ground frost heaving conditions when supplemented with appropriate heaving forces.

Thus, there has not been found a model of pile-soil interaction obtained by a simultaneous solution of three kinds of equations with respect to the pile geometry under frost heaving. These three equations are the static equilibrium equations, thermal conductivity ones and physical equations for stresses. Therefore, the purpose of the research is to develop such a generalized model of the freezing soil-pile system as a closed analytical solution with respect to the pile geometry.

To achieve the purpose, the three kinds of equations were considered, i.e. static equations of force equilibrium with respect to the pile geometry; physical equations in the form of Hooke's law for soil frost heaving stresses; thermal conductivity equation to determine the position of the frost boundary.

2. Methods

The model was built in the framework of continuum mechanics, i.e. the static equilibrium equations of forces, the thermal conductivity equation and physical equations in the form of Hooke's law were used, which provided closed form of the model.

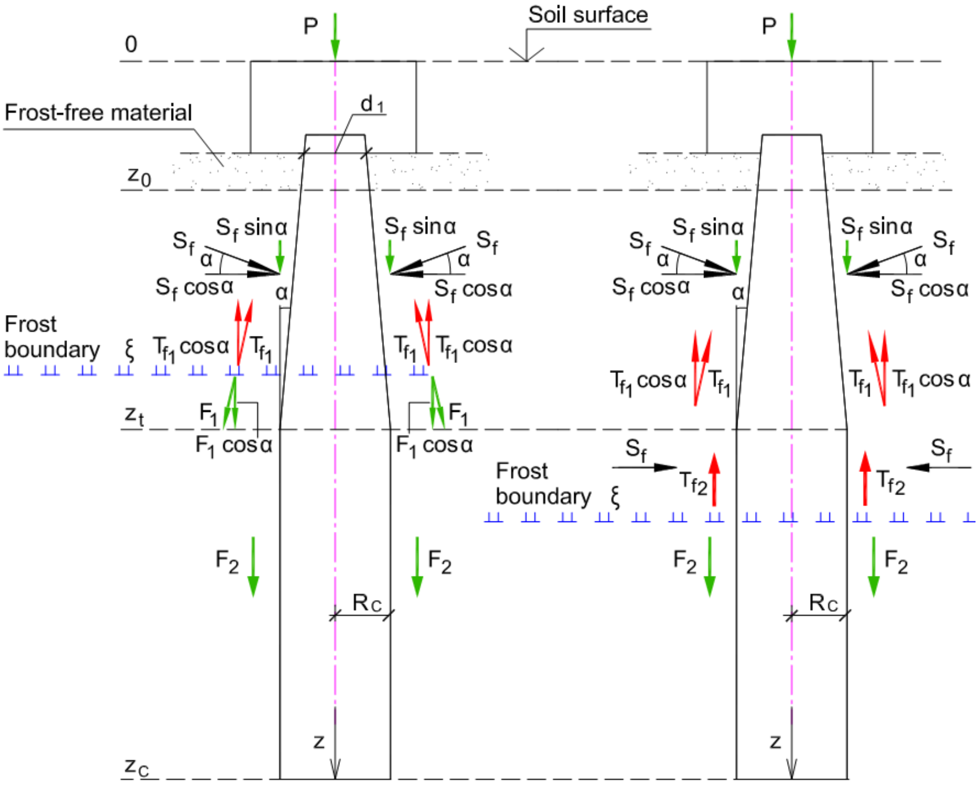
The static equilibrium equations of force at the soil-pile boundary related the foundation geometry with the stress-strain state and the thermal characteristics of the frozen soil. The normal frost heaving stresses were derived from the physical equations in the form of Hooke's law. The thermal conductivity equation allowed to obtain the frost boundary position.

The model was a generalized way of obtaining pile geometry. It was a mathematical expression of equilibrium condition of a pile. This model was reduced to second-order algebraic equations with respect to the geometry of the reverse taper pile. The model was considered by the author in [18], [30], [31].

2.1. Static equations with respect to the pile geometry

Static equilibrium equations with respect to the pile geometry were given for two design schemes: the frost boundary position (1) within the pile variable cross-section and (2) within the pile constant cross-section, as shown in Fig. 1.

In Figure 1 σ_f, τ_{fi}, f_i are the frost heaving normal and tangential stresses, the design value of soil resistance along the pile in the thawed soil, respectively; P- the sum of the external load and the pile's own weight; α is angle of the pile taper surface.



(a) the frost boundary position within the pile variable cross-section

(b) the frost boundary position within the pile constant cross-section

Figure 1. Design schemes of the pile

A pile with an upper reverse taper was considered. Static equilibrium equations of pile forces, acting on the pile under freezing and frost heaving of the soil were as follows (Fig. 1, a, b):

if $z_0 < \xi < z_t$ (Figure 1, a): $-P - S_f \sin\alpha + (T_{f1} - F_1)\cos\alpha - F_2 = 0$, (1)

if $z_0 < z_t < \xi$ (Figure 1, b): $-P - S_f \sin\alpha + T_{f1}\cos\alpha + T_{f2} - F_2 = 0$. (2)

Integral forces on lateral area of the pile (see Figure 1):

if $\xi < z_\kappa$:

$$S_f = \int_0^{\xi} \sigma_f 2\pi R_t(z) dz,$$

$$T_{f_1} = \int_0^{\xi} \tau_{f1} 2\pi R_t(z) dz,$$

$$F_1 = \int_{\xi}^{z_t} f_1 2\pi R_t(z) dz,$$

$$F_2 = \int_{z_t}^{z_c} f_2 2\pi R_c dz.$$

if $\xi > z_t$:

$$S_f = \int_0^{z_t} \sigma_f 2\pi R_t(z) dz,$$

$$T_{f_1} = \int_0^{z_t} \tau_{f1} 2\pi R_t(z) dz,$$

$$T_{f_2} = \int_{z_t}^{\xi} \tau_{f2} 2\pi R_c dz,$$

$$F_2 = \int_{\xi}^{z_c} f_2 2\pi R_c dz.$$

The area of a unit annular strip along the taper perimeter (dF_t):

$$dF_t = 2\pi R_t(z) dz. \tag{3}$$

Taking into account the variable radius of the taper, expression (3) will be written:

$$dFt = 2\pi[R_c - \sin\alpha(z_t - z)]dz. \tag{4}$$

The area of a unit annular strip along the perimeter of cylindrical part of the pile is as follows:

$$dF_c = 2\pi R_c dz. \tag{5}$$

Taking into account the above expressions for the integral forces and (3), (4), (5), if cos α ≈ 1, equations (1), (2) take the form:

if $z_0 < \xi < z_t$ (Figure 1, a):

$$-P - \left(\int_{z_0}^{\xi} \sigma_f dF_t\right)\sin\alpha + \int_{z_0}^{\xi} \tau_{f1} dF_t - \int_{\xi}^{z_t} f_1 dF_t - \int_{z_t}^{z_c} f_2 dF_c = 0, \tag{6}$$

if $z_0 < z_t < \xi$ (Figure 1,б):

$$-P - \left(\int_{z_0}^{z_t} \sigma_f dF_t\right)\sin\alpha + \int_{z_0}^{z_t} \tau_{f_1} dF_t + \int_{z_t}^{\xi} \tau_{f2} dF_c - \int_{\xi}^{z_c} f_2 dF_c = 0. \tag{7}$$

Based on expressions (6), (7) and taking into account the area of the strip of the taper (4), the static equilibrium equations of forces are written as the second-order algebraic equations with respect to the angle of the pile taper.

If frost boundary position is $z_0 < \xi < z_t$:

$$\begin{gathered}\sigma_f (z_t\xi - z_t z_0 - 0.5\xi^2 + 0.5z_0^2)(\sin\alpha)^2 + \\ +[-R_c\sigma_f(\xi - z_0) - \tau_{f_1}(z_t\xi - z_t z_0 - 0.5\xi^2 + 0.5z_0^2) + 0.5 f_1 (z_t - \xi)^2](\sin\alpha) + \\ +R_c[\tau_{f1}(\xi - z_0) - f_1(z_t - \xi) - f_2(z_c - z_t)] - 0.5\pi^{-1}P = 0.\end{gathered} \tag{8}$$

If frost boundary position is $z_0 < z_t < \xi$:

$$\begin{gathered}0.5\sigma_f (z_t - z_0)^2(\sin\alpha)^2 + \\ +[-R_c\sigma_f(z_t - z_0) - 0.5\tau_{f1}(z_t - z_0)^2](\sin\alpha) + \\ +R_c[\tau_{f1}(z_t - z_0) + \tau_{f2}(\xi - z_t) - f_2(z_c - \xi)] - 0.5\pi^{-1}P = 0.\end{gathered} \tag{9}$$

The solution of equations (8) and (9) with respect to the value of sin α, allows to obtain the pile geometric parameters, that is the angle of the pile taper α as shown in Figure 1.

The equations take into account the stress-strain state of the soil and the thermal characteristics of the frozen soil.

2.2. Physical equation in the form of Hooke's law for normal frost heaving stresses

In this model, the static equations of equilibrium of acting forces were supplemented by physical equations in the form of Hooke's law for normal frost heaving stress.

Based on the linear elastic deformability of soils, the author used the theory of elasticity to find the stresses of frost heaving. The equation for normal heaving stresses was written in the form of Hooke's law and took into account the thermal characteristics of the soil. The Young's modulus of elasticity in the equation was replaced by the deformation modulus.

Normal heaving stresses were found in the open system. In paper [32] the author showed that frost heaving occurs when the total volume of frozen and unfrozen water exceeds the volume of pores in the soil. This causes an increase in the soil volume and the development of normal frost heaving stress σ_f. The normal frost heaving stress is developed at the contact of the expanding soil with the foundation surfaces. Thus frost heaving stress is, on the one hand, the function of the soil porosity; on the other hand, it is caused by "excess moisture" resulting in the formation of "excess ice", that exceeds the pore volume.

The expression for normal frost heaving stress at z depth is as follows [32]:

$$\sigma_f = E_f \frac{h_{heave}}{z} \cdot \left[1 - e\left(1 - w_w \frac{\rho_d}{\rho_w} - 1.09w\frac{\rho_d}{\rho_w}\right)\right] k_{ан}, \tag{10}$$

where E_f is the deformation modulus of frozen soil (кN/cm^2), z is a coordinate from the day surface (cm); w is the natural moisture of soil and w_w is the water content due to unfrozen water, e is a porosity ratio; ρ_d is the dry density; ρ_w is the free water density: ρ_d / ρ_w is the coefficient of mass moisture to volumetric moisture; k_{an} is the anisotropy factor taking into account the direction of the heaving forces; h_{heave} is the displacement of soil under frost heaving resulting from water sucking up. According to Konrad's formula [33], [34], the displacement of soil under frost heaving resulting from water sucking up, h_{heave}, is as follows

$$h_{heave} = 1.09 \cdot SP \cdot \tau \cdot grad\ t\ , \tag{11}$$

where SP is the segregation potential of soil, t is the frost heaving time and $grad\ t$ is the temperature gradient.

After substituting equation (11) into equation (10), the formula of normal frost heaving stress as the function of moisture, leading to "excess ice", was obtained.

$$\sigma_f = E_f \frac{1.09 \cdot SP \cdot \theta \cdot grad\ t}{z} \cdot \left[1 - e\left(1 - w_w \frac{\rho_d}{\rho_w} - 1{,}09 w \frac{\rho_d}{\rho_w}\right)\right] k_{ан}, \tag{12}$$

The formula of the tangential frost heaving stresses was given in [18].

$$\begin{aligned} \tau_f &= \left[c_{soil} - 1.09(w + (SP \cdot \theta \cdot grad\ t \cdot k_{an}/z)(c_{soil} - c_{ice})\right] + \\ &+ \eta_{soil}\gamma_{soil} z (tg\varphi_{soil}) \left(1 - 1.09(w + (SP \cdot \theta \cdot grad\ t \cdot k_{an}/z)\right) + \\ &+ \eta_{ice}\gamma_{ice} z (tg\varphi_{ice}) 1.09(w + (SP \cdot \theta \cdot grad\ t \cdot k_{an}/z)), \end{aligned} \tag{13}$$

where c_{soil} is the soil specific cohesion; c_{ice} is the ice shear resistance; η_{soil} and η_{ice} are the coefficients of the lateral pressure of soil and ice; γ_{soil} and γ_{ice} are the volume density of soil and ice; φ_{soil} and φ_{ice} are the angles of internal friction of soil and ice, respectively.

The tangential frost heaving stress can also be found on the base research by Labuz and Zang [35].

2.3. Thermal conductivity Equation and determining the position of the frost boundary

The frost boundary position was determined on the basis of the thermal conductivity equation. It was assumed that the soil within the pile length was homogeneous; the deformation modulus of frozen soil within the pile length was constant; the temperature along the depth of the freezing layer changed linearly. It was also assumed that the temperature did not change along the *x*- and *y*- coordinates, but did so along the *z*-coordinate.

Phase transitions of water into ice were taken into account in theoretical modeling of heat and mass transfer processes in the soil and the Stefan problem was used. As a freezing soil region has a movable phase boundary the freezing boundary movement condition was written as:

$$q_f - q_{thawed} = Q_{phase}\xi^{/} \tag{14}$$

After substituting expressions for heat fluxes ($q = \lambda\ grad\ t$), the equation (14) took the form of the law of motion of the freezing boundary:

$$\lambda_f \frac{\partial t_f(z,\theta)}{\partial z} - \lambda_{thawed} \frac{\partial t_{thawed}(z,\theta)}{\partial z} = Q_{phase} \frac{\partial \xi(\theta)}{\partial \theta}\ , \tag{15}$$

where ξ is the frost boundary position; θ is freezing period; t_f, t_{thawed} are temperature of frozen and thawed zones; λ_f, λ_{thawed} are coefficients of thermal conductivity of frozen and thawed soil; Q_f is specific heat of phase transitions (heat of freezing); q_f, q_{thawed} are heat fluxes in thawed and frozen zones:

$q_f = \lambda_f\ grad\ t_f$, $q_{thawed} = \lambda_{thawed}\ grad\ t_{thawed}$.

Provided that at the transit point temperature t is zero, (z, θ) = const = 0, and, the temperature along the depth of the freezing layer changes linearly, the Stefan condition (15) was written in the form of a differential equation:

$$Q_{phase}\frac{d\xi}{d\theta}=\frac{-\lambda_f t_{surf}}{\xi}, \qquad (16)$$

The solution of the equation was a function that allowed to find the frost boundary position i.e. the depth of freezing at constant temperature on the soil surface:

$$\xi(\theta)=\sqrt{\frac{2\lambda_f\left|t_{surf}\right|\theta}{Q_{phase}}}, \qquad (17)$$

where $|t_{surf.}|$ is the absolute value of negative soil surface temperature.

An expression for the case of variable surface temperature was obtained by averaging it over some intervals (decades or months). A graph of the variation of the soil temperature along the depth of the layer is shown in Figure 2. In the Figure 2: t_f is the soil temperature at the design level along freezing layer; t_{thawed} is the temperature of thawed soil; θ is freezing period.

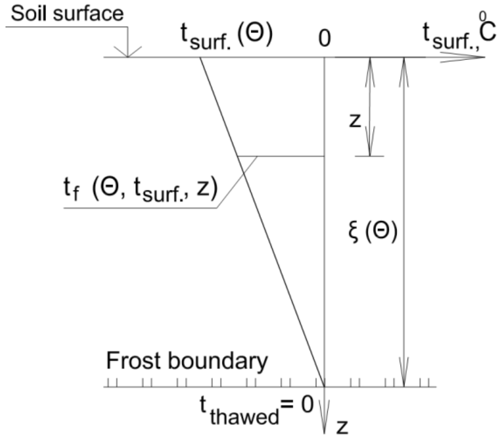


Figure 2 - Graph of the soil temperature variation by the depth of the freezing layer.

From the similarity of the triangles in Figure 2 it followed:

$$\frac{t_f(\theta,z)}{t_{surf}(\theta)}=\frac{\xi(\theta)-z}{\xi(\theta)}=1-\frac{z}{\xi(\theta)}, \qquad (18)$$

Then the soil temperature in the freezing layer was determined from the expression:

$$t_f(\theta,z)=t_{surf}(\theta)\left[1-\frac{z}{\xi(\theta)}\right]. \qquad (19)$$

The derivative of this function on the coordinate was then as follows:

$$\frac{\partial t_f(\theta,z)}{\partial z}=-\frac{t_{surf}(\theta)}{\xi(\theta)}. \qquad (20)$$

Just as in the case of constant surface temperature, it was assumed that Stefan's condition was valid for the movable phase boundary during soil freezing (15).

Since at the phase boundary t_f < 0, and t_{thawed} = 0, we wrote down the expression for heat flux in thawed zone as follows: $\lambda_{thawed}\ (\partial t_{thawed}(z,\theta)/\partial z) = 0$

Then Stefan's condition (15) took the form:

$$\lambda_f\frac{\partial t_f(\theta,z)}{\partial z}=Q_{phase}\frac{\partial\xi(\theta)}{\partial\theta}. \qquad (21)$$

Then Stefan's condition (15) became:

$$\lambda_f\left(-\frac{t_{surf}(\theta)}{\xi(\theta)}\right)=Q_{phase}\frac{\partial\xi(\theta)}{\partial\theta} \;\Rightarrow\; \frac{\partial\xi(\theta)}{\partial\theta}=-\frac{\lambda_f}{Q_{phase}}\frac{t_{surf}(\theta)}{\xi(\theta)}. \tag{22}$$

After separating the variables, it became:

$$\xi d\xi=-\frac{\lambda_f}{Q_{phase}}t_{surf}(\theta)d\theta. \tag{23}$$

Then it was assumed that the law of temperature change in time was known (see Figure 2) and in the time interval from 0 to θ the surface temperature was constant ($t_{surf\ (i+1)}$ = const = θ). Integrating expression (23) in the time interval from 0 to θ and in coordinates from 0 to ξ resulted in:

$$\int_0^{\xi}\xi d\xi=-\frac{\lambda_f}{Q_{phase}}\int_0^{\theta}t_{surf}d\theta=-\frac{\lambda_f t_{surf}}{Q_{phase}}\int_{\theta}^{\theta}d\theta \;\Rightarrow\; \frac{\xi^2}{2}=-\frac{\lambda_f t_{surf}}{Q_{phase}}\theta \tag{24}$$

Then the equation for determining the frost boundary position at a constant step in time from θ_1 to θ_{i+1} with the corresponding temperature interval took the form:

$$\xi=\left[-\frac{2\lambda_f t_{surf}}{Q_{phase}}\theta\right]^{0.5}. \tag{25}$$

Assuming that within a step from θ_1 to θ_{i+1} the surface temperature was constant and equal to the average value ($t_{surf\,(i+1)}$ = const), expression (25) was integrated

$$\int_{\xi_i}^{\xi_{i+1}}\xi d\xi=-\frac{\lambda_f}{Q_{phase}}\int_{\theta_i}^{\theta_{i+1}}t_{surf}(\theta)d\theta, \tag{26}$$

where i is step numbers, i = 1,2,3 ..., into which the considered time period was divided.

Replacing $t_{surf}(\theta)$ with a step function with values of t_{surf} averaged over months, it was obtained:

$$\int_{\xi_i}^{\xi_{i+1}}\xi d\xi=-\frac{\lambda_f}{Q_{phase}}\int_{\theta_i}^{\theta_{i+1}}t_{surf\ (i+1)}d\theta=-\frac{\lambda_f t_{surf\ (i+1)}}{Q_{phase}}\int_{\theta_i}^{\theta_{i+1}}d\theta, \tag{27}$$

Equation (27) was transformed:

$$\frac{\xi_{i+1}^2-\xi_i^2}{2}=-\frac{\lambda_f t_{surf\ (i+1)}}{Q_{phase}}(\theta_{i+1}-\theta_i), \tag{28}$$

Hence, the equation for step-by-step determination of the frost boundary position at each new step in time θ_{i+1} became as follows:

$$\xi_{i+1}=\left[\xi_i^2-\frac{2\lambda_f t_{surf\ (i+1)}}{Q_{phase}}(\theta_{i+1}-\theta_i)\right]^{0.5}, \tag{29}$$

θ_i is the value of the frost boundary position at the previous step.

The position of the frost boundary under buried structures could also be determined by two parameters: the internal temperature in the structure and the radius of thermal influence of the structure on the soil. This radius was considered to be the distance from the structure at which the natural temperature of the soil was preserved. The position of the frost boundary was determined from the condition of linear temperature distribution at that distance. The internal temperature in the tunnel was found from the heat balance equation. The radius of thermal influence of the structure on the soil was obtained from the equation:

$$R = \sqrt{\frac{2\lambda_{red}(t_{int} - t_z)}{Q}}\ , \qquad (30)$$

where Q is the average annual vertical heat flux in the soil around the structure; λ_{red} is the reduced coefficient of thermal conductivity; "t_{int}" is the internal temperature in the structure; t_z is the temperature of free soil at the considered depth.

The numerical complex developed by Ulitsky, Kudryavtsev, Paramonov, and Sakharov [36], [37] could also be used to determine the position of the frost boundary. Models of the thermal conductivity of frozen soils to find the position of the frost boundary were given in [38].

3. Results and Discussion

The model developed by the author had the following assumptions:

- The frost boundary, which was horizontal, was movable. As it advanced, the thawed soil transitioned to the frozen state.

- In frozen and thawed soil, elastic physical relationships were true.

- The temperature at the soil surface was variable.

- The delay in soil cooling and freezing described in the second Fourier law was not taken into account in this paper, and the temperature of frozen soil at a certain depth was considered at the same time as that at the surface.

The limitations of the model for the case of a reverse taper pile were as follows:

- A vertical friction pile was considered.

-The length of the cylindrical part of the pile was assumed to be not less than the length that provided the bearing capacity of the pile in thawed soil.

-The length and angle of the taper were limited by the diameter of the pile at the place of its embedding in the grillage (d_1), as shown in the figure 1.

The model was a system of equations:

Static equilibrium equations of forces if $z_0 < \xi < z_t$:

$$-P - \left(\int_{z_0}^{\xi} \sigma_f\, dF_t\right)\sin\alpha + \int_{z_0}^{\xi} \tau_{f1}\, dF_t - \int_{\xi}^{z_t} f_1\, dF_t - \int_{z_t}^{z_c} f_2\, dF_c = 0\,.$$

Static equilibrium equations of forces if $z_0 < z_t < \xi$:

$$-P - \left(\int_{z_0}^{z_t} \sigma_f\, dF_t\right)\sin\alpha + \int_{z_0}^{z_t} \tau_{f_1}\, dF_t + \int_{z_t}^{\xi} \tau_{f2}\, dF_c - \int_{\xi}^{z_c} f_2\, dF_c = 0.$$

The area of a unit annular strip along the taper perimeter

$$dF_t = 2\pi[R_c - \sin\alpha(z_t - z)]dz\,,$$

The area of a unit annular strip along the perimeter of cylindrical part of the pile

$$dF_c = 2\pi R_c dz,$$

Static equilibrium equations of forces with respect to the angle of the pile taper if frost boundary position $z_0 < \xi < z_t$:

$$\begin{aligned}&\sigma_f(z_t\xi - z_t z_0 - 0.5\xi^2 + 0.5z_0^2)(\sin\alpha)^2 + \\ &+[-R_c\sigma_f(\xi - z_0) - \tau_{f_1}(z_t\xi - z_t z_0 - 0.5\xi^2 + 0.5z_0^2) + \\ &+0.5 f_1(z_t - \xi)^2](\sin\alpha) + R_c[\tau_{f1}(\xi - z_0) - \\ &- f_1(z_t - \xi) - f_2(z_c - z_t)] - 0.5\pi^{-1}P = 0\,,\end{aligned}$$

if frost boundary position $z_0 < z_t < \xi$:

$$0.5\sigma_f(z_t - z_0)^2(\sin\alpha)^2 +$$
$$+[-R_c\sigma_f(z_t - z_0) - 0.5\tau_{f1}(z_t - z_0)^2](\sin\alpha) +$$
$$+R_c[\tau_{f1}(z_t - z_0) + \tau_{f2}(\xi - z_t) - f_2(z_c - \xi)] - 0.5\pi^{-1}P = 0.$$

Frost heave normal stresses:

$$\sigma_f = E_f\frac{1.09 \cdot SP \cdot \theta \cdot grad\ t}{z} \cdot \left[1 - e\left(1 - w_w\frac{\rho_d}{\rho_w} - 1.09w\frac{\rho_d}{\rho_w}\right)\right]k_{an}.$$

Frost heave tangential stresses:

$$\tau_f = [c_{soil} - 1.09(w + (SP \cdot \theta \cdot grad\ t \cdot k_{an}/z)(c_{soil} - c_{ice})] +$$
$$+\eta_{soil}\gamma_{soil}z(tg\varphi_{soil})\left(1 - 1.09(w + (SP \cdot \theta \cdot grad\ t \cdot k_{an}/z)\right) +$$
$$+\eta_{ice}\gamma_{ice}z(tg\varphi_{ice})1.09(w + (SP \cdot \theta \cdot grad\ t \cdot k_{an}/z)),$$

Frost boundary position:

$$\xi_{i+1} = \left[\xi_i^2 - \frac{2\lambda_f t_{surf\ (i+1)}}{Q_{phase}}(\theta_{i+1} - \theta_i)\right]^{0.5}$$

The frost boundary position under embedded structures can also be found by two parameters: the internal temperature in the structure and the radius of thermal influence of the structure on the soil.

Design schemes for the model are shown in Figure 1.

The interpretation of the resulting model was done as in the example. A shallow tunnel partly located in seasonally freezing layer was considered. Based on the data of standard engineering-geological surveys and loads from the tunnel, geometric parameters of piles with upper reverse taper under the tunnel columns were determined. The position of frost boundary was found from the tunnel floor level. Length of piles was 3.2 m., diameter of cylindrical part of pile was 0.5 m. Tangential frost heaving stresses were taken according to geotechnical survey data. Normal horizontal frost heaving stresses were calculated at the level of the center line of the taper. The angles of the pile tapers were calculated from the static equations of equilibrium of acting forces. The static equations were solved taking into account the variability of the pile cross section and were written in the form of second-order algebraic equations with respect to the sine of the pile taper angle. The results of the calculations are shown in Table 1.

Table 1. Calculation results for the author's model.

Pile model	Frost boundary position, m	Frost heaving normal stress, кN/m^2	Pile geometric characteristics		Pile volume, m^3
			Taper angle, degrees	Taper length, m	
№1	2.1	57	7.5	1.1	0.543
№2	2.1	52	5.5	1.28	0.528
№3	2.1	48	4.2	1.46	0.516
№4	2.1	45	3.4	1.6	0.519

Thus, the example above showed that the author's model allowed to determine the required geometric parameters of piles from the condition of equilibrium of acting forces. This ensured the absence of vertical displacements of piles and the stability of the overlying structures in the given climatic and soil conditions.

The equations developed in the previous works by Repetsky, Yushkov [24], and other researchers made it possible to check the specified geometric parameters of the piles in the conditions under consideration. If the parameters did not satisfy, it would be necessary to set new parameters of the piles. The criterion was the vertical displacement of the piles, which required comparison with the limit values. Note that such values were also found from the joint calculation of the pile and the overlying structure. That increased the labor intensity of the solution process. In the author's model, the required parameters of the pile were determined in one step – when solving equilibrium equations.

The dimensions and working conditions of the piles in the author's model corresponded to the experimental models of the piles in Repetsky's thesis research. The results of the author's calculation coincided with the experimental data with deviations of 3.2–17.66%, as shown in Table 2. The upward deviations of calculated pile taper angles from the experimental data from 3.2 to 17.7 % proved Repetsky's resulting data. The downward deviations of calculated angles from 8.2 to 13.6 % characterized the equilibrium of the experimental piles in the soil, which also coincided with the data in his research.

Table 2. Comparison of the author's analytical model with experimental data.

Pile model	Taper angle, degrees		Deviations of calculated angles from experimental, %	Vertical displacement of experimental pile, mm,
	Experiment	Calculation		
№ 1	6.0	7.5	+17.66	1.2
№ 2	5.0	5.5	+3.21	0.85
№ 3	4.5	4.2	–8.15	0.55
№ 4	4.0	3.4	–13.6	0.4

The author's model was implemented in an automated calculation module [39]. The module made it possible to reduce the labor intensity of calculations; provided a systematic approach to solving the problem, taking into account the multifactor calculations, and gave opportunities to add new modules in the development of calculation capabilities.

4. Conclusions

Thus, the author proposed a model of the freezing soil-pile system as a closed analytical solution with respect to geometric parameters of the pile.

The model included the solution of static equilibrium equations of forces with respect to geometric parameters of the pile; physical equations in the form of Hooke's law for soil frost heaving stresses; thermal conductivity equations for determining the position of frost boundary.

The obtained solutions made it possible to get geometric parameters of the pile required for pile equilibrium in the soil. That means that its vertical displacements during freezing and frost heaving were eliminated which ensured the integrity of the overlying structures.

Besides, the author's model made it possible to get a number of variants for geometric parameters of the pile, satisfying the condition of equilibrium. Among them, solutions with minimum material capacity were selected as the most economically efficient ones..

Further research directions could be the adaptation of the model to piles of other configurations, application of the model to bridge structure supports in cold regions, and development of an automated calculation module.

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Contacts:

***Olga Tretiakova**, PhD in Technical Science*
E-mail: olga_wsw@mail.ru

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