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Modelling of thin-walled members with restrained torsion considering the section properties

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Abstract. Various engineering structures include lightweight or thin-walled beam structures that are used in a complex loading situation which includes restrained torsion of closed or open section. The importance of restrained torsion of thin-walled cross-sections is significant as the deformations and stresses caused by torsion affect the behaviour of the structures with open as well as closed section. The aim of this study is to demonstrate and compare different methods used to develop stiffness matrix for the finite element beam calculation of open and closed thin-walled sections with restrained torsion. The beam stiffness matrices are presented and graphically compared in order to choose the most convenient method for advanced structural analysis of thin-walled 3D beams with restrained torsion. The interpolation functions containing hyperbolic and approximate functions are considered, which satisfy the governing differential equation for torsion, with different value of characteristic number of torsion (ka). Comparing both methods, we can conclude that both are similar for small value of ka and this is commonly considered for open thin-walled section as their value of ka is small. The percentage of error between the results obtained by two methods of element stiffness matrix development for torsion with restrained warping is given graphically. Based on this study, numerical examples are considered and compared with results obtained by different finite element software. The examples include restrained and free torsion which are nonuniform and uniform torsion, respectively.

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1. *Introduction*

Steel members are widely used in civil engineering because of its high strength, excellent ductility, fast construction, and effective space partition. Normally steel members are manufactured as thin-walled structures. The behaviour of a thin-walled bar in torsion depends strongly on the topology of its section. If the network of the section does not contain any loop, the section is called open whereas if the network of the section contains at least one loop, the section is called closed. This geometric property is used to develop special methods for the computation of their torsion and warping constants. It will be shown that the torsion properties of open thin-walled sections differ significantly from those of closed thin-walled sections. This difference has an effect for the structural response of the beam, most importantly when it comes to shear and torsion behaviour. The importance of restrained torsion of thin-walled cross-sections is significant as the deformations and stresses caused by torsion affect the behaviour of the whole structure. The present study deals with beam member subjected to torsion and uses different finite element approach.

The governing equations are solved to calculate displacements and internal forces and moments for the structure.

It is well known that the effect of non-uniform torsion must be considered in structural analysis of thin-walled beams with open cross-sections. It was unusual to check the influence of torsion on load carrying structural elements, but today it is quite often that the stresses and deformations caused by torsion will determine the studs of the structures. The importance of restrained torsion of thin-walled cross-sections has grown significantly as the deformations and stresses caused by torsion will affect the behaviour of the structures with open as well as closed section [1]. It is well known that the effect of non-uniform torsion must be considered in structural analysis of thin-walled beams with open cross-sections. It was unusual to check the influence of torsion on load carrying structural elements, but today it is quite often that the stresses and deformations caused by torsion will determine the studs of the structures. Thin-walled sections do not behave according to the law of the plane sections employed by Euler-Bernoulli-Navier however, the general theory of thin-walled section is developed by Vlasov and In addition, thin-walled structures like plates and shells associated with finite element formulations are the most common construction elements in nature and technology [2–4]. When a thin-walled beam is undergoing flexure and torsion simultaneously, transverse and torsional shear deformations would be coupled [5].

If the structures are designed using only the effect of Saint Venant torsion resistance thus the analysis may ignore the torsion part in the members and the design may be underestimated. To overcome this inaccuracy, several researchers tried to develop stiffness matrix with four degrees of freedom for a member subjected to a restrained torsion [6–11]. This additional stiffness matrix considers the warping degree of freedom at the ends of the member with thin-walled section. This study deals with the member finite element method subjected with torsion and it is done by considering beam element and equation which are necessary for the computing deformations will be derived thus to calculate the displacements and internal forces and moments for the structures.

A Number of researchers have dealt with restrained torsion of thin-walled beams with open and closed cross sectional types [12–17]. Different thin-walled torsion hypotheses have been developed to account for the shear deformation due to restrained torsion of open thin-walled section with the assumption in which the derivative of shear stress in middle surface is constant along the length of the member as they much used in engineering structures [18–23]. A finite element model is studied based on mixed variational formulation in order to improve convergence and provide an explicit way to calculate internal forces and stresses in thin- walled bar [24]. Also, different studies are developed a numerical method of thin-walled bar systems design by different theories and formulated matrixes of the stiffness of thin-walled finite elements [25–27]. The bending and torsion behaviour of cold-formed steel bars was studied experimentally based on the strengths of unbraced cold-formed steel channel beams loaded eccentrically [28–29]. Many design methods have been developed to deal with or without restrained torsions. There are different commercial softwares commonly consider with two degrees of freedom at each node of a member for a space frame without considering the effect of warping restraint at the ends of the members [30–33]. The warping part of the first derivative of the twist angle has been considered as the additional degree of freedom in each node at the element ends which can be regarded as part of the twist angle curvature caused by the warping moment [34–35]. Numerous studies developed the 4×4 member stiffness matrix including warping as an additional degree of freedom and commonly with open thin-walled section [11, 36, 37].

In this contribution, which is an extension of previous studies on thin-walled structure different 3D finite elements for open and closed sections are compared based on their section properties. Currently, many residential, social and sports buildings are being built using a steel frame, the application field of steel frame is expanding. Results of this research are applicable for software implementation, that is an advantage in respect to analogous jobs. There is lack of papers devoted to numerical solutions of the problem, that proves insufficient development of the topic and so that the investigation is relevant.

So, the first objective of this study is to consider the stiffness matrices including additional degrees of freedoms at the nodes and added to member displacement vector, apply these matrices for analysis and assess them by considering several examples and validate by comparing different finite element software.

One more objective is to analyze the effect of changing interpolation function, the same test example numerical experiments are conducted. First interpolation function type considered is hyperbolic, it contains hyperbolic functions of deflection. Such function satisfies the governing differential equation for torsion exactly and use above derivatives of stiffness matrix. The alternative approach is considered too, which is using approximate shape function. The second approach has an advantage for varying cross sections and non-linear problems and compared results with different studies [20, 38].

The last objective of the study is conducting of verification examples by commercial software. The examples of open and closed section members under uniformed and non-uniformed torsion are considered and compared to results obtained by the new approaches introduced in this paper. Comparing both methods, we can conclude that both are similar for small value of ka and this is commonly considered for open thin-walled section as their value of ka is small. The percentage of errors between the two methods of element stiffness matrix for torsion with restrain torsion are given graphically.

2. Methods

2.1. Governing equations for 3D thin-walled beams with restrained torsion

The differential equation of restrained torsion of thin-walled beams can be obtained by the following expression and the total free warping torsion and total restrained torsion are corresponding to uniform warping rotation and nonuniform or restrained rotation, respectively. The governing equation for non-uniform torsion is used to study the behaviour of a bar with restrained torsion and it is derived for bars with thin-walled sections with local coordinate systems y_1, y_2, y_3 as shown in Fig. 1.

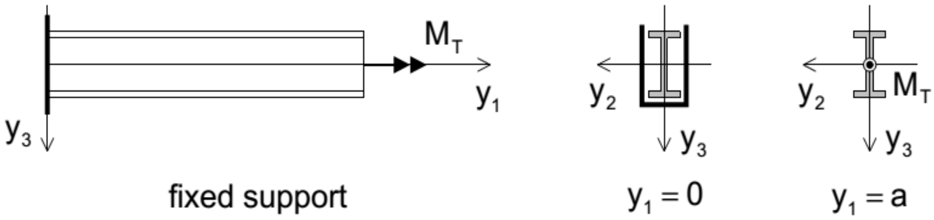


Figure 1. Prismatic bar subjected to torsion

$$EJ_{\omega}\frac{d^4\theta_1}{dy_1^4}-GJ_t\frac{d^2\theta_1}{dy_1^2}=0,$$
$$\frac{d^4\theta_1}{dy_1^4}-\left(k\right)^2\frac{d^2\theta_1}{dy_1^2}=0, \quad (1)$$
$$ka:=a\sqrt{\frac{GJ_t}{EJ_{\omega}}},$$

when θ_1 is angle of twist; E,G are elastic constant; J_{ω} is warping constant; ka is characteristic number for torsion; J_t is torsion constant; m_T is twisting load per unit length of bar; a is length of the bar.

The general solution for the homogeneous equation (1) is satisfied by the following assumed twisting angle function $\theta_1\left(y_1\right)$ and it yields to the exact solutions of the angle of twists of a node as it is defined in the following expression:

$$\theta_1=c_1 sinhky_1+c_2 coshky_1+c_3 ky_1+c_4. \quad (2)$$

The governing equations for a member and frame are derived by applying the principle of virtual work to the beam. The differential governing equations for the generalized member displacements are satisfied for arbitrary virtual displacements and expressed as follows:

$$EJ_{\omega}\theta_{1,1111}-GJ_t\theta_{1,11}-m_1-m_{\omega,1}=0. \quad (3)$$

The sum over the members of the virtual work δW_m of the inner forces in (1) equals the sum over the members of the virtual work δW_{md} of the member loads.

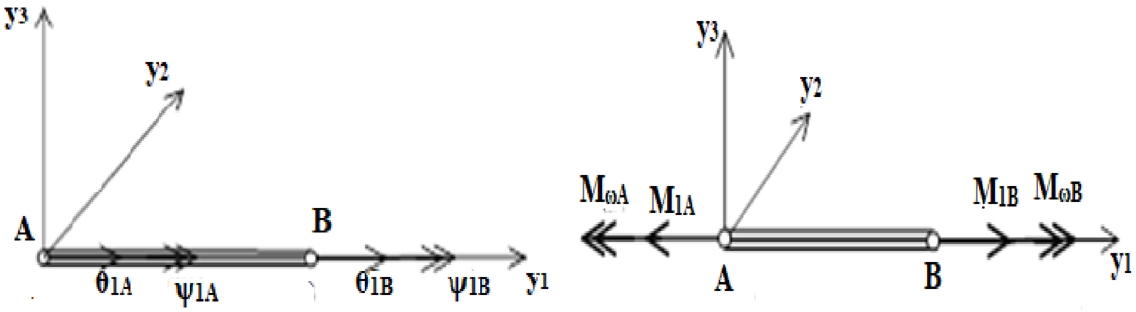
$$\sum_{m=1}^{M}\delta W_m=\sum_{m=1}^{M}\delta W_{md}+\delta W_n\int_0^a\left(EI_{\omega}\delta\psi_{,1}\psi_{,1}+GI_t\delta\theta_{1,1}\theta_{1,1}\right)dy_1. \quad (4)$$

The nodal displacement and corresponding nodal moments vectors can be expressed as follow:

$$v_m{}^T = \begin{bmatrix} \theta_{1A} & \psi_A & \theta_{1B} & \psi_B \end{bmatrix},$$
$$q_m{}^T = \begin{bmatrix} M_{1A} & M_{\omega A} & M_{1B} & M_{\omega B} \end{bmatrix},$$

where M_1 and M_ω are the torsion moments and the warping moments respectively at the nodal points.

The positive directions of the member end moments and node rotations are shown in Fig. 2.



a) b)

Figure 2. Positive nodal displacement (a) and nodal loads (b) of the bar.

As it is known, the relationship between the generalized force vector $\boldsymbol{q}_m$ and displacement vector $\boldsymbol{v}_m$ is established by the stiffness matrix $\boldsymbol{K}_m$ of the element as show below.

$$\boldsymbol{q}_m = \boldsymbol{K}_m \cdot \boldsymbol{v}_m, \tag{5}$$

where $\boldsymbol{K}_m$ is the stiffness matrix of current torsion element and for non-uniform torsion.

A trigonometric interpolation and approximation solutions of rotation θ_1 are used as an initial parameter and finally compared based on the nature of section type. For non-uniform torsion, a trigonometric interpolation of rotation β_1 is used as an initial parameter and finally compared with the approximation solution. To consider the warping of the restrained member, additional degrees of freedoms are introduced at the nodes and added to member displacement vector. An interpolation function containing hyperbolic functions of y_1, which satisfies the governing differential equation for torsion considered as given below:

$$\theta_1(y_1) = \boldsymbol{g}(y_1)^T \boldsymbol{b}$$
$$\boldsymbol{g}^T = \begin{bmatrix} g_1(y_1) & g_2(y_1) & g_3(y_1) & g_4(y_1) \end{bmatrix}$$
$$\boldsymbol{b}^T = \begin{bmatrix} \theta_{1A} & \theta_{1,1A} & \theta_{1B} & \theta_{1,1B} \end{bmatrix}$$

$$\beta_1 = \boldsymbol{h}_\omega^T C$$
$$h_\omega^T = \begin{bmatrix} \sinh ky_1 \\ \cosh ky_1 \\ y_1 \\ 1 \end{bmatrix} \qquad C = \begin{bmatrix} C_1 & C_2 & C_3 & C_4 \end{bmatrix}$$

The interpolation functions are substituted into the left-hand side of (4) and the integration over the length of the member is performed for axial and bending loads but separately considered for torsion as it developed based on the two different methods. The contribution of torsion to the internal virtual work of the governing differential equation is given as the following expressions:

$$\int_0^a (E J_\omega \delta\theta_{1,11}\theta_{1,11} + G J_t \delta\theta_{1,1}\theta_{1,1})\, dA = \delta\boldsymbol{b}^T (\boldsymbol{K}_{\omega 1} + \boldsymbol{K}_{\omega 2})\boldsymbol{b}$$

where $\boldsymbol{K}_{\omega 1}$ is component of stiffness matrix with warping restrained

$\boldsymbol{K}_{\omega 2}$ is component of stiffness matrix without warping restraint.

The Stiffness matrices components $\boldsymbol{K}_{\omega 1}$ and $\boldsymbol{K}_{\omega 2}$ are added to the member stiffness matrix $\boldsymbol{K}_m$ in the usual manner.

$$\boldsymbol{K}_T = \frac{EJ_\omega}{a^3}\begin{bmatrix} k_{T1} & k_{T2} & k_{T3} & k_{T4} \\ k_{T2} & k_{T6} & k_{T7} & k_{T8} \\ k_{T3} & k_{T7} & k_{T11} & k_{T12} \\ k_{T4} & k_{T8} & k_{T12} & k_{T16} \end{bmatrix},$$

$$K_{T1} = K_{T11} = S * ka\,sinh\,ka,\;\; K_{T6} = K_{T16} = S*(cosh\,ka - \frac{sinh\,ka}{ka})*a^2, \tag{6}$$

$$K_{T2} = K_{T4} = S*(cosh\,ka - 1)*a,\;\; K_{T8} = S*\left(\frac{sinh\,ka}{ka} - 1\right)*a^2,$$

$$S = \left(\frac{ka^2}{Q}\right),\;\; Q = 2\left(1 - cosh\,ka\right) + ka\,sinh\,ka, \quad K_{T3} = -K_{T1}, \quad K_{T7} = K_{T12} = -K_{T2}.$$

An alternative to the above derivatives of stiffness matrix can be replaced by using approximate shape function and it has an advantage for varying cross sections and non-linear problems. The element stiffness matrix for torsion with restrain warping can be used by divided into two matrices. The parameters K_{T1}, K_{T2}, K_{T6} and K_{T8} can be replace by approximation as shown below:

$$\begin{aligned} K_{T1} &= 12 + \frac{6}{5}*ka^2, \quad & K_{T2} &= 6 + \frac{1}{10}*ka^2 \\ K_{T6} &= 4 + \frac{2}{15}*ka^2, \quad & K_{T8} &= 2 - \frac{1}{30}*ka^2 \end{aligned} \tag{7}$$

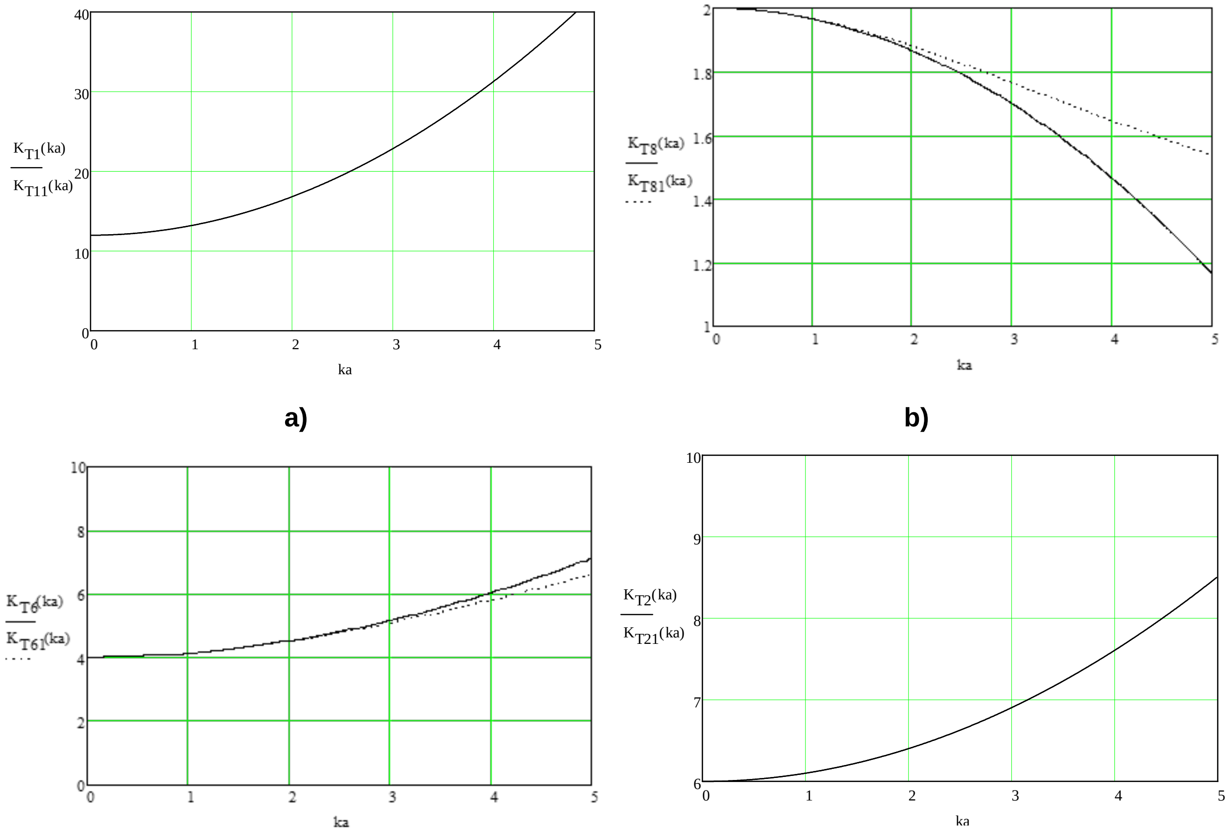
Considering the above series expressions, the alternative matrices can express as shown below:

$$\boldsymbol{K}_T = \frac{E\,C_\omega}{a^3}\begin{bmatrix} 12 & -6a & -12 & 6a \\ 6a & 4a^2 & 6a & 2a^2 \\ -12 & -6a & 12 & 6a \\ 6a & 2a^2 & 6a & 4a^2 \end{bmatrix} + \frac{G\,J}{30a}\begin{bmatrix} 36 & -3a & -36 & -3a \\ -3a & 4a^2 & 3a & -a^2 \\ -3a & 3a & 36 & 3a \\ -3a & -a^2 & 3a & 4a^2 \end{bmatrix} \tag{8}$$

3. Results and Discussions

3.1. The assessment of the exact and approximate methods for different value of ka

Comparing both methods, we can conclude that both are similar for small value of ka, and this is commonly considered for open thin-walled section as their value of ka is small as shown in Fig. 3. Referring equation 6, the components of stiffness matrix are generated based on the trigonometric and approximate methods. The percentage of error between the two methods of element stiffness matrix for torsion with restrain warping is given graphically as shown below in Fig. 3.



c) **d)**

Figure 3. Evaluation of the exact and approximate methods for different value of ka.

Referring Fig. 4, for ka =1 and 2 the errors range between 6.7 % to 9.7 % which is considered reasonable and both methods are acceptable for open thin-walled sections. So, the lengths of the member can be limited based on the section type and should be chosen for small value of ka.

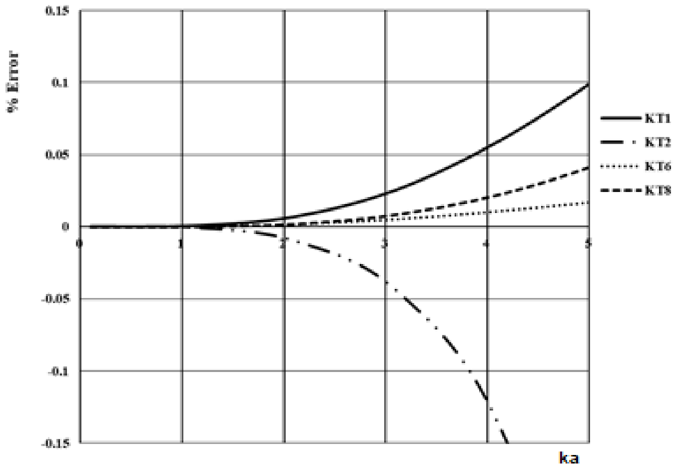


Figure 4. Approximation error graph for the values of K_{T1}, K_{T2}, K_{T6} and K_{T8}.

Considering the results of Fig. 3 and 4, the lengths of the member can be limited based on the section type and should be chosen for small value of ka = 2 as given below.

$$a \leq 0.1925 * \sqrt{\frac{J_t}{J_\omega}}.$$

If the member is free to warp, C_w = 0 and the torsional moment is carried by St Venant's torsion which is considered as uniform torsion. Considering Expression 8, only the second part of the matrix or the uniform torsion stiffness matrix can be used as given below.

$$\boldsymbol{K}_T = \frac{G\,J_t}{a}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \quad \text{or} \quad \boldsymbol{K}_T = \frac{G\,J_t}{30a}\begin{bmatrix} 36 & -3a & -36 & -3a \\ -3a & 4a^2 & 3a & -a^2 \\ -3a & 3a & 36 & 3a \\ -3a & -a^2 & 3a & 4a^2 \end{bmatrix}$$

The local element stiffness matrix after formation must be transformed to global coordinate system. The transformation is performed by extended transformation matrix and can be formally expressed as follow:

$$\boldsymbol{K}_{mx} = \boldsymbol{R}_m \boldsymbol{K}_{my} \boldsymbol{R}_m^T,$$

where R_m is member rotation matrix.

The member load vector is transformed by analogy to the member displacement vector:

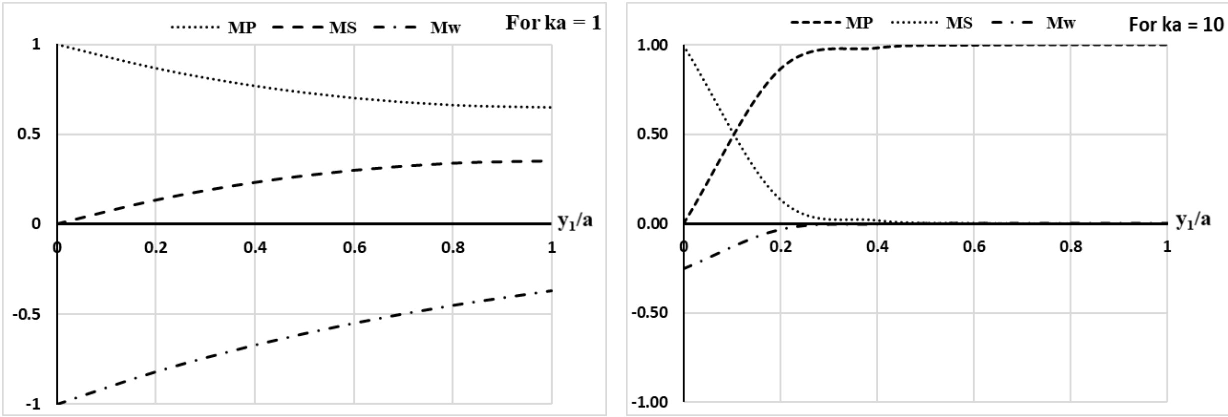
$$\boldsymbol{q}_{mx} = \boldsymbol{R}_m \boldsymbol{q}_{my},$$

where $\boldsymbol{q}_{mx}$ is member load vectors refereed to the global coordinate system; $\boldsymbol{q}_{my}$ is member load vectors refereed to the member coordinate system.

A system displacement vector $\boldsymbol{u}_s$ and a system load vector $\boldsymbol{q}_s$ are defined for the frame, which contain the nodal displacement and load coordinates in an order that is favorable for the solvers of the algebraic system equations. The member displacement and load vectors are related to the corresponding system vectors with topology matrices $\boldsymbol{T}_m$:

$$\boldsymbol{u}_m = \boldsymbol{T}_m \boldsymbol{u}_s, \quad \boldsymbol{q}_m = \boldsymbol{T}_m \boldsymbol{q}_s.$$

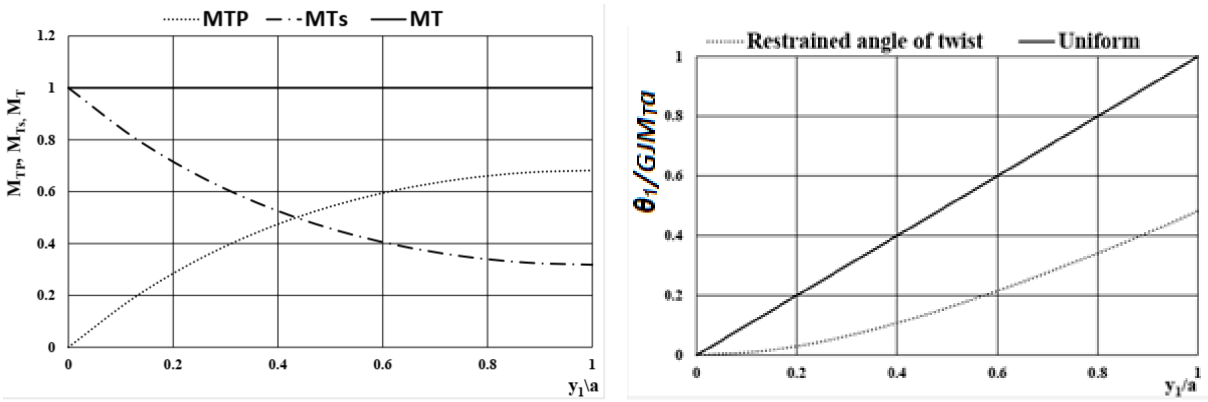
This study considers several section types, including the I section, rectangular hollow section, and channel section. Considering the prismatic bar of Fig. 1 as it is fixed at vertex $y_1 = 0$ and subjected to a twisting moment of M_T without warping restraint at vertex $y_1 = a.$ The section properties, displacements, rotations, stresses are to be compared for all cases and compared their distribution within the span based on the required value of ka. The value of ka for the closed and open sections differs largely, accordingly for closed section the shear stress is constant while for open section varies its direction and magnitude across the thickness. The section property for open and closed sections varies with respect to torsional and warping constants thus the value of ka differs. For small value of ka both torsion mechanisms contribute to M_T throughout the beam in both cases but with the increasing of the value of ka the influence of the twisting moment differs. For value of ka greater than 10, the total torsional moment is restricted to small length (approximately *0.2a*) near to the support and its magnitude changes rapidly in both cases. A combined graphs for M_{TP}, M_{TS}, and M_ω for the values of $ka = 1$ and $ka = 10$ are shown in the Fig. 5. Referring Fig. 5, the values of M_{TP}, M_{TS}, and M_ω differ for $ka = 1$ and for $ka = 10.$ For $ka = 1$, the total torsional moment components are extent throughout the span of the beam as shown in Fig. 5 (a) and its magnitude changes gradually as the value of ka is small. If the value of ka is small, it is most common for open sections. For $ka = 10$, the total torsional moment is restricted to small length near to the support and its magnitude changes rapidly as shown in Fig. 5 (b). If value of ka is large, it is most common for closed sections.



a) b)

Figure 5. Combined M_{TP}, M_{TS} and M_{ϖ} for value of ka = 1 (a), and ka = 10 (b).

The variation of M_{TP}, M_{TS}, and M_T on the axis of the bar are shown in Fig. 6. Both M_{TP} and M_{TS} contribute to M_T through the span of the beam and the torsional stresses are due to St Venant shear stresses and the restraint of warping. The applied twisting moment is resisted entirely by the secondary twisting moment at support $(y_1 = 0)$ and entirely by the primary twisting moment at $y_1 = a.$ In addition, the variation of the uniform angle of twist, non-uniform angle of twist, M_{TP}, M_{TS}, and M_T on the axis of the bar are shown in Fig. 6. The rotation at vertex $y_1 = a$ due to non-uniform torsion is around 50 percent of the rotation for uniform torsion of an I-section. Both M_{TP} and M_{TS} contribute to M_T through the span of the beam and the torsional stresses are due to St Venant shear stresses and the restraint of warping. The applied twisting moment is resisted entirely by the secondary twisting moment at support $(y_1 = 0)$ and entirely by the primary twisting moment at $y_1 = a.$ The distribution of the total moment between uniform and non-uniform torsion at intermediate points is shown in Fig. 6.



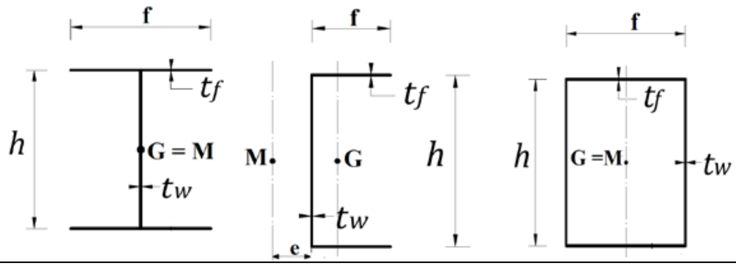
a) b)

Figure 6. The normalised graphs of θ_1, (a) M_{TP}, M_{TS} and M_T of restrained I-beam section (b).

3.2. Numerical Examples

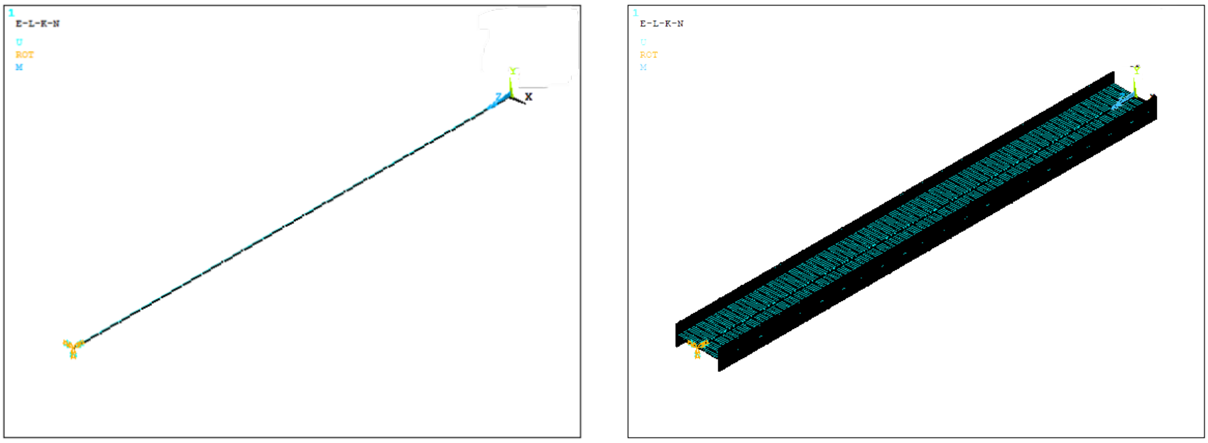
Based on the studies of the stiffness matrix for finite element procedure, the two methods are considered for the analysis of the uniform and non-uniform torsion behaviour of steel beam. The theory of restrained torsion is applied to a cantilever beam of length L subjected to external distributed torque. The two approaches of stiffness matrixes for the finite element methods compared in the previous sections, several problems of thin-walled beams with external torque applied on shear centre are presented to demonstrate the applicability of the methods. Three different sections of thin-walled beams loaded with external torque applied on shear centre are considered and compared with different methods as shown in Table 1. The accuracy of the presented model is illustrated by three examples using most commonly finite element software. The twisting effects and stresses are calculated, and the angle of twisting are presented graphically furthermore we considered the axial stress σ_z variation within individual section types.

Table 1. Different section types considered in this study



Section type			
h(mm)	400	400	400
f(mm)	180	180	180
t_f(mm)	11	11	11
t_w(mm)	8	8	8
$E = 200*10^6$ kN/m^2	$M_T = 1.0$ kN m	$G = 77*10^6$ kN/m^2	

For the above numerical examples, we considered different methods for comparison and validation of the results for uniform and nonuniform torsions. The verification of the model was conducted by means of FEM software ANSYS, Abaqus, Mathcad and current theory. Six models were created for three restrained and three uniformly twisted beams with I-section, Channel, and rectangular sections. The finite element BEAM 189 was used for Ansys because it can consider warping by entering special key options. Similarly, the non-homogeneous torsion is evaluated in ABAQUS by using of a shell model. A shell model is used because a thin-walled spatial beam is investigated and it works with the reduced number of finite elements in a way that less equations to solve. The following model is considered using ANSYS software and each model consists of 100 linear elements, the section is divided by 10 points in each direction and the results for each one is available. Such meshing is sufficient for the purposes of this analysis. One end of the cantilever beam is fixed for all degree of freedom and the torque moment is applied at another end by M_T command in the shear centre of the beam center.



a) b)

Figure 7. The load and support scheme for an I section.

The section properties, displacements, rotations, stresses are to be calculated for all cases and compared their distribution with the span based on the required value of ka. The variation of the uniform and non-uniform angle of twist (restrained torsion) on the axis of the three section types of the bar are shown in Table 2. The rotations at vertex $y_1 = a$ due to non-uniform torsion are around 49, 39 and 95 percent of the rotation for uniform torsion of the I section, channel section and rectangular hollow sections respectively. The rotation at vertex $y_1 = a$ due to uniform and non-uniform torsion is almost the same for the rectangular hollow cross section and its magnitude is negligible.

Table 2. Comparison of uniform and uneven torsion for different types of cross sections.

Methods	Applied torsion	I-Section	Chunnel Section	Rectangular Section

Current theory	Restrained	1.09E-01	47.7%	9.32E-02	38.9%	3.27E-04	98.2%
	Uniform	2.28E-01		2.40E-01		3.33E-04	
Ansys	Restrained	1.12E-01	49.3%	9.10E-02	38.7%	3.36E-04	91.8%
	Uniform	2.27E-01		2.35E-01		3.66E-04	
Mathcad 7DOF FEM	Restrained	1.19E-01	52.4%	9.21E-02	38.8%	3.31E-04	94.7%
	Uniform	2.28E-01		2.38E-01		3.50E-04	
Abaqus	Restrained	1.10E-01	48.1%	9.21E-02	38.6%	3.33E-04	97.6%
	Uniform	2.28E-01		2.39E-01		3.41E-04	

The variation of the non-uniform angle of twist (restrained torsion) on the axis of the three different cross-sections of the bar are shown in Table 2. The values of rotation angle in each node of restrained and uniformed cases were used for constructing comparison diagram. The rotation at vertex $y_1 = a$ due to non-uniform torsion is around 50 percent of the rotation for uniform torsion of an I-section. For I-cross-section as the value of ka is small and the magnitudes of the angle twist are extent throughout the span of the beam and its magnitude changes gradually as the value of ka is small as shown in Fig. 8 and 9.

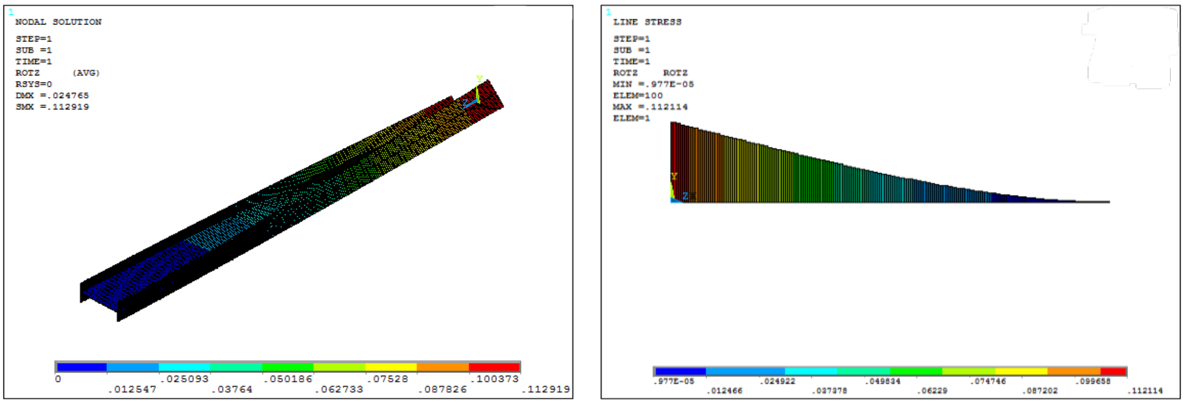


Figure 8. Variation of angle of twisting for an I-section of thin wall structure.

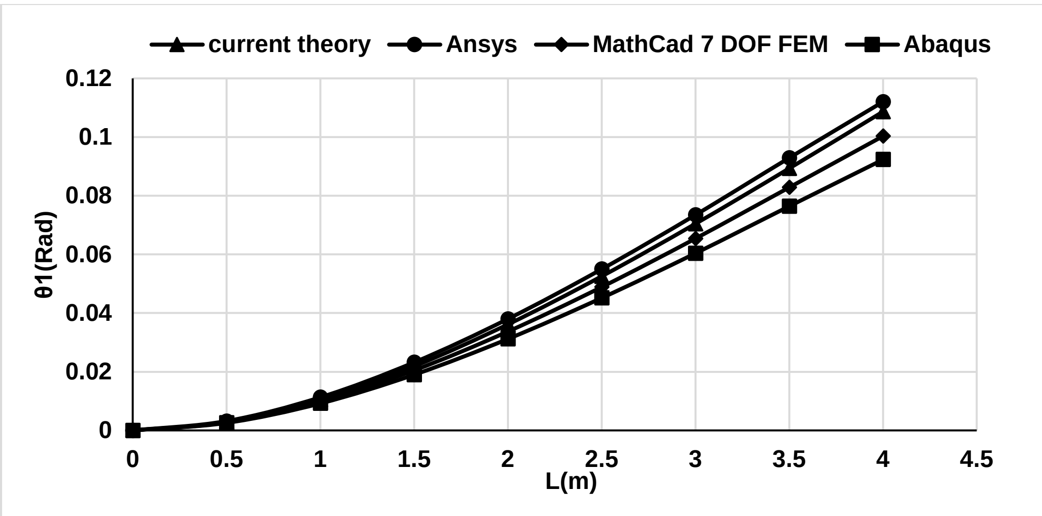


Figure 9. Rotations along the longitudinal direction of the I-section of thin wall structure.

For rectangular hollow cross-section as the value of ka is large and the angle of twist is very small in its magnitude. The angle of twist at points nearer to the support has significant magnitude as shown in the Fig. 10 and 11 and suddenly drops to a small value at the free end.

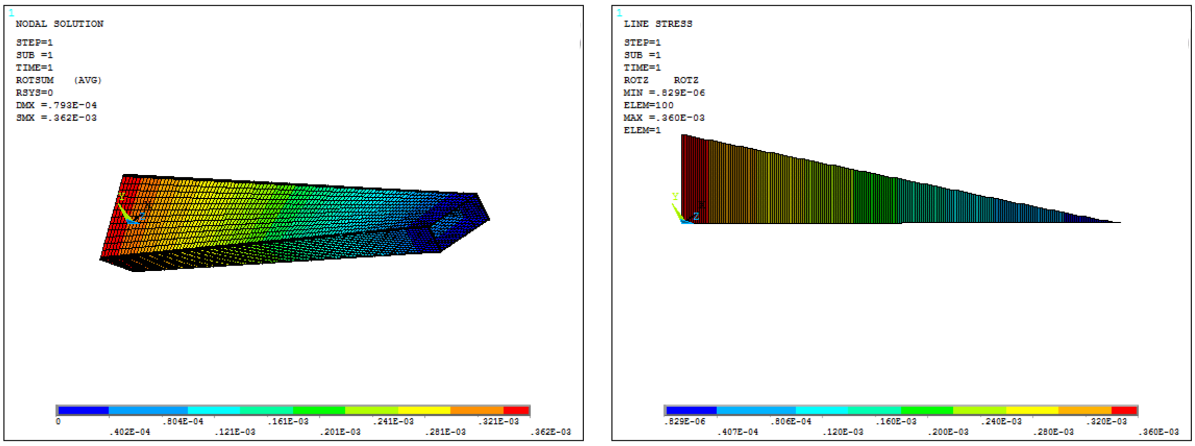


Figure 10. Variation of angle of twisting for rectangular section of thin wall structure.

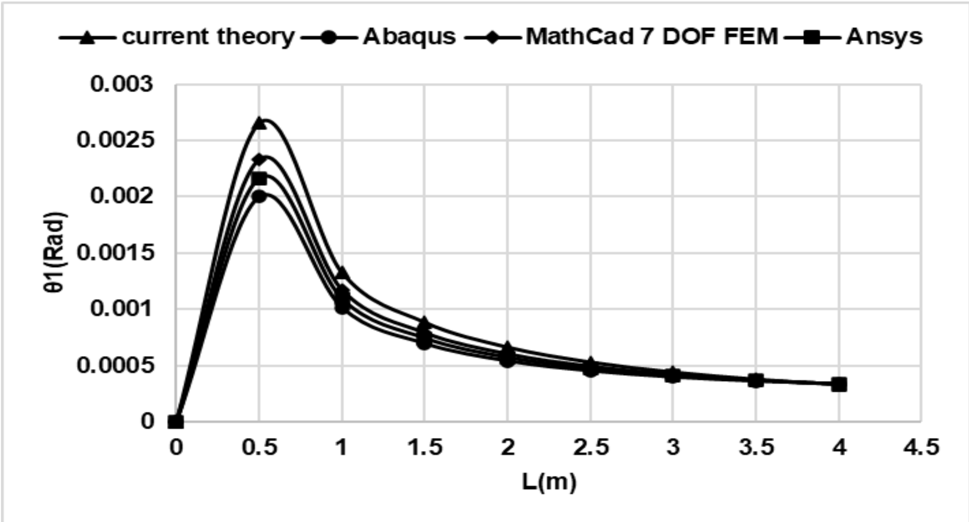


Figure 11. Rotations along the longitudinal direction of rectangular section of thin wall structure.

The variation of the non-uniform angle of twist (restrained torsion) on the axis of the channel section of a bar is shown in Fig. 12 and 13. The rotation at vertex $y_1 = a$ a due to non-uniform torsion is around 40 percent of the rotation for uniform torsion of the channel section.

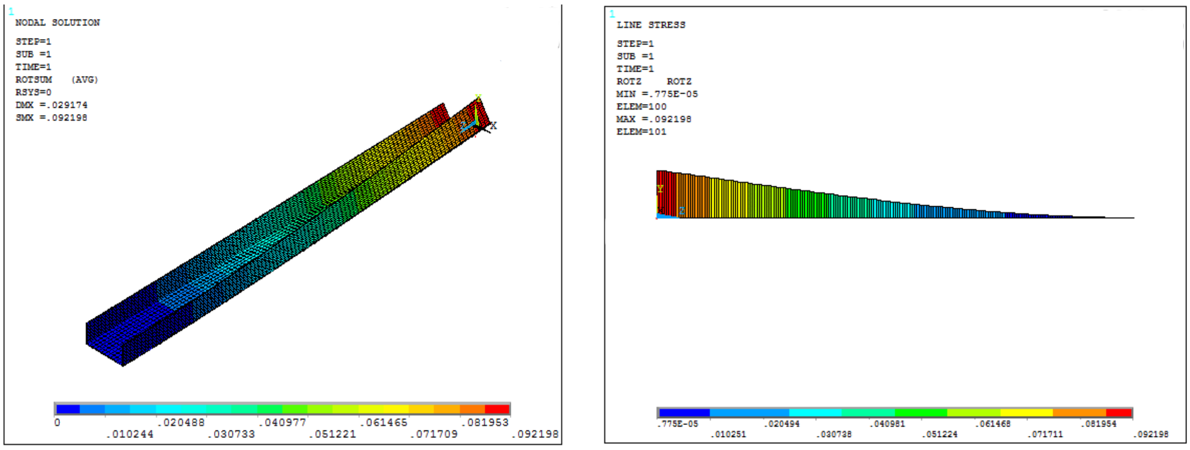


Figure 12. Variation of angle of twisting for channel section of thin wall structure.

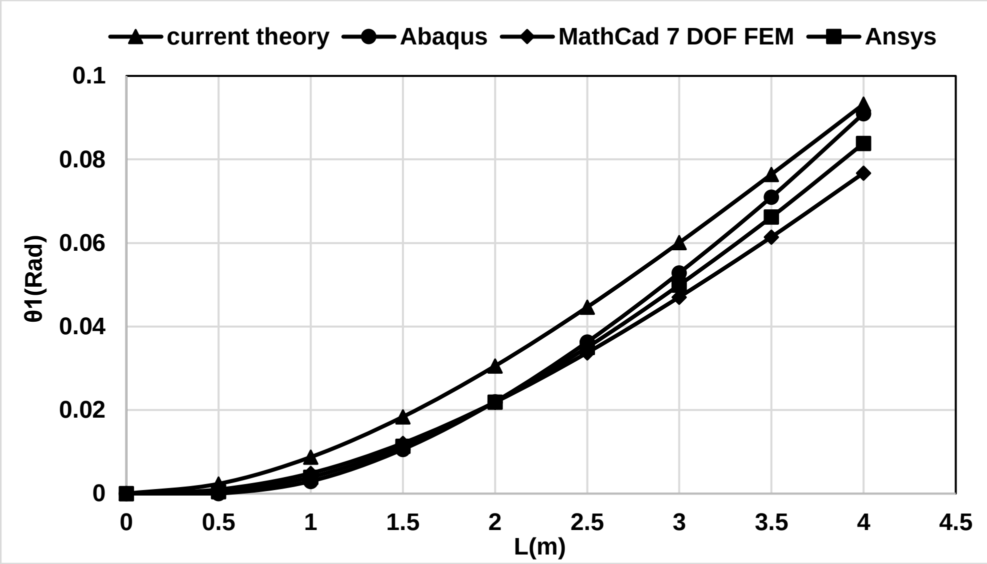


Figure 13. Rotations along the longitudinal direction of channel section of thin wall structure.

The study is compared with different research works which are considered for open thin-walled sections. The section properties are considered as the main criteria to compare the two different stiffness matrix and presented their comparison graphically. Associating both methods, we can achieve that both are similar for small value of ka and this is commonly considered for open thin-walled section as their value of ka is usually small and all results are compared with different studies [6, 10, 13, 18, 24, 38, 40]. For closed sections, the rotations are very small and are considered negligible but comparing to different studies, it shows that the effect of warping must be considered in the case of non-uniform torsion of closed-section beams [38–39].

4. Conclusion

In this study, two methods of stiffness matrix for the thin-walled structures with restrained torsion are used based on the exact and approximate methods. Based on different studies and design practices the two methods are successfully applied for the design of thin-walled structures with restrained torsion as part of finite element methods. According to this study, it is concluded that:

1. Comparing both methods, we can conclude that both are similar for small value of ka and this is commonly considered for open thin-walled section as their value of ka is small.

2. The percentage of error between the two methods of element stiffness matrix for torsion with restrain warping is negligible for the value of ka less than 2.

3. As the variation of the total torsional components depends on the value of ka and we can consider different section types. For ka = 1 and 2 the errors range between 6.7 % to 9.7 %, which is considered reasonable and both methods are acceptable for open thin-walled sections. So, the lengths of the member can be limited based on the section type and should be chosen for small value of ka.

4. For small value of ka both torsion mechanisms contribute to M_T throughout the beam in both cases but with the increasing of the value of ka the influence of the twisting moment differs. For value of ka greater than 10, the total torsional moment is restricted to small length (approximately 0.2*a*) near to the support and its magnitude changes rapidly in both cases.

5. For ka = 1, the total torsional moment components are extent throughout the span of the beam as shown in Fig. 4 and its magnitude changes gradually as the value of ka is small. If the value of ka is small, it is most common for open sections.

6. The angle of twist of non-uniform torsion differs from uniform torsion by 50, 1.8, and 41 percent for the I-section, rectangular tube, and channel section, respectively.

7. For ka = 10, the total torsional moment is restricted to small length near to the support and its magnitude changes rapidly. If value of ka is large, it is most common for closed sections.

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