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Numerical study of the process of unsteady flow in a three-layer porous medium

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Abstract. The unsteady fluid flow in a three-layer porous medium is numerically investigated and is an important and topical problem. An analytical solution of the equation for the pressure fluid layer is obtained on the basis of the theory of elastic regime, taking into account the overflow from the coating and the low-permeability layer into the low-permeability bulkhead and external sources that greatly affect the liquid level change. In the main bounded aquifer only horizontal liquid migrations prevail, and in the cover and low-conductive layers only vertical migrations prevail, allowing for horizontal components of the flow rate to be omitted here. Evaporation from the surface of a liquid in a porous medium is considered. Evaporation from the surface of a liquid in a porous medium significantly affects the distribution of overflows in the layers and the distribution of the liquid level in a porous medium. Therefore, when designing vertical drains for enhanced oil recovery in multilayer reservoirs and designing fluid flows in reservoirs, evaporation must be taken into account. The computational experiments have established that the dynamics of changes in the liquid level in a porous medium depends significantly on the evaporation parameter. With an increase in its value, the liquid level in a porous medium decreases proportionally over time. The accuracy of the numerical solution using the balance equation showed that the error does not exceed 1.3%.

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1. Introduction

The pressureless flow of fluid through a multilayer porous medium is found in many technical applications. Historically, such flows were considered concerning hydraulic structures. Later, the obtained results and approaches were applied to filter technology, chemical processes and apparatuses, oil production, climate technology, and the analysis of fluid movement in the elements of buildings and structures.

In particular, the paper [1] proposes a mathematical model and a numerical algorithm for monitoring and forecasting groundwater and surface water migrations using geofiltration models.

The problem analytical solution aims to study the interaction of surface and subsurface water flows. The paper [2] considers two-dimensional steady subsurface water flow in the vertical plane. In that article, the aquifer is idealized as an infinite band, and the channel is modeled as a horizontal equipotential function.

The article [3] proposes a mathematical model and a numerical algorithm for solving the problem of unsteady free flow of groundwater filtration considering well galleries in heterogeneous porous media where the wells differ by their hydrogeological characteristics. The authors of article have compiled an analytical solution to the specified problem to get a linearized system describing groundwater free filtration using the Laplace transform with variable t. To create isolated areas preventing the spread of harmful liquids and to protect groundwater in the interlayers, ratios to support the water heads there have been derived, as well as formulas for determining interlayer water tables in the corresponding zones of the groundwater filtration area [4].

In the article [5], the authors have developed a general mass transfer model based on the infiltration model of shore intake considering water exchange between groundwater and surface water. The mass transfer model describes the salt transfer in groundwater and the kinetics of salt exchange in dry soil. Some attention in modeling the process is paid to developing a methodology for numerical modeling of the groundwater level regime in the zones of shore intake influence. This will ensure reliable calculations when forecasting the operation of wells and estimating groundwater reserves of water intakes in difficult hydrogeological conditions. Numerical algorithms and software are developed to perform calculations on computer systems to solve inverse problems of aquifers' hydrogeological parameters and identify water exchange parameters. The general mass transfer model proposed by the authors of the articles [6, 7] will make it possible to consider the main processes comprehensively and to assess the groundwater mineralization degree and the transfer of contaminants, including hydrocarbon ones, by interacting flows of surface water and groundwater.

The article [8] deals with filtration processes: with a stationary filtration mode on an interfluvial soil body with constant levels; with a nonstationary filtration mode at backwater on an interfluvial soil body; influenced by a rise in the water level on the right border. Depression curves are made for stationary and nonstationary modes, and the correctness of computer program calculations has been proved using the Dupuit formulation. As noted in the paper, the convergence of the actual level values and the model ones has been found unsatisfactory at levels of 0.40 m and 0.25 m and when applying the stationary mode. However, should the level increase to 0.35 m, there are almost no or small discrepancies. The numerical model demonstrates how the level change rate depends on the transmissivity level: the higher the transmissivity level, the faster the stationary filtration mode starts running.

The article [9] provides the results of mathematical modeling of the water table distribution in the underflow talik depending on the intensity of water intake from wells and pits. As concluded by the article's authors on the calculation results, the selected layout of wells along the talik zone of the river and the water intake regime will provide the water volumes required for the development of the mining and processing plant. it is necessary to operate five additional production wells To ensure the estimated demand for drinking water during 2013-2017. The hydrodynamic impact of the pit on the designed water intake wells is also insignificant. By the 31st (2044) year of development, an additional drop in the water level from 0.8 m (well 42) to 8.9 m (well 127) is forecasted, which does not exceed the permissible 65 m.

A mathematical model and a numerical algorithm are developed to forecast the groundwater table on the slope of the river valley [10] To provide hydrogeological forecasts. The article's authors note that it characterizes soils' geological structure, underground flow boundary conditions, and water-conductivity parameters. Hydrogeological parameters and boundary conditions intensity degree are identified by the multivariate numerical modeling. Using the results obtained from it, water flows to drainage structures, and their dependence on technogenic infiltration have been estimated. Parameters of shallow red-brown clays are determined at which the clays promote the local rise of the groundwater level. The article considers the hydrodynamic processes in the mouth of the Temernik River on the right bank of the Don River to give detailed data on the soil body's hydrogeological structure and assess the filtration parameters and boundary conditions. The authors used the results to determine the water flow to the in-situ drainage structures in the seepage zone. In the studies performed by the authors, the method used is based on numerical hydrogeological modeling that systematically includes the interrelated geofiltration parameters and makes it possible to cover the geoecological risk factors to develop effective solutions to combat flooding.

In the paper [11], the author modeled subsurface water filtration through a homogeneous earth dam with vertical slopes on a non-conductive base. The GEO-SLOPE GeoStudio hydrodynamic calculation software package has identified the main characteristics of the filtration flow and constructed the depression curves. To confirm the adequacy of the results of the numerical calculations, the author of the work has compared them with the results calculated using a proven method; the conclusion provides recommendations for using boundary conditions to make modeling more accurate.

The paper [12] considers steady plain filtration in a rectangular dam with a partially non-conductive vertical wall during evaporation from the water table. A mixed multiparameter boundary equation has been formulated for the analytical function theory to study the evaporation effect; it was solved using the method of P.Ya. Polubarinova-Cochina. The proposed model provides the basis for an algorithm for calculating the filtration characteristics of the flow. The results of a hydrodynamic analysis of the flow rate dependencies are presented, as well as the ordinate of the point where the depression curve starts concerning all physical parameters of the scheme. The exact values of the specified characteristics are compared with the known approximate values obtained by other authors out of evaporation process conditions. The results of the study give an idea (at least a qualitative one) on the possible dependence of the migration characteristics when considering the problem of filtration to an imperfect pump well.

The article [13–14] describes a numerical study of the problems of free nonlinear filtration in a trapezoidal and rectangular soil dam with the horizontal layers of different coefficients: filtration, partial saturation zone, vertical dam core, and horizontal drain canals. A generalization to the three-dimensional case is given as well. The proposed grid calculation method can be used to conduct multifactorial studies of the environmental aspects affecting water transfer in in-situ multilayer porous media.

The article [15] compares the solutions for two methods of calculating anisotropic dam filtration; numerical experiments have been performed for several profiles of soil dams, their elements, and antifiltration devices (downstream or upstream shell, screen, and core). Further, the following methods of solving anisotropic problems have been compared: through an imitated hydrodynamic filtration grid made by stretching an orthogonal hydrodynamic grid previously constructed by the EGDA method for a distorted isotropic model of the dam; and through a finite element numerical method supported by the local variation method.

The paper [16] considers the numerical solution of the anisotropic filtration problem for the soil dam. The results of computational experiments are given that were performed on a computer when solving the filtration problem and calculating the stability of the soil dam slope taking into account the anisotropy.

In the dissertation [17], the implementation methods are developed and compared concerning one-dimensional models of water runoff along river slopes and the channel network based on the finite difference methods and the finite element method; the behavior patterns are proposed, implemented, and studied for the difference schemes of numerical integration of two-dimensional models of water runoff along the surface of slopes with a topography of different complexity; the numerical integration method has been developed and verified for equations of vertical water transfer in soils based on a four-point implicit scheme; the model of water erosion during rainfall floods is proposed and implemented, which describes the processes of drip and plane erosion and the transfer of soil particles by water flow along the surface of river slopes and the channel network. The advantages of applying finite element schemes are also demonstrated concerning real catchments; effective algorithms for its application have been developed, various methods of combining models of rainwater runoff formation at various catchment schemes are proposed.

In the dissertation [18], a steam-assisted gravity drainage model is developed for the first time taking into account the law of filtration with the ultimate pressure gradient, which allows for describing the main development stages of the steam chamber, namely, its growth to the top of the stratum, horizontal expansion, and expansion of the steam chamber towards the stratum bottom; a method has been developed for determining the anisotropic stratum filtration parameters according to vertical interference test; a mathematical model has been developed for studying the stationary fluid flow to the radial system of horizontal wells in the anisotropic stratum taking into account the influence of hydraulic pressure losses on friction in wellbores; a semi-analytical model is created to describe the process of nonstationary fluid flow to a multisectional horizontal well equipped with inflow control valves and pressure sensors in isolated sections; short-time tests are designed for vertical wells with hydraulic fracture, imperfect wells, and horizontal wells; complex transfer functions are expressed, and amplitude-frequency and phase-frequency specifications are determined for the following systems: 'porous-fractured vertical well' and 'hydraulic fracture in layer of finite conductivity.'

According to the results obtained, many conceptual mathematical and computational models have been developed to forecast the process of groundwater filtration and migration of variable-saturated liquids in multilayer porous media. In contrast to the above works, in this research, the unsteady liquid flow in a three-layer porous media considers external sources that greatly affect the liquid level change.

2. Methods

For a numerical study, specific conditions have been considered. The unsteady flow to the vertical drain canals in a three-layer boundary stratum considers evaporation from the liquid stream's upper surface and the elastic regime in a low-conductive layer. The main bounded aquifer lies under a low-conductive cover stratum and has a low-conductive layer below, facilitating its connection with the bed rock. It is assumed that the A.N. Myatiyev-G.N. Girinskii theory is true for these conditions. It should be noted that in the main bounded aquifer only horizontal liquid migrations prevail, and in the cover and low-conductive layers only vertical migrations prevail, allowing for horizontal components of the flow rate to be omitted here.

Considering the above, the continuity equation for the cover layer has the following form:

$$\frac{\partial V}{\partial z} = 0. \tag{1}$$

For this purpose, V is the vertical component of the flowrate in the porous media.

Further, applying the Darcy Law and considering the condition of the continuity of the liquid heads on the cover layer bottom, gives the following:

$$\mu_в \frac{\partial H_1}{\partial t} = -K_в \frac{H - H_1}{H_1} + q + F; \tag{2}$$

and the following equation for the bounded aquifer based on the elastic regime theory including liquid migration from the cover and low-conductive layer

$$\mu \frac{\partial H}{\partial t} = \frac{\partial}{\partial x}\left(T \frac{\partial H}{\partial x}\right) + \frac{\partial}{\partial y}\left(T \frac{\partial H}{\partial y}\right) + K_в \frac{H_1 - H}{H_1} - K_n \frac{\partial H_2\left(x, y, -m, t\right)}{\partial z}; \tag{3}$$

with a low-conductive dam taking into account the elastic filtration mode will be written as follows:

$$a_n \frac{\partial H_2}{\partial t} = \frac{\partial^2 H_2}{\partial z^2}. \tag{4}$$

Here, H_1 is a liquid head in the top layer; $\mu_в$ is a free liquid loss or lack of saturation; $K_в$ is a filtration coefficient in the cover layer; H is a liquid head in the intralayer; q is the total infiltration characterizing the actual infiltration and evaporation from the liquid level in a porous medium; x and y are coordinates of the horizontal plane, μ is an elastic liquid loss coefficient; F is an external source; $T = mk$ is filtration conductivity; K is a filtration coefficient; m is the thickness of the middle aquifer layer; K_n is a filtration coefficient; $H_2(x,y,z,t)$ is a liquid head in the lower layer; z is a vertical coordinate; $a_n \frac{K_n m_n}{\mu_n}$ is a piezoelectric conductivity coefficient; μ_n is an elastic liquid loss coefficient; m_n is the thickness of the low-conductive layer (Fig. 1).

3. Results and Discussion

In contrast with the study performed by F.B. Abutaliyev [19] and other authors, the external *F* source on the cover layer of the porus media was considered.

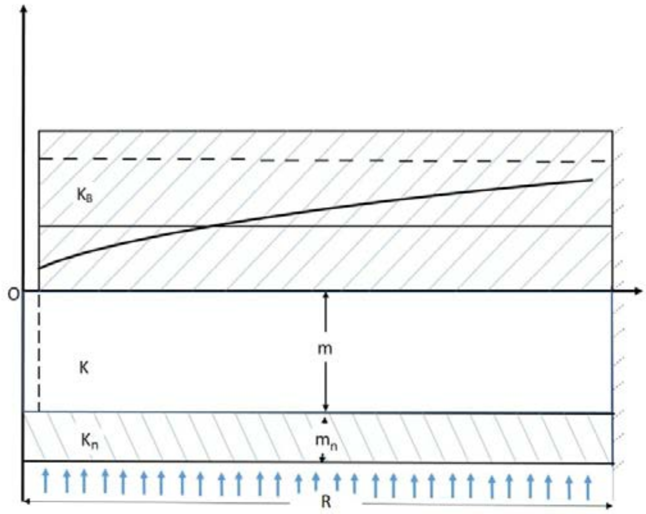

Figure 1. Schematic model of the flow in the three-layer porous medium.

Thus, the system of differential equations in partial derivatives (2)–(4) describes the flow process in a three-layer porous medium.

It should be noted that in the article [16], an analytical solution to the problem was given, with the linearization of equations (2) and (3) and condition $q = const$. However, it is necessary to emphasize that $q(x,y,H_1,t)$ is a function of coordinates x, y of the level (H_1) and time (t).

This function depends on H_1 as follows. If the liquid level in a porous medium is on the upper horizontal surface of a porous medium, then the function takes maximum values. If the liquid level in a porous medium drops below a critical depth, then the function equals zero.

This monotonically decreasing function reaches its maximum value when H_1 coincides with the upper horizontal surface of a porous medium and asymptotically tends to zero at $H_1 \to H_{kp}$, where H_{kp} is a critical value of the liquid level in a porous medium. Thus, $q \equiv 0$ below this critical depth. The value q in such assumptions denotes evaporation from the liquid level in a porous medium.

In the paper [20], the following relation is used to calculate the value q:

$$q = q_0\left(1 - \frac{m_b - H_1}{m_b - H_{k\rho}}\right)^n. \tag{5}$$

Here, q_0 is the intensity of evaporation on the upper horizontal surface of a porous medium; H_{kp} is the critical subsurface liquid depth; n is an exponent that depends on external factors and the subsurface liquid depth.

It should be noted that, in the article [21], evaporation from the liquid level in a porous medium for a single-layer stratum model was considered at $n = 1$.

According to the study, the general case of the evaporation task in form (4) for the multilayer stratum model has no solution. It should be noted that, in general, it is difficult and rather impossible to formulate an analytical solution to the problem of unsteady filtration for the system (2)–(3). Therefore, it is advisable to apply the finite difference method to integrate the nonlinear system of equations (2)–(4), taking into account the evaporation in the form (5), and the external source F.

Let us consider the liquid flow to the vertical drainage well drilled into the main aquifer in a limited circular stratum with $r \leqslant R_k$, considering the evaporation in the form (5)

It is assumed that the well is located in the stratum center. Then, due to the flow symmetry in (2)–(4), the system of equations considering the liquid intake to the upper surface of the liquid flow:

$$F+q_0\left(1-\frac{m_b-H_1}{m_b-H_{k\rho}}\right)^n-\mu_b\frac{\partial H_1}{\partial t}=K_b\frac{H_1-H}{H_1}; \quad (6)$$

$$\frac{1}{a}\frac{\partial H}{\partial t}=\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial H}{\partial r}\right)+\frac{K_b}{T}\frac{H_1-H}{H_1}-\frac{K_b}{T}\frac{\partial H_2\left(r,-m,t\right)}{\partial z}; \quad (7)$$

$$\frac{1}{a_n}\frac{\partial H_2}{\partial t}=\frac{\partial^2 H_2}{\partial z^2}. \quad (8)$$

With initial and boundary conditions based on the following:

$$H_1\left(r,0\right)=H\left(r,0\right)=H_2\left(r,z,0\right)=H_0; \quad (9)$$

$$\frac{\partial H\left(R_c,t\right)}{\partial r}=\frac{Q_c}{2\pi TR_c}; \quad (10)$$

$$\frac{\partial H\left(R_k,t\right)}{\partial r}=0; \quad (11)$$

$$H_2\left(r,-m,-m_n,t\right)=H_0; \quad (12)$$

$$H_2\left(r,-m,t\right)=H\left(z,t\right). \quad (13)$$

To integrate the system (6)–(8) numerically into the conditions (9)–(13), the nondimensional variables by the following formulas proceeded:

$$U=\frac{H_1}{H_{xa\rho}};\ V=\frac{H}{H_{xa\rho}};\ W=\frac{H_2}{H_{xa\rho}};\ s=\ell\,\mathrm{n}\frac{r}{R_k};\ z=m_b\zeta;$$

$$t=\frac{{R_k}^2}{a}\tau;\ Q=2\pi TH_{xa\rho}Q^*.$$

Together with the equations (6)–(13), the equations became as following:

$$F+P_0\left(1-\frac{1-\lambda U}{1-U_{k\rho}}\right)^n-\gamma\frac{\partial U}{\partial\tau}=\frac{U-V}{U}; \quad (14)$$

$$\frac{\partial V}{\partial\tau}=\ell^{-2s}\frac{\partial^2 V}{\partial s^2}+\alpha\frac{U-V}{U}-\beta\frac{\partial W\left(s,-\frac{m}{m_b},\tau\right)}{\partial\zeta}; \quad (15)$$

$$\delta\frac{\partial W}{\partial\tau}=\frac{\partial^2 W}{\partial\zeta}; \quad (16)$$

$$U\left(s,0\right)=V\left(s,0\right)=W\left(s,\zeta,0\right)=W_0; \quad (17)$$

$$\frac{\partial V\left(s_c,\tau\right)}{\partial s}=Q^*; \quad (18)$$

$$\frac{\partial V\left(0,\tau\right)}{\partial s}=0; \quad (19)$$

$$W\left(s,-\frac{m+m_n}{m_b},\tau\right)=W_0; \tag{20}$$

$$W\left(s,-\frac{m}{m_b},\tau\right)=V\left(s,\tau\right); \tag{21}$$

where

$$\alpha=\frac{K_b R_k{}^2}{TH_{ka\rho}};\ \beta=\frac{K_n}{T}\frac{R_k{}^2}{m_b};\ \gamma=\frac{a\mu_b}{K_b R_k{}^2}H_{kar};\ \delta=\frac{am_b{}^2}{a_n R_k{}^2};\ \lambda=\frac{H_{xa\rho}}{m_b};$$

$$\rho_0=\frac{q_0}{K_b};\ W_0=\frac{H_0}{H_{xa\rho}};\ s_c=\ell n\frac{R_c}{R_k};\ Q^*=\frac{Q}{2\pi TH_{xa\rho}}.$$

According to the analysis of material balance numerical solutions, it is found that the nondimensional spatial variable *s* must be taken in the form indicated above since only in this case, the solution outcomes in the vicinity of the well R_c are correctly considered. For example, if $s=\frac{r}{R_k}$, then the error in the balance ratio can be about 20 %.

An implicit finite difference scheme is applied to numerically solve the system of nonlinear equations (14)–(16) with additional conditions (16)–(21).

The segment (s_0, 0) was split into m equal parts with an increment of Δs. Then $s_i = s_0 + i\Delta s$, i = 0, 1, 2, …, m.

Due to condition (21), equation (16) must be solved $m-1$ time along the lines parallel to the *z*-axis passing through the points s_i, *I = 0, 1, 2, …, m-1*. To solve this equation with the finite difference method, the segment

$$\left(-\frac{m}{m_b},-\frac{m+m_n}{m_b}\right)$$

is to be divided into ℓ equal segments with the points

$$\zeta_j=-\frac{m+m_n}{m_b}+j\Delta\zeta;\ j=0,1,2,\ldots,l,\ \Delta\zeta=const.$$

The uniform time increment $\Delta\tau$ was introduced. As for the system of equations (14)–(16) with any of the spatial and temporal points $\{s_1,\zeta_j,k\Delta\tau\}$, $i = 0, 1, 2, \ldots, m-1$, $j = 0, 1, 2, \ldots, l-1$, $k = 1, 2, \ldots$, a stable implicit finite difference scheme with accuracy was compiled as follows

$$o\left[\left(\Delta s\right)^2+\left(\Delta\zeta\right)^2+\Delta\tau\right]$$

$$V_{i,k}-V_{i,k-1}=\theta\ell^{-2S_i}\left(V_{i+1,k}-2V_{i,k}+V_{i-1,k}\right)+\alpha\Delta\tau\frac{U_{i,k}-V_{i,k}}{U_{i,k}}+\frac{\beta\Delta\tau}{2\Delta\zeta}\left(-3W_{i,\ell,k}+4W_{i,\ell-1,k}-W_{i,\ell-2,k}\right); \tag{22}$$

$$W_{i,j,k}-W_{i,j,k-1}=\chi\left(W_{i,j+1,k}-2W_{i,j,k}+W_{i,j-1,k}\right); \tag{23}$$

$$\frac{\gamma}{\Delta\tau}\left(U_{i,k}-U_{i,k-1}\right)=\rho_0\left(1-\frac{1-\lambda U_{i,k}}{1-U_{k\rho}}\right)^n-\frac{U_{i,k}-V_{i,k}}{U_{i,k}}. \tag{24}$$

Here

$$\theta = \frac{\Delta\tau}{(\Delta s)^2};\ U_{i,k} = U(s_i, k\Delta\tau);\ \chi = \frac{\Delta\tau}{(\Delta\zeta)^2}$$

$$V_{i,k} = V(s_i, k\Delta\tau);\ W_{i,j,k} = W(s_i, \zeta_j, k\Delta\tau);$$

$$j = 1, 2, \ldots, \ell - 1,\ i = 1, 2, \ldots, m - 1,\ k = 1, 2, \ldots.$$

Considering (20), the equation (23) was written as follows

$$W_{i,j,k} = A_{i,j+1,k} W_{i,j+1,k} + B_{i,j+1,k}, \tag{25}$$

where the sweep coefficients are defined as follows:

$$A_{i,j+1,k} = \frac{\chi}{1 + \left(2 - A_{i,j,k}\right)\chi};\ B_{i,j+1,k} = \frac{W_{i,j,k-1} + \chi B_{i,j,k}}{1 + \left(2 - A_{i,j,k}\right)\chi};$$

$$A_{i,1,k} = 0;\ B_{i,1,k} = W_0;$$

$$j = 1, 2, \ldots, \ell - 1.$$

In the equation (22), *W* was substituted with its value from (25), taking into account the conditions (18) and (22), assuming $U_{i,k}$ known. The equation will be rewritten in the following form

$$V_{i,k} = C_{i+1,k} V_{i+1,k} + D_{i+1,k}. \tag{26}$$

Where

$$C_{i+1,k} = \frac{\theta \ell^{-2S_i}}{1 + \left(2 - C_{i,k}\right)\theta \ell^{-2S_i} + \frac{\alpha\Delta\tau}{U_{i,k}} - \frac{\beta\Delta\tau}{2\Delta\zeta}\left[\left(4 - A_{i,\ell-1,k}\right)A_{i,\ell,k} - 3\right]};$$

$$D_{i+1,k} = \frac{V_{i,k-1} + \alpha\Delta\tau + \frac{\beta\Delta\tau}{2\Delta\zeta}\left[\left(4 - A_{i,\ell-1,k}\right)B_{i,\ell,k} - B_{i,\ell-1,k}\right] + \theta \ell^{-2S_i} D_{i,k}}{1 + \left(2 - C_{i,k}\right)\theta \ell^{-2S_i} + \frac{\alpha\Delta\tau}{U_{i,k}} - \frac{\beta\Delta\tau}{2\Delta\zeta}\left[\left(4 - A_{i,\ell-1,k}\right)A_{i,\ell,k} - 3\right]};$$

$$C_{i,k} = 1 - \frac{\ell^{2S_1}}{2\theta}\left\{1 + \frac{\alpha\Delta\tau}{U_1} - \frac{\beta\Delta\tau}{2\Delta\zeta}\left[\left(4 - A_{i,\ell-1,k}\right)A_{i,\ell,k} - 3\right]\right\};$$

$$D_{1,k} = -\Delta s Q^* + \frac{\ell^{2S_1}}{2\theta}\left\{V_{i,k-1} + \alpha\Delta\tau + \frac{\beta\Delta\tau}{2\Delta\zeta}\left[\left(4 - A_{i,\ell-1,k}\right)B_{i,\ell,k} - B_{i,\ell-1,k}\right]\right\};$$

$$i = 1, 2, \ldots, m - 1.$$

Out of equation (26), considering condition (19), the formula got like this:

$$V_{m,k} = V_{m-1,k} = \frac{D_{m,k}}{1 - C_{m,k}}. \tag{27}$$

Thus, once $C_{i,k}$ and $D_{i,k}$ are calculated according to the recurrent ratios above and $V_{m-2,k}, \ldots, V_{0,k}$ is sequently determined, it will be possible to finally solve the following based on the formula (25)

$$W_{i,j,k} : W_{i,\ell,k}, \ldots, W_{i,1,k}.$$

It is assumed that $U_{i,k}$. Then, for example, assuming that $U_{i,k} = U_{i,k-1}$, the first approximation $V_{i,k}^{(I)} = W_{ijk}^{(I)}$ is known. The $U_{i,k}^{(I)}$ is found by substituting $V_{m,k}^{(I)}$ into equation (14) and integrating it, for example, by the Adams-Störmer method over $[(k-1)\Delta\tau,\ k\Delta\tau]$. The second approximation $V_{i,k}^{(2)}$ is determined by it substituted into (26).

The process will be finished when the following iterative process reproducibility condition is met

$$\max\left|V_{i,k}^{(p+1)} - V_{i,k}^{(p)}\right| < \varepsilon,$$

where $\varepsilon > 0$ is a small value of the calculation error.

Analysis of the numerical calculations performed according to the above algorithm demonstrates that it is advisable to apply damping according to the sequence $V_{i,k}^{(p)}$ with regard to the formula

$$V_{i,k}^{(\rho)} = \upsilon \overline{V}_{i,k}^{(\rho)} + (1-\upsilon) V_{i,k}^{(\rho-1)}, \tag{28}$$

where $\overline{V}_{i,k}^{(p)}$ is the solution of (2.33). That is, if the sequence $\overline{V}_{i,k}^{(p)}$ tends to be 'inconsistent,' the equation (28) stabilizes it.

To assess the accuracy of the numerical solution, the method of balance equations is used. This equation is deriveted as follows. The equation (7) can be written in nondimensional form

$$\xi\frac{\partial V}{\partial\tau} = \frac{\partial}{\partial\xi}\left(\xi\frac{\partial V}{\partial\xi}\right) + \alpha\rho_0\xi\left(1-\frac{1-\lambda U}{1-U_{k\rho}}\right)^n - \alpha\gamma\xi\frac{\partial U}{\partial\tau} - \beta\xi\frac{\partial W\left(\xi,-\frac{m}{m_b},\tau\right)}{\partial\xi}, \tag{29}$$

where $\xi = \dfrac{r}{R_k}$

The equation (29) is integrated throughout the pore space of the aquifer stratum. Then, due to *m* = *const*, the integrated form is as follows

$$\int_{\xi_c}^{1}\xi\frac{\partial V}{\partial\tau}d\xi = -Q^* + \alpha\rho_0\int_{\xi_c}^{1}\xi\left(1-\frac{1-\lambda U}{1-U_{k\rho}}\right)^n d\xi - \alpha\gamma\int_{\xi_c}^{1}\xi\frac{\partial W\left(\xi,-\frac{m}{m_b},\tau\right)}{\partial\xi}d\xi. \tag{30}$$

The notation was introduced:

$$V_{c\rho}(\tau) = \int_{\xi_c}^{1}\xi V(\xi,\tau)\,d\xi; \tag{31}$$

$$U_{c\rho}(\tau) = \int_{\xi_c}^{1}\xi U(\xi,\tau)\,d\xi; \tag{32}$$

Then equation (30) can be written as follows

$$\frac{dV_{c\rho}}{dt} = -Q^* + q_{ucn}(\tau) - \alpha\gamma\frac{dU_{c\rho}}{dt} - q_n. \tag{33}$$

For this purpose,

$$q_{ucn}(\tau) = \alpha\rho_0\int_{\xi_c}^{1}\xi\left(1-\frac{1-\lambda U}{1-U_{k\rho}}\right)^n d\xi; \tag{34}$$

$$q_n(\tau) = \beta\int_{\xi_c}^{1}\xi\frac{\partial W\left(\xi,-\frac{m}{m_b},\tau\right)}{\partial\xi}d\xi. \tag{35}$$

Integrating (33) by τ gives the follows

$$V_{c\rho}(\tau)=V_{c\rho}(0)-Q^{*}\tau+\int_{0}^{\tau}q_{ucn}(\tau)d\tau-\alpha\gamma\left[U_{c\rho}(\tau)-U_{c\rho}(0)\right]-\int_{0}^{\tau}q_{n}(\tau)d\tau. \qquad (36)$$

That the ratio is called the balance equation. The following is proceed to assess the accuracy of the numerical solution to this equation. The numerical solution corresponding to the moment of time (τ) is averaged by formulas (31) and (32). Then the evaporation and liquid migration is calculated by formulas (34) and (35) further substituting these values in (36). Then an approximate equation denoting the accuracy of the approximate solution averaged over the entire porous space is as follows

$$V_{c\rho}(\tau)+Q^{*}\tau+\alpha\gamma U_{c\rho}(\tau)-\int_{0}^{\tau}q_{ucn}(\tau)d\tau+\int_{0}^{\tau}q_{n}(\tau)d\tau\approx V_{c\rho}(0)-\alpha\gamma U_{c\rho}(0). \qquad (37)$$

The problem is considered with the following data to illustrate the abovementioned algorithm:

R_0 = 10 cm, R_K = 500 m, m_b = 40 m,	Q_c = 1,250 m^3/day,
m = 100 m, m_n = 10 m, H_0 = 39 m,	a = 10^6 m^2/day,
n = 0, 1, 2, 3;	a_n = 10^2 m^2/day,
q_0 = 0, 0.0036, 0.036 m/day	K = 0 m/day,
T = 10^3 m^2/day, μ_b = 0.1,	K_n = 0.01 m/day
H_{kp} = 37 m	K_b = 1 m/day

In this example, the vertical filtration coefficients vary greatly. The ratio of the filtration coefficients in the cover and high-conductive layers is $\frac{K}{K_6}=10$, while the ratio of these coefficients in the high-conductive layer and the low-conductive dam is $\frac{K}{K_6}=1,000$. Until recently, this fact suggested that low-conductive layers containing liquid are incompressible or, at best, have elastic reserves. However, it is assumed them to be completely negligible. Studies [22] have shown that layers can give a significant amount of liquid, even the low-conductive ones. The liquid saved against other reserves reduces the efficiency of vertical drain canals (wells), increasing the running time at the liquid level in a porous medium.

Figures 2–4 show the results of calculations regarding the liquid level in the cover layer for different parameters n and evaporation q_0 values. These figures illustrate how the parameter n and evaporation q_0 affect the distribution of the liquid level in a porous medium.

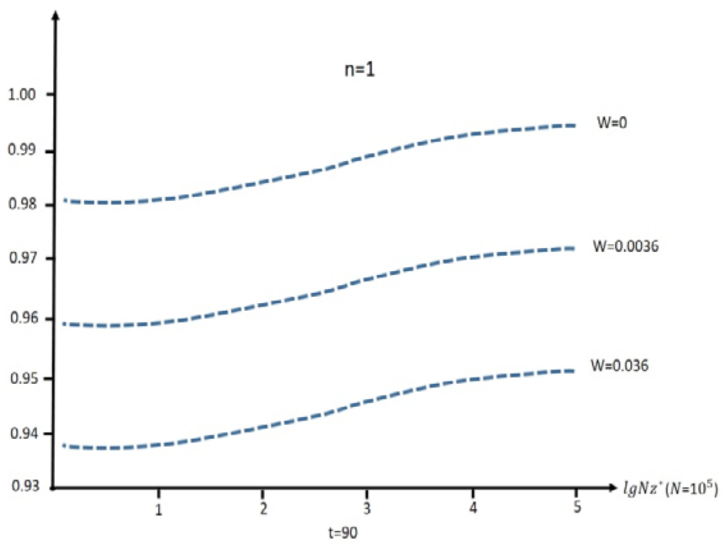

Figure 2. Change in the value of the liquid level in a porous medium at the cover layer at *n = 1* and different evaporation values q_0.

According to the curves in Fig. 2, the change in the liquid table value significantly depends on the evaporation value. As the evaporation value increases, the liquid level in a porous medium drops proportionally over time.

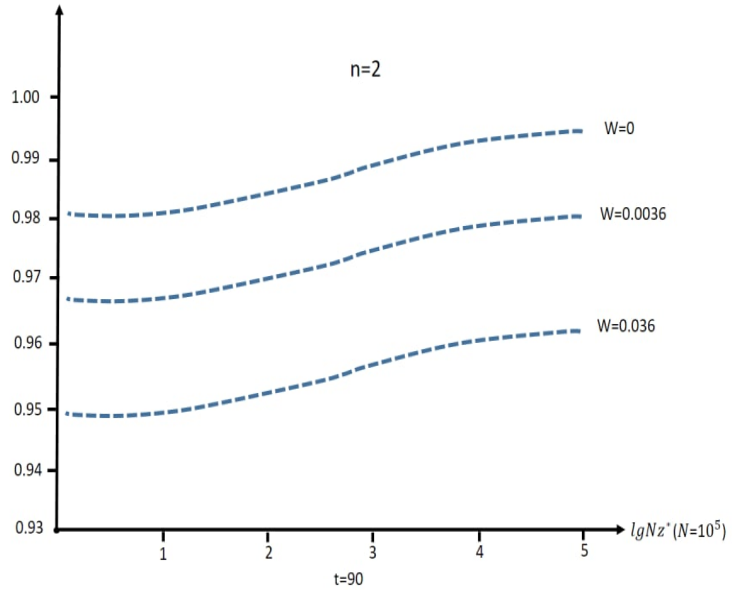

Figure 3. Change in the value of the liquid level in a porous medium in the cover layer at *n = 2* and different evaporation values q_0.

According to the numerical calculations obtained and the curves in Fig. 3 and 4, it can be seen that the change in the value of the liquid level in a porous medium also depends on the change in the parameter *n*. As it increases, the liquid level in a porous medium drop at different evaporation values [23].

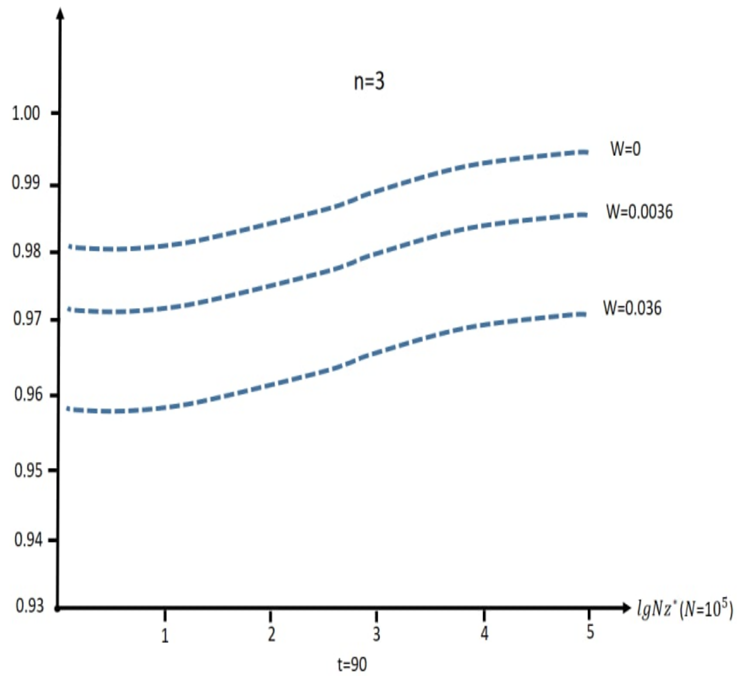

Figure 4. Change in the value of the liquid level in a porous medium in the cover layer at *n = 3* and different evaporation values q_0.

Fig. 5–7 represent flow curves reduced to an area unit with a cover layer and a low-conductive layer for different evaporation parameters q_0 and *n* values. Liquid migrations depend on evaporation parameters.

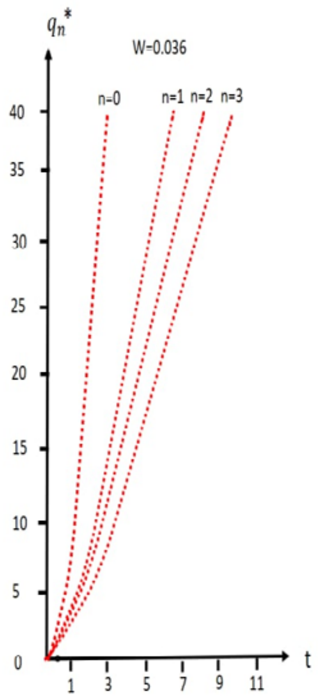

Figure 5. Flow curves reduced to an area unit with a cover and low-conductive layers at q_0 = 0.036.

Fig. 2–7 conclude that evaporation from the liquid level in a porous medium significantly affects the liquid migration distribution in the layers and the liquid level in a porous medium distribution. Thus, one must consider evaporation when designing pumped-well drain canals in multilayer medium to improve lands and study liquid flows in layers.

The paper verified the accuracy of the numerical solution according to the balance equation. It appears that the error margin is limited by 1.3 %.

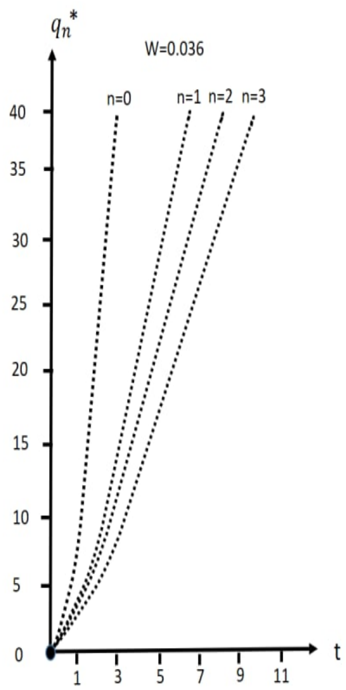

Figure 6. Flow curves reduced to an area unit with a cover and low-conductive layers at q_0 = 0.0036.

4. Conclusions

The computational experiments have established that the dynamics of changes in the liquid level in a porous medium depends significantly on the evaporation parameter. With an increase in its value, the liquid level in a porous medium decreases proportionally over time.

An analysis of the obtained numerical calculations showed that the change in the liquid level in a porous medium depends significantly on the parameter n. As its value increases, the liquid level in the porous medium decreases at different values of the evaporation parameter.

The analyzing results of numerical calculations show that evaporation from the surface of a liquid in a porous medium significantly affects the distribution of overflows in the layers and the distribution of the liquid level in a porous medium. Therefore, when designing vertical drains to improve fluid selection in multilayer reservoirs and designing liquid flows in layers, it is essential to take into account evaporation.

Checking the accuracy of the numerical solution using the balance equation showed that the error does not exceed 1.3 %.

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