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## Spatial stress-strain state of earth dams

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**Abstract.** The study is devoted to the stress-strain state (SSS) of various earth dams analyzed using a spatial model. The article provides a detailed review of well-known scientific publications on the SSS assessment of dams. In this article, a mathematical model was developed to assess the SSS of earth dams using a spatial model based on the Lagrange variational equation, considering real geometry, properties of the material, and non-homogeneous design features of structures. A technique was developed for solving spatial problems to assess the SSS of earth dams by the finite element method using the program developed by the authors and the ABAQUS software. The adequacy of the mathematical model and the accuracy of the results obtained were verified by solving test problems. The SSS of the Ghissarak ( $H = 138.5$  m), Sokh ( $H = 87.3$  m), and Pachkamar ( $H = 70.0$  m) earth dams under the action of body forces and hydrostatic water pressure was studied. It was established that the greatest displacements were observed on the crest and in the zone of the dam core; an account for nonhomogeneous design features significantly affects the resulting displacement field in the core zone; a spatial deformed state of the structure occurs near the banks; positive stress (not considered in a plane model) arises in a small area in the upper part of the core near the crest, caused by the crest indentation by the lateral surcharge.

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### 1. Introduction

When designing earth dams in seismic regions, the problem of choosing a computational model for studying their SSS (SSS), and the dynamic behavior, in general, arises. In most cases, when studying such types of structures, simplified plane calculation models are used.

Simplified calculation models cannot describe many of the effects of the spatial work of real structures. This causes an overrun of the material and the inability to provide the required margin of safety and reliability of the structure.

The solution to this problem, considering the above factors, can be most fully and accurately obtained using numerical methods, for example, the finite element method (FEM) or the finite difference method (FDM) [1–6] using a spatial model of structures.

To date, there are a number of scientific papers devoted to the risk of damage and the assessment of the SSS of earth dams in plane and spatial settings.

Despite the increased requirements for dam safety in the process of design, construction, and operation, there is a certain risk of accidents in earth dams.

It was noted in [7] that the risk factors for earth-rock dam breach change with time during the operation period. At that, a static risk analysis limited to a certain time cannot satisfy the needs of a comprehensive assessment and early warning. Therefore, in this article, a model of a dynamic Bayesian network was developed to study the dynamic characteristics of the probability of the breach of earth dams.

In [8], the risks associated with the failure of a rock-earth dam were analyzed from two points of view: the probability of a dam failure and the associated loss of life. To calculate the probability of dam failure, an analysis based on fuzzy event tree method was proposed. The proposed method and model were applied to a dam of the reservoir in Jiangxi Province.

In [9], the results of studies on numerical modeling of the process of earth-rock dams breach were summarized. However, there is still a significant gap in the versatility of computer software and technology for visualizing the breach process of a dam. It is expected that more efforts should be made in the future to investigate a detailed, physically based numerical model of a rockfill dam, and more attention should be paid to the application of visualization technology in simulating the process of dam failure.

In [10], the assessment of deformations of a compacted embankment dam partially supported by a tailing dam was considered using a two-dimensional and three-dimensional finite element model with PLAXIS and TOCHNOG programs, respectively. The modelling was done to simulate the complete stage of dam construction with compacted embankment and filling of the dam reservoir from a tailings dump. The results obtained made it possible to present and understand the deformation mechanisms that occur during the construction of the overpass and the filling of the reservoir.

In [11], using a specially developed computer program, a study was conducted on the impact of the stressed state of the dam on the stability coefficients of its slopes under the main and special combination of loads. The method of circle-cylindrical sliding surfaces was used. The study was conducted for a rock-earth dam with a height of 133 m. It was found that under high seismic impacts due to the formation of zones of soil decompaction, the stability margin of the upper slope was lower than the normative one.

In [12], an attempt was made to give an insight into the 3D probabilistic analysis of a dam by examining the reliability of a real earth dam using field measurements and comparing the 3D results with the 2D ones. It was established that the use of a three-dimensional computational model in a probabilistic analysis could give smaller estimates of the probability of dam failure compared to an analysis based on a two-dimensional model. The results showed that the use of a coarse mesh could lead to an underestimation of the failure probability, especially for 3D cases.

In [13], a new method for analyzing the failure risk caused by surges, landslides, and coast instability was proposed, taking into account the spatial variability of material parameters. Based on the theory of random fields, a method for modeling the spatial variability of material parameters was proposed and the most dangerous sliding surfaces of the slope of the reservoir banks with a minimum value of safety factors were determined.

Reference [14] analyzes the data of time measurements of the horizontal and vertical displacements of the dam under its own weight and water pressure in the reservoir at Ikpoba River dam. The calculation and adjustment of each measurement were conducted at different time points and separately. Analysis of the results showed that nine points of the dam moved to a horizontal position, while at 10 points the horizontal displacements did not change. Vertical displacement occurred in seven points, while in 13 points the vertical displacements did not change, both outside and along the crest of the dam.

In [15], the seismic response of the Pacoima dam site was estimated by performing a three-dimensional analysis using the boundary element method. The nature of the displacement and force caused by the scattering of seismic waves in the dam section was assessed. The calculation results were compared with the registered ground motion. A 4.3-magnitude earthquake recorder was used to analyze site response at Pacoima Dam in 2001. Estimation of the displacement of various points of the dam on both sides of the canyon in time showed a decrease in the amplitude of motions with an increase in the height of the points under consideration. The results obtained showed that the spatial nature of the dam point motion with time was due to the effect of the relief along the canyon.

The results of studies of the stress state in the longitudinal and transverse directions of various sections of a concrete dam, taking into account the sequence of dam erection, were presented in [16]. The purpose of the study was to search for the optimal SSS of the dam in terms of the formation of the minimum stress state of concrete.

In [17], on the basis of detailed monitoring data of the project of the high arch dam, the characteristics of the spatiotemporal evolution and the relationship of deformation of the valley and the dam were analyzed. The effect of valley narrowing deformation on dam deformation was modeled using the 3D Finite Element Method (FEM). A model for monitoring the safety of the deformation of an arch dam was developed,

considering the impact of the deformation of the valley. At the same time, the deformation of the dam was expanded into components of water pressure, temperature, valley deformation, and time effect.

In [18], the models of the global system of the Koyna dam were constructed using the ABAQUS software, considering dam-reservoir-foundation interaction. Results of uncoupled models showed overestimated predicted stability and damage detection parameters in the dam. The effect of clay and stone foundations on the resistance to crack propagation in the dam was assessed.

In [19], the SSS and dynamic characteristics of models of two different earth dams were studied. The studies conducted have shown that for some types of earth dams it is possible to use a plane model for the preliminary assessment of the stress state. Some features of the stress state in the spatial case were revealed, indicating dangerous areas with the highest stresses, and the nature of eigen oscillations that cannot be described using a plane model.

In [20], the SSS of earth dams under the action of harmonic load was numerically studied. A two-dimensional problem for the cross-section of a dam was studied using the equation of state, considering structural changes in the moisture properties of soil. The problem was solved numerically – by the method of finite differences. The results were presented in the form of graphs and were analyzed.

The study in [21] presents a comparative method for assessing the SSS of earth dams under the action of static load using plane and spatial models. The results of the assessment of the SSS of earth dams using these models were presented. Some features of the stress state in the spatial case were revealed, which cause the emergence of dangerous zones with the highest stresses.

Articles [22, 23] give a detailed analysis of well-known publications devoted to various models and methods, and the results of studying the stress state of earth dams, taking into account the elastic-plastic properties of soil. A mathematical model, methods and algorithm were constructed to assess the SSS of earth dams in a plane statement under various impacts, with account for the elastic-plastic properties of soil, non-homogeneous structural features and the level of reservoir filling. The strength of earth dams with a height of 195 m was estimated using the Coulomb-Mohr strength theory.

The paper [24] presents the results of studies of natural oscillations and harmonic responses of the elements of the Kenyir dam power station in Terengganu (Malaysia). Dynamic characteristics of the power plant were determined using the ANSYS software. The amplitude-frequency response for the points of the structure was obtained in a large frequency range by applying a force to the structure. A real-scale 3D model of the Kenyir dam power station was built using Solid Works software and ANSYS software. The maximum deviation of 0.90361 m in the z direction was obtained at a resonant frequency of 5.4 Hz.

The results of explosive tests of the highest Masjed Soleiman earth dam in Iran were presented in [25]. During the in-situ tests, 23 eigenfrequencies and 16 dam vibration modes were determined. Along with this, a numerical analysis was conducted, and the comparison of its results with the test results showed that the best matches were obtained at low vibration modes for a dam model with a massive foundation, and at higher vibration modes, the best results were obtained for a dam model with a massless foundation.

In [26], for numerical simulation of the SSS of the impervious element in the body of the dam of the Gotsatinskaya HPP, a mechanical model of soil materials taking into account the elastic-plastic properties of soil was used. Various numerical studies were conducted to assess the SSS of the systems under consideration.

The study in [27] presents the analysis data of methods for modeling the SSS of hydro-technical structures using computer systems. Quantitative evaluation was performed using the RSCI database in terms of both the number of publications and their citation, a qualitative analysis of abstract information, and full-text publications.

The study in [28] presents the results of the numerical simulation of the SSS of the diaphragm of a rock-fill cofferdam made of concrete by the method of bored piles. A feature of the operation of the diaphragm in the body of the cofferdam, which distinguishes it from the working conditions in the body of the dam, was revealed. The greatest danger to the strength of the concrete diaphragm is the bending strain. It was established that hydrostatic pressure causes displacement in the diaphragm, which, in turn, causes tensile stresses that are dangerous for concrete.

The research performed in [29] is devoted to the results of numerical studies of the SSS under seismic impact of an intensity of 9 points on a 16-m-high rock-fill cofferdam, the impervious element of which is a concrete diaphragm made of bored piles. It was revealed that the presence of a rigid concrete diaphragm in the center of the dam worsens the seismic resistance of the cofferdam and leads to the stability violation of the upper slope of the cofferdam.

The above review of studies on the SSS and dynamic behavior of earth dams in a spatial setting is very limited.

Despite their shortcomings and advantages, all these publications have their scientific and practical significance.

Therefore, the study and assessment of the SSS of earth dams in a spatial formulation, taking into account various features of the structure and comparison of the results with the ones obtained in a plane formulation, and determining the possibility of their use in assessing the strength of dams, are urgent problems.

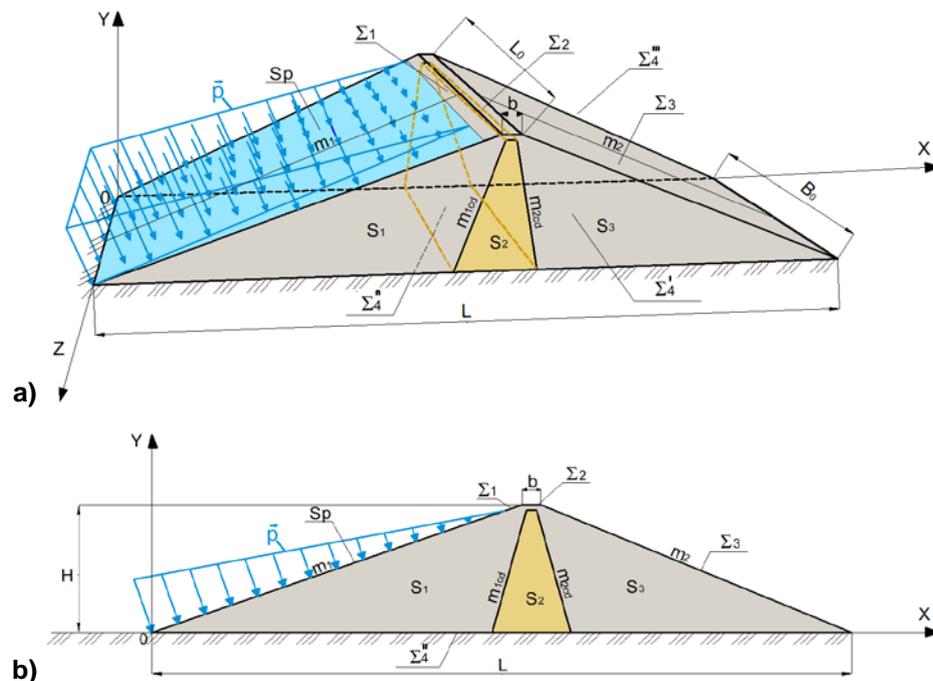
The purpose of this study is to develop a mathematical model and methods for solving spatial problems and studying the SSS of various earth dams and assess the possibility of using a plane model of a structure, which can adequately describe the SSS of earth dams.

## 2. Methods

### 2.1. Mathematical model

A non-homogeneous spatial system is considered, i.e. an earth dam (Fig. 1a) occupying the volume  $V = V_1 + V_2 + V_3$ . The surfaces of the lower slope  $\Sigma_3$  and the crest  $\Sigma_2$  are stress-free. This system (Fig. 1a) is under the action of body forces  $\vec{f}$ ; hydrostatic water pressure  $\vec{p}$  is applied to surface  $\Sigma_1$  on site  $S_p$ .

It is necessary to determine the components of the displacement vector, the strain and stress tensors in the body of the spatial system occurring under the action of its own weight and hydrostatic pressure of the reservoir water (Fig. 1a).



**Figure 1. Non-homogeneous system: a) a spatial model of the mid-section of the dam; b) a plane model of the mid-section of the dam.**

Here:  $\Sigma_4^{\prime\prime}$  is the area of foundation,  $\Sigma_1$ ,  $\Sigma_3$  are the areas of upper and lower slopes, respectively,  $\Sigma_2$  is the crest area,  $\Sigma_4^{\prime}$ ,  $\Sigma_4^{\prime\prime\prime}$  are the area of a dam connected to the mountain range,  $S_p$  is the area where the hydrostatic water pressure  $\vec{p}$ , acts,  $V_1$ ,  $V_3$  are the volumes of the upper and lower prisms,  $V_2$  is the volume of the core,  $L_0$  is the length of the dam crest,  $b$  is the width of the crest,  $B_0$  is the width of the foundation along the alignment,  $L$  is the width of the foundation cross-section,  $m_1$ ,  $m_2$  are the upper and lower slope ratios, respectively,  $m_{1cd}$ ,  $m_{2cd}$  are the slope ratios of upper and lower parts of the core.

To simulate the SSS occurring in a spatial system (Fig. 1a), the principle of virtual displacements is used; according to it, the sum of virtual works of all active forces acting on the system during virtual displacements is zero [5], i.e.:

$$\delta A = - \int_V \sigma_{ij} \cdot \delta \varepsilon_{ij} dV + \int_V \vec{f} \cdot \delta \vec{u} dV + \int_{S_p} \vec{p} \cdot \delta \vec{u} dS = 0, \quad i, j = 1, 2, 3 \quad (1)$$

Physical properties of the body are described by the relations between stresses  $\sigma_{ij}$  and strains  $\varepsilon_{ij}$  in the following form [30]

$$\sigma_{ij} = \lambda \varepsilon_{kk} \delta_{ij} + 2\mu \varepsilon_{ij} \quad (2)$$

and the Cauchy relations [30]

$$\varepsilon_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right), \quad (3)$$

kinematic boundary condition is also used:

$$\vec{x} \in \sum_4^{\cdot} + \sum_4^{\cdot\cdot} + \sum_4^{\cdot\cdot\cdot} : \vec{u} = 0, \quad (4)$$

Here  $\vec{u}$ ,  $\varepsilon_{ij}$ ,  $\sigma_{ij}$  are the components of displacement vector, strain and stress tensors;  $\delta \vec{u}$ ,  $\delta \varepsilon_{ij}$  are the isochronous variations of the components of displacement vector and strain tensors;  $\rho$  is the density of the body material;  $\vec{f}$  is the vector of body forces;  $\vec{p}$  is the hydrostatic water pressure acting on the surface  $S_p$ ;  $\lambda$  and  $\mu$  are the Lamé constants;  $\theta = \varepsilon_{kk}$  is the volumetric strain;  $\vec{u} = \{u_1, u_2, u_3\} = \{u, v, w\}$  are the components of displacement vector of the body point;  $\vec{x} = \{x_1, x_2, x_3\} = \{x, y, z\}$  are the coordinates of the body point for solving spatial problems  $i, j, k = 1, 2, 3$ , and plane problems  $i, j, k = 1, 2$ .

It is necessary to determine in the body of the dam (Fig. 1a) the functions of displacements  $\vec{u}(\vec{x})$ , strains  $\varepsilon_{ij}(\vec{x})$  and stresses  $\sigma_{ij}(\vec{x})$  arising under the action of body ( $\vec{f}$ ) and surface ( $\vec{p}$ ) forces that satisfy equations (1)-(3) and boundary conditions (4) for arbitrary virtual displacements  $\delta \vec{u}$ .

The purpose of solving this problem is to determine how adequately the solution to the problem for a plane strain state for a given system (Fig. 1b) reflects the processes occurring in spatial systems (Fig. 1a).

The considered variational problem (1)-(4) is solved by the finite element method [31]. To solve the spatial problem, a finite element in the form of tetrahedra with 68493 degrees of freedom is used, and for the plane problem, triangular finite elements with 5734 degrees of freedom of the system are used.

The procedure of the finite element method allows us to reduce the considered variational problem (1)-(4) to a system of non-homogeneous high-order algebraic equations, i.e.:

$$[K]\{u\} = \{P\}, \quad (5)$$

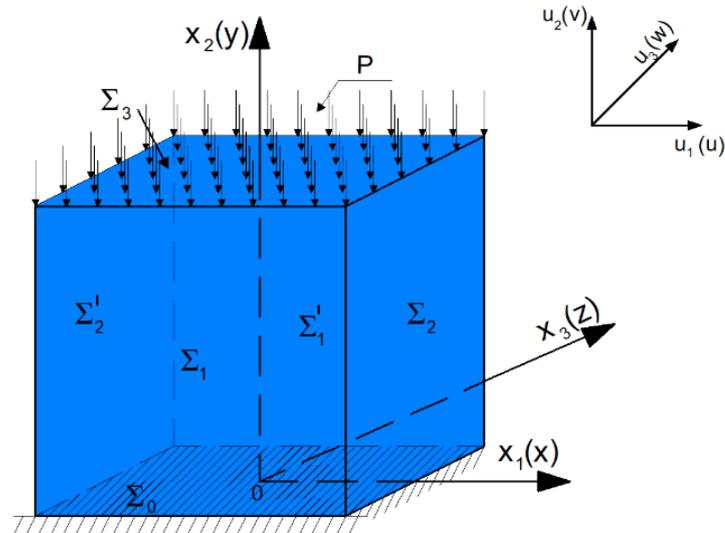
Here:  $[K]$  is the stiffness matrix for the considered body (Fig. 1);  $\{u\}$  are the sought-for components of displacement vectors in the nodes of the finite element (after partitioning the body into finite elements);  $\{P\}$  are the components of external (body and surface) forces acting on the nodes of the finite element.

When solving the above tasks, the computer calculation program developed by the authors was used for solving plane problems, besides, the well-known ABAQUS software was used to solve both plane and spatial problems.

### 3. Results and Discussion

To check the adequacy of the mathematical model and the accuracy of the results obtained, a test problem was solved using plane and spatial models.

The stress state of a rectangular long parallelepiped (Fig. 2) is considered under the action of distributed load  $P$ . The parallelepiped rests on an absolutely rigid and sliding base:  $y = 0$ ;  $u = 0$ ;  $\sigma_{12} = 0$ ;  $\sigma_{13} = 0$ .



**Figure 2. Deformable parallelepiped located on a sliding base.**

It is necessary to determine the components of displacement vector  $(u, v, w)$  and the components of stress tensor  $(\sigma_{11}, \sigma_{22}, \sigma_{12}, \sigma_{23}, \sigma_{33}, \sigma_{13})$  at any point of the body (Fig. 2).

The exact solution to this problem in a plane formulation is given in [32], and the solution has the following form:

$$u = \frac{\nu(1+\nu)}{E}x; \quad v = -\frac{(1-\nu^2)}{E}y; \quad \sigma_{22} = -P; \quad \sigma_{12} = \sigma_{21} = \sigma_{11} = 0.$$

The numerical solution to this problem was obtained using the computer program developed by the authors and the ABAQUS software using plane and spatial models, the results obtained were compared with exact solutions [32].

When solving this problem, the following initial data were used:  $P = 1.0$ ;  $a = b = 1.0$ ;  $E = 1.0$ ;  $\nu = 0.25$ . A comparison of the numerical and exact solutions obtained using a plane model is given in Table 1, and the solutions obtained using spatial models are given in Table 2.

**Table 1. Comparison of numerical and exact solutions of plane problems for some points of the body.**

№	Coordinates of the point		Exact solution [32]		Numerical solution obtained by the authors	
	x	y	u	v	u	v
1	0.25	0.0	0.07812	0.000	0.07750	0.000
2	0.50	1.0	0.15625	-0.9375	0.15600	-0.9290
3	0.50	0.375	0.15650	-0.3515	0.15300	-0.3650
4	0.50	0.75	0.15650	-0.7031	0.15500	-0.7100
5	-0.375	1.0	-0.11718	0.9875	-0.09975	-1.000
6	-0.375	0.0	0.11718	0.000	-0.09375	0.000
7	0.840	0.66	0.02667	-0.6191	0.02651	-0.6184
8	-0.058	0.64	-0.01843	-0.6017	-0.01847	-0.6017

**Table 2. Comparison of numerical and exact solutions obtained using spatial models.**

№	Name	$\sigma_{11}$	$\sigma_{22}$	$\sigma_{33}$	$\sigma_{12}$	$\sigma_{13}$	$\sigma_{23}$
1	Exact solution [28]	0	-1	0	0	0	0
2	Numerical solution obtained by the authors	-55 E-18	-1	-222 E-18	-44 E-18	-266 E-18	-310 E-18

A comparison of the results obtained (i.e. numerical and exact solutions) given in Tables 1 and 2 shows that the numerical solutions in both cases were obtained with high accuracy. Consequently, the results obtained with these models and techniques make it possible to estimate the stress state of various bodies not only of a simple structure but also of rather complex structures with sufficiently high accuracy.

The SSS (SSS) of three different dams under the action of body forces and hydrostatic water pressure was studied using plane and spatial models: Ghissarak, Sokh and Pachkamar earth dams located on the territory of the Republic of Uzbekistan. When studying the effect of water on the SSS of the dams, various levels of reservoir filling were considered.

The SSS of these dams was studied using the above mathematical models and methods under the action of hydrostatic water pressure in the upstream head, the own weight of the structure, considering the actual physical and mechanical characteristics of soils and design features of the dams under consideration.

The hydrostatic water pressure in the upstream head of the dam was taken as an external influence, according to the following formula [33]:

$$\vec{p} = \rho_0 \cdot g \cdot h, \quad (6)$$

Here,  $\rho_0$  is the density of water;  $h$  is the level of reservoir filling.

The study is conducted for three dams, built on the territory of highly seismic regions of Uzbekistan, with the following main geometric and design parameters:

1. **Ghissarak dam.** Rock-earth dam with a height of  $H = 138.5$  m is located on the Aksu River in the Kashkadarya region of Uzbekistan. The laying of the upstream slope is  $m_1 = 2.2$ , and of the downstream slope  $m_2 = 1.9$ ; the slope ratio of the loamy core is  $m_{1cd} = m_{2cd} = 0.15$ .

Retaining prisms are laid from rock mass with the following physical and mechanical parameters:  $E = 3600$  MPa, specific gravity of soil  $\gamma = 1.9$  tf/m<sup>3</sup>, Poisson's ratio  $\nu = 0.3$ . The core is laid from loam with physical and mechanical parameters:  $E = 2400$  MPa, specific gravity of soil  $\gamma = 1.7$  tf/m<sup>3</sup>, Poisson's ratio  $\nu = 0.35$ . The transition zone is laid of sandy-gravelly soil. The width of the dam crest is  $b = 16$  m and length  $L_0 = 660$  m, the width of the foundation is  $L = 634$  m, and the width of the foundation along the alignment is  $B_0 = 140$  m.

2. **Sokh earth dam** ( $H = 87.3$  m high) was built on the Sokh River in the Fergana region, with the slope ratio  $m_1 = 2.5$ ,  $m_2 = 2.2$ .

Retaining prisms are laid from pebbles with the following physical and mechanical parameters:  $E = 3550$  MPa specific gravity of soil  $\gamma = 2.1$  tf/m<sup>3</sup>, Poisson's ratio  $\nu = 0.35$ . The core is laid from loam with the slope ratio  $m_{1cd} = m_{2cd} = 0.25$  and with physical and mechanical parameters:  $E = 2400$  MPa, specific gravity of soil  $\gamma = 1.75$  tf/m<sup>3</sup>, Poisson's ratio  $\nu = 0.35$ . The dam crest is  $b = 10$  m wide and  $L_0 = 487.5$  m long; the width of the foundation is  $L = 530$  m, and the width of the foundation along the alignment is  $B_0 = 210$  m.

3. **Pachkamar earth dam** with a height of  $H = 70$  m was erected in the Kashkadarya region, with slope ratio  $m_1 = m_2 = 2.25$ .

Retaining prisms are laid from sand-pebbles with physical and mechanical parameters:  $E = 3600$  MPa, specific gravity of soil  $\gamma = 2.25$  tf/m<sup>3</sup>, Poisson's ratio  $\nu = 0.3$ . The core is laid from loam

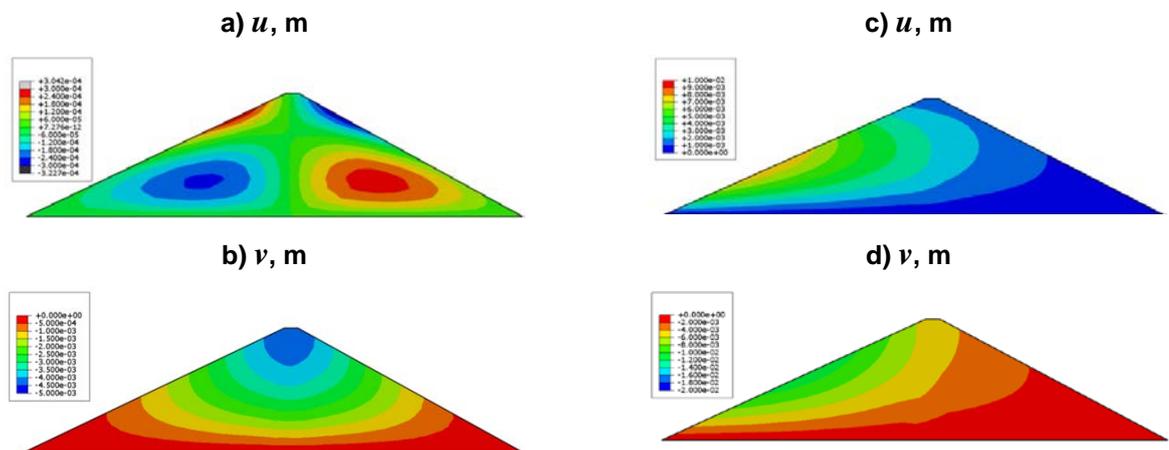
with slope ratio  $m_{1cd} = m_{2cd} = 0.5$  with physical and mechanical parameters:  $E = 2400$  MPa, specific gravity of soil  $\gamma = 1.78$  tf/m<sup>3</sup>, Poisson's ratio  $\nu = 0.35$ . The crest of the dam is  $b = 8$  m wide and  $L_0 = 573$  m long; the width of the foundation is  $L = 634$  m, and the width of the foundation along the alignment is  $B_0 = 217$  m.

The calculation results for these dams are the estimation of the components of displacement vectors  $u$ ,  $v$ ,  $w$ , the components of strain  $\varepsilon_{ij}$  and stress  $\sigma_{ij}$  tensors for all points of the structure.

The problem is solved using a spatial model and the results obtained are compared with the results of the problem obtained for a plane model in the case of a plane-deformed state of the central cross-section of the structure.

For the convenience of result analysis in the characteristic longitudinal and transverse sections of the dam, isolines (i.e., lines of equal values) of the components of displacements, strains, and stresses are plotted.

Figure 3 shows lines of equal values of horizontal (Fig. 3 a, c) and vertical (Fig. 3 b, d) displacements of the Ghissarak dam, obtained using the plane model under the action of its own weight and hydrostatic pressure of reservoir water, with completely filled reservoir (Fig. 3 c, d) and with empty reservoir (Fig. 3 a, b).

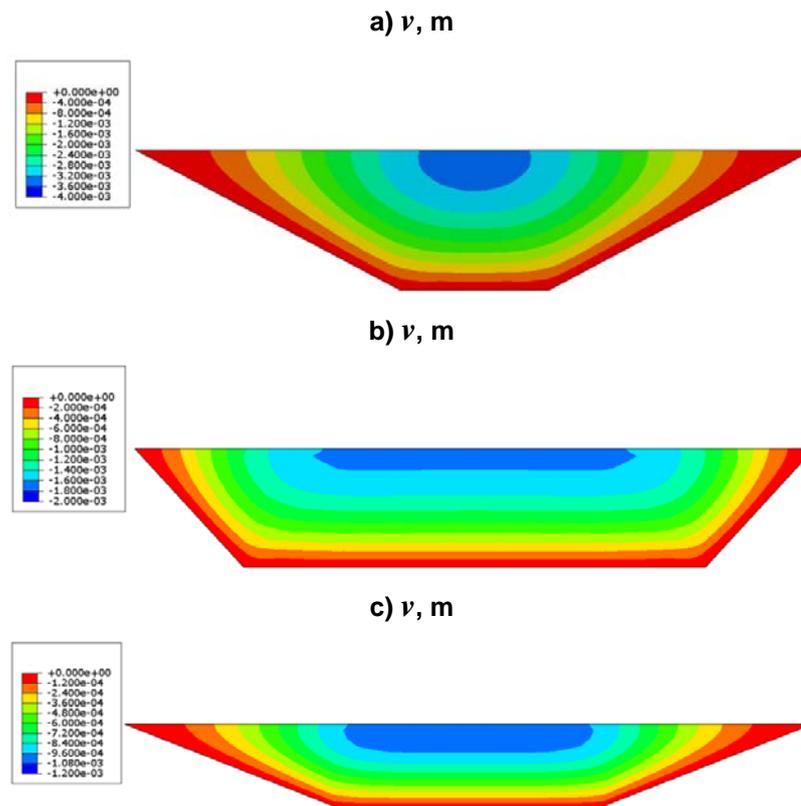


**Figure 3. Lines of equal values of displacement fields ( $u$  and  $v$ ) of the Ghissarak dam with completely filled reservoir (c, d) and with empty reservoir (a, b) obtained using a plane model.**

An analysis of the results obtained in a plane formulation (Fig. 3 a, b) shows that the displacements (absolute strain) of points in the body of the dam are approximately symmetrical with respect to the center of the dam. In the core of the dam, the values of horizontal displacements are close to zero, and their values increase towards the centers of the upper and lower retaining prisms. In this case, the displacement of the point in the vertical direction prevails. This is explained by the fact that the calculations were made considering only the own weight of the dam (Fig. 3 a, b). At the points located in the upper levels of the structure, the displacement values are greater than the ones at the points of the lower levels. The highest displacements are observed on the crest and in the zone of the dam core. Consideration of design features, namely the presence of a loam core significantly affects the displacement field only in the core zone.

The value of displacements (absolute strain) of the points of the Ghissarak dam profile (Fig. 3 c, d) in the case of consideration of hydrostatic pressure and body forces significantly changes the strain pattern in the upstream head. At that, the symmetrical distribution of strain relative to the center of the dam is violated in the body of the dam, and the absolute strain of the dam significantly depends on the level of water filling; this is especially evident when calculating the completely filled reservoir (Fig. 3 c, d). The same characteristic pattern was observed in the study of the SSS of the Sokh and Pachkamar dams.

Fig. 4 shows the distribution of equal values of vertical displacements (i.e., isolines)  $u_2$  (along the  $x_2$ -axis) in the longitudinal section of the Ghissarak (a), Sokh (b), and Pachkamar (c) dams, obtained using a spatial model under the action of body forces.



**Figure 4. Isolines of equal values distribution of vertical displacements  $v$  – (along the y-axis) in the longitudinal section of the Ghissarak (a), Sokh (b), and Pachkamar (c) dams, obtained using a spatial model under the action of body forces**

Analysis of the results obtained shows that, in all dams, the values of vertical displacements from the foundation of the dam to the upper levels gradually decrease, and the amplitude values of these displacements depend on the height of the dam.

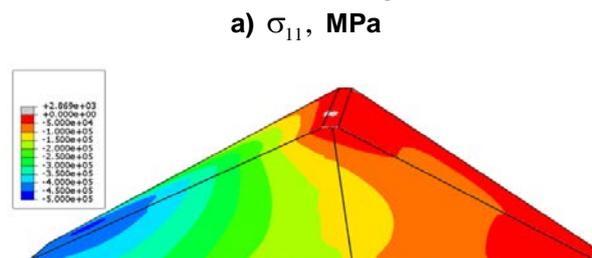
Along with this, the results in Fig. 4 show that the distribution of equal values of vertical displacements in the longitudinal section of the dam in the Ghissarak dam (Fig. 4 a) differs significantly from the distribution of the same displacements in the Sokh (Fig. 4 b) and Pachkamar (Fig. 4 c) dams.

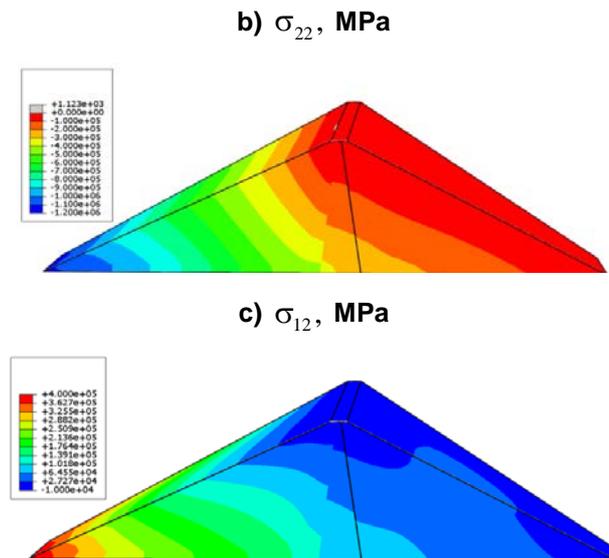
Having analyzed the distribution of vertical displacements along the length of the longitudinal section, we can see that in the Ghissarak dam the displacement changes dramatically from the middle of the longitudinal length to the left and right banks (Fig. 4 a). This shows that the banks have a significant influence on the dam strain since the geometric parameters of the dam are more consistent with a three-dimensional body.

As for the Sokh (Fig. 4b) and Pachkamar (Fig. 4 c) dams, the banks do not have a significant effect on strains (in the figures this is shown in blue color), i.e. here the use of a plane design model for calculations can be considered justified.

It can be concluded that when modeling the strain state of various dams, a plane calculation model does not always lead to a reliable result.

Figure 5 shows lines of equal values of horizontal  $\sigma_{11}$  (a), vertical  $\sigma_{22}$  (b) and tangential  $\sigma_{12}$  stresses of the Ghissarak dam under the action of its own weight and hydrostatic water pressure when the reservoir is completely filled. These results were obtained for all earth dams under consideration using plane and spatial models at different levels of reservoir filling with water.





**Figure 5. Lines of equal values of horizontal  $\sigma_x$  (a), vertical  $\sigma_y$  (b), and tangential  $\tau_{xy}$  stresses for the Ghissarak earth dam according to a spatial model at completely filled reservoir.**

The results obtained using the spatial model show that in the central part of the Sokh and Pachkamar dams, the conditions of plane strains of the theory of elasticity are observed since in this part there are only three strain components  $\varepsilon_{11}$ ,  $\varepsilon_{22}$ ,  $\varepsilon_{12}$  and they depend only on the coordinates of the point in the  $xOy$  plane.

A comparison of the results obtained for the central cross-section of the Ghissarak dam using a spatial model (Fig. 5) with the results obtained by a plane model shows that both statements give an almost similar qualitative pattern of the stress state, but the difference lies in the quantitative overestimation of the stresses obtained with the spatial model, namely, stresses  $\sigma_{11}$ ,  $\sigma_{22}$  increase by more than 30 %, the overestimated value of shear stresses  $\sigma_{12}$  is up to 10 %, compared with the plane calculation. At that, calculations by the spatial model show the value of stresses  $\sigma_{11}$ ,  $\sigma_{22}$  in the upper part of the dam with a positive value (Fig. 5 a, b), not observed using the plane model. The distribution of stresses along the longitudinal axis of the dam, obtained by the spatial model, shows that a more complex character takes place in the crest and near the end parts of the dam.

For example, in the Ghissarak dam, under the action of body forces, the upper part of the core near the crest rises, i.e. tensile strain occurs. This is apparently explained by the following process: the upper and lower surcharges of denser material and rigidity, press on the cores composed of less rigid material, this, in turn, intends the core and the upper part of the core near the crest rises (in Fig. 5, the positive value of the stress shows this phenomenon).

To verify this phenomenon, a test problem was solved, i.e., this problem was solved using a spatial model for two options of characteristics of the dam core material with modified Poisson's ratios  $\nu = 0.499$  and  $\nu = 0.001$ . The result was as follows: at  $\nu = 0.499$ , the indentation of the upper part of the core near the crest increased considerably and at  $\nu = 0.001$  no such indentation was observed.

In a certain part of the core near the crest of the Ghissarak dam, we observe the process of bulging of relatively soft soil of the core upwards, apparently due to the fact that the dam is located in a narrow alignment, the left and right banks of the alignment actively affect the SSS of the central part of the dam and soft soil of the core is pressed in.

This phenomenon manifests itself only when using a spatial calculation model. In contrast to the Sokh and Pachkamar dams, the Ghissarak dam is a three-dimensional body in terms of geometric dimensions. Therefore, such a pronounced effect is manifested only in the Ghissarak dam.

In the dams considered in the study, all components of the strain and stress tensor are manifested in the end parts; this shows a spatial SSS of the structures in this part of the dam.

This means that the use of a plane model to assess the SSS of earth dams does not always lead to an adequate description of the SSS occurring in them.

## 4. Conclusions

1. A detailed review of scientific publications devoted to the assessment of the SSS of various dams was given.
2. A mathematical model and methods were developed for assessing the spatial SSS of earth dams under various static effects, taking into account structural non-homogeneities and the actual geometry of structures using the variational principle and the finite element method.
3. The adequacy of the mathematical model and the accuracy of the results obtained were verified by solving a test problem and the spatial SSS of the Ghissarak, Sokh, and Pachkamar earth dams was studied using a spatial model under the action of body forces and hydrostatic water pressure.
4. It was stated that:
  - the greatest displacements were observed on the crest and in the zone of the dam core; an account for the non-homogeneous features of the structure significantly affected the resulting displacement fields in the core zone, and a spatial deformed state of structures occurred near the banks;
  - the SSS of real earth dams significantly depends on the commensurability of geometric dimensions of the retaining prisms and the slope ratios, and on the slopes of the dam banks;
  - in the upper part of the core (in the central cross-section) of the Ghissarak dam calculated by the spatial model, the distribution pattern of vertical stresses  $\sigma_y$  and strains  $\varepsilon_y$  changes significantly, compared with the results obtained by the plane model.
  - in a small area in the upper part of the core near the crest of the Ghissarak dam, positive stress arises due to the indentation of the core by side surcharges; this is apparently due to the fact that the dam is located in a narrow alignment, the left and right banks of the alignment actively affect the SSS of the central parts of the dam.

## References

1. Zareckij, Ju.K., Lombardo, V.N. Statika i dinamika gruntovyh plotin [Statics and dynamics of earth dams]. M.: Energoizdat, 1983. 256 p. (rus)
2. Krasnikov, N.D. Sejsmostojkost' gidrotehnicheskikh sooruzhenij iz gruntovyh materialov [Seismic resistance of hydro-technical structures made of earth materials]. M.: Energoizdat, 1981. 240 p. (rus)
3. Ljahter, V.M., Ivashenko I.N. Sejsmostojkost' gruntovyh plotin [Seismic resistance of earth dams]. M.: Nauka, 1986. 233 p. (rus)
4. Konstantinov, I.A. Dinamika gidrotehnicheskikh sooruzhenij [Dynamics of hydro-technical structures]. Part 2. L.: Ed. LPI, 1976. 196 p. (rus)
5. Mirsaidov, M.M. Teorija i metody rascheta gruntovyh sooruzhenij na prochnost' i sejsmostojkost [Theory and methods for calculating earth structures for strength and seismic resistance]. Tashkent: Fan, 2010. 312 p. (rus)
6. Mirsaidov, M.M., Sultanov, T.Z. Ocenka dinamiceskoy prochnosti gruntovyh plotin s uchetom nelinejnogo deformirovaniya [Evaluation of the dynamic strength of earth dams taking into account nonlinear deformation]. Tashkent: "Adabiyet uchkunlari", 2018. 258 p. (rus)
7. Li, Z., Wang, T., Ge, W., Wie, D., Li, H. Risk analysis of earth-rock dam breach based on dynamic bayesian network. Water. 2019. 11 (11). 2305. DOI: 10.3390/w11112305
8. Fu, X., Gu, Ch.Sh., Su, H.Zh., Qin, X.N. Risk Analysis of Earth-Rock Dam Failures Based on Fuzzy Event Tree Method. Int J Environ Res Public Health. 2018. 15 (5). 886. DOI: 10.3390/ijerph15050886
9. Zhong, Q., Shan, Y., Liu, J. Earth-Rock Dams' Breach Modelling. Dam Engineering. 2020. DOI: 10.5772/intechopen.92893
10. Sidnei Helder Cardoso Teixeira, Paulo Roberto de Paiva, Tennon Freire de Souza Junior, Eduardo Conte. Deformations on a Talings Dam Embankment Due to Its Heightening and Reservoir Filling. Civil Engineering. 2021. DOI: 10.21203/rs.3.rs-528290/v1
11. Sainov, M.P., Gapeev, D.S., Kudrjavcev, G.M. Vlijanie naprjazhjonogo sostojanija kamenno-zemljanoj plotiny na ustojchivost' ejo [Influence of the stress state of a rock-and-earth dam on the stability of its slopes] Science of Science Internet Journal. 2017. 9(6).
12. Xiangfeng, G., Dias, D., Carvajal, C., Peyras, L., Breul, P. Three-dimensional probabilistic stability analysis of an earth dam using an active learning metamodeling approach. Bulletin of Engineering Geology and the Environment. 2022. 81. 40.
13. Kai, D., Zefa, L., Lu, X., Chen, Ch., Jinbao, Sh., Jiankang, Ch., Zhenyu, W. Analysis of dam overtopping failure risks caused by landslide-induced surges considering spatial variability of material parameters. Front. Earth Sci. 2021. 9-2021. DOI: 10.3389/feart.2021.675900
14. Ehiorobo, J., Ehigiator-Irughe, R. 3-d spatial analysis of deformation at Ikpoba dam from gps data. Journal of the Nigerian Association of Mathematical Physics. 2011. 19. Pp. 493–498.
15. Sohrabi, B.A., Isari, M., Tarinejad, R., Maghami, Sh. Topography effects in pacoima dam site using time-domain three-dimensional bem. Bulletin of earthquake science and engineering Spring. 2019. 6(1). Pp. 23–34.
16. Konstantinov, I., Kuzmin, S., Savchenko, A., Boychenko, P., Nagornya, D. Allowance for sequence of mass concrete dam erection on soils during its stress-strain analysis. MATEC Web of Conferences. 2016. 73(3). 01005. DOI: 10.1051/mateconf/20167301005
17. Zheng, F., Yaqi, L. Deformation characteristics and a safety monitoring model of high arch dam affected by valley narrowing deformation. 11<sup>th</sup> Conference of Asian Rock Mechanics Society IOP Conf. Series: Earth and Environmental Science. 2021. 861. 042040. DOI: 10.1088/1755-1315/861/4/042040

18. Nariman, N.A., Lahmer, T., Karampour, P. Uncertainty quantification of stability and damage detection parameters of coupled hydrodynamic-ground motion in concrete gravity dams. *Struct. Civ. Eng.* 2019. 13(2) .Pp. 303–323. DOI: 10.1007/s11709-018-0462-x
19. Mirsaidov, M.M., Toshmatov, E.S. Spatial stress state and dynamic characteristics of earth dams. *Magazine of Civil Engineering.* 2019. 89(5). Pp. 3-15. DOI: 10.18720/MCE.89.1
20. Khusanov, B., Khaydarova, O. Stress-strain state of earth dam under harmonic effect. *E3S Web of Conferences.* 2019. 97. 05043. DOI: 10.1051/e3sconf/20199705043
21. Sultanov, T.Z., Yuldoshev, B., Toshmatov, E., Yarashov, J., Ergashev, R., Mirsaidov, M. Strength assessment of earth dams. *MATEC Web of Conferences.* 2019. 265. 04015. DOI: 10.1051/matecconf/201926504015
22. Mirsaidov, M.M., Sultanov, T.Z., Yarashov, J.Y. Strength of earth dams considering elastic-plastic properties of soil. *Magazine of Civil Engineering.* 2021. 108 (8). Article No. 10813. DOI: 10.34910/MCE.108.13
23. Mirsaidov, M., Sultanov, T., Yarashov, J., Urazmukhammedova, Z. Estimation of the earth dam strength with inelastic soil properties. *IOP Conference Series Materials Science and Engineering.* 2020. 883. 012021. DOI: 10.1088/1757-899X/883/1/012021
24. Arbain, A., Mazlan, A.Z., Zawawi, M.H., MohdRadzi, M.R. Vibration analysis of kenyer dam power station structure using a real scale 3d model. *Civil and environmental engineering reports.* 2019. 29(3). Pp. 048–059. DOI: 10.2478/ceer-2019-0023
25. Jafari, M.K., Davoodi, M. Dynamic characteristics evaluation of Masjed Soleiman Dam using in situ dynamic tests. *Canadian Geotechnical Journal.* 2006. 43(10). Pp. 997–1014. DOI: 10.1139/t06-059
26. Prokopovich, V.S., Velichko, A.S., Orishchuk, R.N. Stress-strain state of an earth dam with a clay-cement-concrete diaphragm on the example of the earth dam of the Gotsatlinskaya HPP. *News of the B.E. Vedeneev VNIIG.* 2016. 282. Pp. 87–98.
27. Kulik, K.N. et.al. Analysis of modeling methods of the stress-strain state of the hydro-technical structures for reclamation purposes applying computer systems. *Ekologiya & Stroitelstvo.* 2018. 1. Pp. 21–26.
28. Sainov, M.P., Tolstikov, V.V., Tarasov, A.A. Study of the stress-strain state of rock-fill cofferdam concrete diaphragm at hinge connection with foundation. *The Eurasian Scientific Journal.* 2018. 10(1). URL: <https://esj.today/PDF/75SAVN118.pdf> [reference date: 29.03.2022].
29. Sainov, M.P., Shaimiardianov, I.R. (2018). Study of seismic stability of rock-fill cofferdam with concrete diaphragm. *The Eurasian Scientific Journal.* 2018. 3 (10). <https://esj.today/PDF/03SAVN318.pdf>
30. Aleksandrov, A.V., Potapov, V.D. *Osnovy teorii uprugosti i plastichnosti [Fundamentals of the theory of elasticity and plasticity].* M.: Higher school, 1990. 400 p. (rus)
31. Bate, K., Vilson, E. *Chislennye metody analiza i MKJe [Numerical methods of analysis and FEM].* Moscow: Stroyizdat, 1982. 448 p. (rus)
32. Rekach, V.G. *Rukovodstvo k resheniju zadach po teorii uprugosti [Guide to solving problems in the theory of elasticity].* M.: Higher school, 1977. 215 p. (rus)
33. Chugaev, R.R. *Gidravlika [Hydraulics].* Energoizdat, 1982. L. 203 p. (rus)

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