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Prismatic face slope piles operating under frost heaving

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Abstract. The effectiveness of piles with reverse surface slope in frost heaving of soil has been the subject of discussion in many papers. In the previous works, the author considered cylindrical piles with an upper reverse taper and calculation method for the piles under these conditions. However, in order to extend the area of their use it is necessary to consider other configurations of piles as well. In this study, prismatic face slope piles are modeled for soil frost heaving conditions; equilibrium equations for calculating prismatic piles with four, six and eight faces are derived. The equilibrium equation for prismatic face slope piles in general form is also given. The equations make it possible to determine geometric parameters of piles ensuring their stability in soil under the action of tangential frost heaving forces. The author analyzes material capacity of cylindrical taper piles and prismatic face slope piles. The piles have the same bearing capacity in thawed soil and operate under the same geological and climatic conditions set before. The square pile with a sloping face shows the lowest material capacity. The proposed approach can be used for prismatic piles with a different number of faces in various conditions.

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1. Introduction

The effectiveness of a pile under frost heaving is characterized by its stability, which can be achieved through the equilibrium of the acting forces. This equilibrium prevents the pile from being lifted by frost heaving forces, which contributes to the integrity of the structure above. The pile uplift can be avoided in various ways. The first way is to increase the load on the pile by changing the weight of the structures above. However, this might lead to unjustified cost increases. Another way is to use piles that reduce the impact of frost heaving forces on the structures. The design of such piles involves creation of surface slope resulting in additional restraining forces. The latter counteract the frost forces that cause the piles to rise.

The surface slope for a cylindrical pile can be created by means of a truncated taper at the top of the pile. The bottom base of the taper corresponds to the diameter of the pile, which creates the surface slope where forces are formed to counteract the frost heaving. The author of the paper [1] has developed a pile with an upper reverse taper.

If the pile is prismatic and has a square cross-section, the necessary counter-bulging forces can be obtained by shaping the upper section of the pile into a pyramid, with the upper square base being smaller. This makes it possible to create a slope on the side faces of the pile resulting in counter-bulging forces. Prismatic piles can have a cross-section with a different number of faces.

The effectiveness of surface slope piles has been verified by numerous experimental studies and practical use. V.F. Zhukov [2] described the practice of using piles with an expanded base in harsh climatic conditions of Magadan. K.A. Linell and E.F. Lobacz [3] reviewed the practice of applying foundations in frozen soil. They gave recommendations on designing trapezoidal foundations with expansion in the lower part and on pile foundations with a developed base in the form of an anchor plate. P.A. Abbasov and A.A. Kovalevskii [4] developed piles with ribbed surfaces, which they recommended to use in frost heaving soil conditions. F.G. Gabibov [5] confirmed the effectiveness of such piles in soils increasing in volume. O.P. Medvedeva, I.N. Kazakov and N.F. Bulankin [6] analyzed the performance of pyramidal-prismatic piles in conditions of Siberian climate. The prismatic part of the pile made it possible to use such piles in conditions of freezing and frost heaving. B.S. Yushkov and his colleagues [7–8] conducted research into the performance of the double taper pile (‘double cone pile’) in seasonally freezing soils. They proved the effectiveness of such a pile under soil frost heaving due to the upper reverse taper. S.V. Feshchenko, A.V. Veshkurtsev, G.B. Barskaya [9] proposed to increase the cross-section of the lower part of the pile in order to anchor it in permafrost. L. Domaschu k[10–11] studied foundations with expanded bases and their performance in heaving soils. In his paper [10] he considered a tower foundation consisting of inclined elements forming a truncated pyramid. The test results indicated that the influence of frost heaving on the inclined elements decreased as the angle of inclination increased. X. Huang, Y. Sheng and others [12–13] tested bell-shaped piles in freezing soil. They found increased resistance of such piles to lifting caused by tangential forces of frost heaving. M. Schafer and S.P. Madabhushi [14] conducted experimental studies of small pile models with an expanded base in soils increasing in volume. The results showed that the expanded base increased the resistance of piles to uplift, which is efficient under frost heaving. Z. Zhu, L. Han [15] proposed a taper-cylindrical foundation for a tower in frozen soil. D.C. Sego, K.W. Biggar and G. Wong [16] extended traditionally used investigation methods for straight piles to bell-shaped piles under permafrost. They pointed out that the bearing capacity of a pile in permafrost can be increased by base expansion. Thus, experimental studies reliably confirm the effectiveness of surface slope piles under frost heaving conditions.

However, practical design of such piles requires calculation methods taking into account frost heaving forces and other factors. The latter include climatic and hydrogeological conditions and also the thermal regime of the soil. The existing calculation regulations are based on the theory of elasticity and Fourier laws related to thermal conductivity of frozen soil. Calculation models of soil frost heaving have been developed by Russian and foreign authors. V.M. Ulitsky, I.I. Sakharov, V.N. Paramonov and S.A. Kudryavtsev [17–18] published a series of papers where they proposed a mathematical model to determine the thermal characteristics and stress-strain state of freezing soil. S. Nishimura and others [19] presented a thermo-hydro-mechanical formulation for frost heaving suitable for assessment of foundation stability. J.H. Dong, X.L. Wu and others [20] developed a computational model of horizontal frost heaving. A.G. Alekseev [21] compared the simulation results obtained by means of various software tools with the analytical solution of the soil frost heaving problem. Based on their work, the author of the paper [22] has developed a method for calculating normal frost heaving forces. However, calculation of piles of complex configurations, including surface slope piles, under frost heaving requires consideration of additional factors.

Many scientists have worked on calculation methods for piles with reverse surface slope and other configurations. Among them are A.Z. Ter-Martirosyan and Z.G. Ter-Martirosyan [23], who used analytical and numerical methods to calculate interaction of a pile with an enlarged base with the surrounding soil. They obtained the dependence of stress-strain state of soil on pile geometric parameters. Their approach can be applied to evaluate pile operation both in thawed and in frozen soil. V.S. Sazhin [24] analyzed frost heaving forces and deformations for shallow foundations. He investigated foundations of trapezoidal section and truncated pyramid-shaped blocks [25]. V.S. Sazhin presented the sloping faces of a trapezoidal foundation as stepped ones in the calculation scheme. To calculate heaving deformations, V.S. Sazhin proposed a design model of freezing soil in the form of a cylinder, with a taper representing a pyramidal pile being placed in it [26]. He used the equation of axisymmetric thermal stress of the soil cylinder. Z.G. Ter-Martirosyan [27] considered the stress-strain state of a freezing cylindrical soil element surrounding the pile. X.Y. Xu and others [28] suggested calculating foundations by the finite difference method, taking into account the stiffness of the foundation that limits soil heaving. G.Q. Kong and others [29, 30] conducted analytical study of a taper pile with an expanded base. They analyzed side resistance of the pile, taper angle and diameter of the lower base by numerical simulation methods and obtained increased load-bearing capacity of the taper pile for its lifting and under the action of negative soil friction. This is of interest for piles under conditions of soil frost heaving and soil thawing. M. Schafer and S.P. Madabhushi [14] mathematically described the behaviour of expanded base piles in layered soils increasing in volume. This mechanism of pile-soil interaction may find its application in frost heaving conditions. P. He et al [31] obtained analytical solution for calculation of trapezoidal channel in frost heaving. H. Jiang and others [32] developed a method for designing a parabolic channel in frozen soil using digital technology. Wu Y. and others [33] calculated stresses and strains of soil around a single bridge pile during freezing and used non-linear finite-element model of pile-soil interaction. S. Jianzhong and others [34] presented a numerical three-dimensional model of a bridge pile foundation in permafrost. It is evident from their research that calculation methods for piles of complex configurations have received attention in scientific practice.

Piles with reverse surface slope, on the one hand, are effective in conditions of soil frost heaving due to their configuration, on the other hand, they require more complex approaches to their calculation than piles of constant section. B.S. Yushkov and others [35–36] gave an analytical quantitative assessment of the effect of the angle of surface slope on pile displacement. However, the angle in each case was not calculated, but was set from a limited number of values, as the pile was manufactured in factory conditions. So this method of selecting geometric parameters of piles is time-consuming. The author of the paper [37–38] has developed a method for calculating the pile with the upper reverse taper, which makes it possible not to select, but to calculate the geometric parameters of the pile with the least time and labor consumption. As the upper reverse taper pile is only one of the variants of piles with reverse surface slope, other pile configurations need to be considered for design practice.

The aim of the study is to extend the taper pile design method developed by the author to prismatic piles with sloping faces (prismatic face slope piles). To achieve the aim, the following tasks have been set:

- to model square, hexagonal and octagonal prismatic piles with the upper part in the form of truncated pyramid;
- to obtain equations with respect to geometric parameters of piles in the soil under the action of heaving forces;
- to estimate material capacity of piles.

2. Methods

A cylindrical pile with the upper reverse taper in frost heaving conditions was developed by the author [37–38] in previous works. However, a special case of a taper is a pyramid, i.e. taper is a pyramid with an infinite number of faces. This paper describes behavior of a cylindrical reverse taper pile and several prismatic face slope piles under frost heaving, the cross-sections of the prismatic ones being square, hexagonal and octagonal. The upper part of the prismatic piles is designed as a truncated pyramid. Calculation schemes for two cases of frost boundary position along the height of the piles are shown in Fig. 1.

In Fig. 1 S_f, T_{fi} are the frost heaving normal and tangential forces and F_i is the frictional force on the side of the pile in the thawed soil, respectively; P is the sum of the external load and the pile weight and α is the pile surface angle. The figure also shows the position of the inherent sections of the pile: z_0 is the bed of non-heaving material under the grillage; $z_{tp(pr)}$ is the bottom base of the taper (pyramid), z_{pl} is the base of the pile.

Table 1 shows the input data for the pile design, where the soil conditions are represented by a stiff clay loam with a liquidity index of 0.4.

Table 1. Initial data for piles calculation

№	Parameter name	Parameter designation	Parameter value
1	Frost heaving tangential stresses	τ_f	100 kPa
2	Frost heaving normal stresses	σ_f	200 kPa
3	Sum of the external load on the pile and the pile weight	P	130 kN
4	Side resistance of the pile in the thawed soil	f_i	26 kPa
5	Frost boundary position	ξ	2.1 m

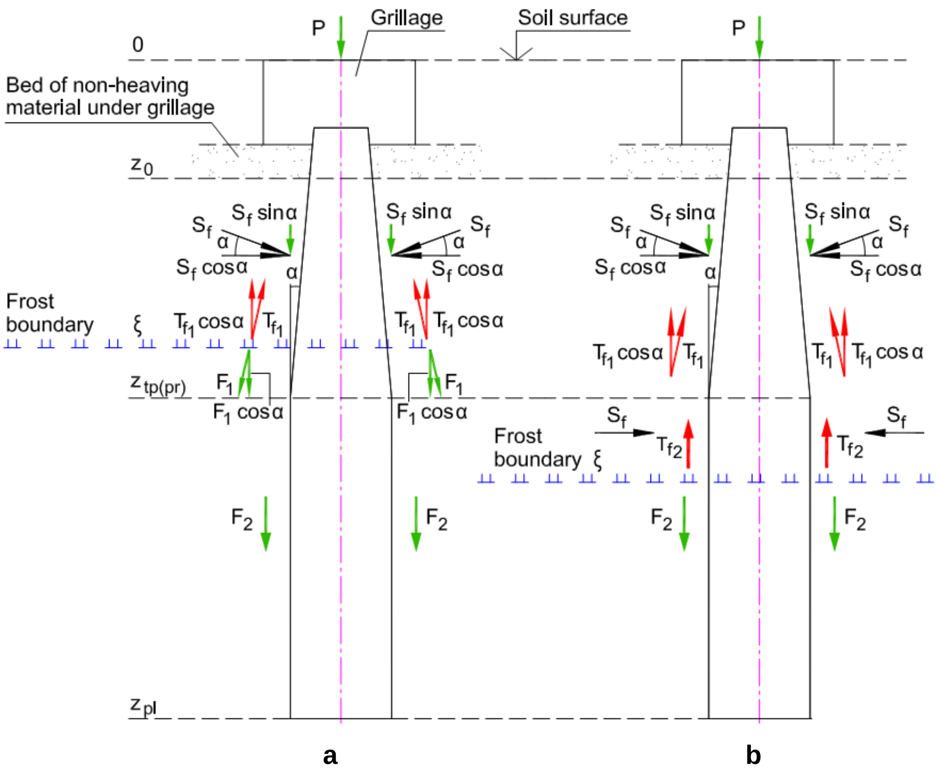

Figure 1. Two cases of frost boundary position of the pile:
a) frost boundary within the part of the pile with a surface slope;
b) frost boundary within the part of the pile of constant section.

2.1. Cylindrical pile with upper reverse taper

The bearing capacity of the cylindrical pile (Fig. 2a) for tangential frost heaving forces is 198.6 kN and tangential frost heaving forces is 231.2 kN. Thus, the tangential frost heaving forces that cause pile uplift exceed its bearing capacity and the condition of pile stability is not met. In this case we propose to specify the taper of the pile and determine the taper angle required for pile stability in the soil, the latter referring to the absence of vertical displacement, i.e. lifting. The taper angle is determined from the equation (1) obtained by the author [37–38].

Fig. 2b illustrates the cylindrical pile with an upper reverse taper and the coordinates of inherent sections. Fig. 2b shows a part of the taper L_{tp}, subject to the influence of frost heaving forces. R_{cl} is the radius of the cylindrical part of the pile.

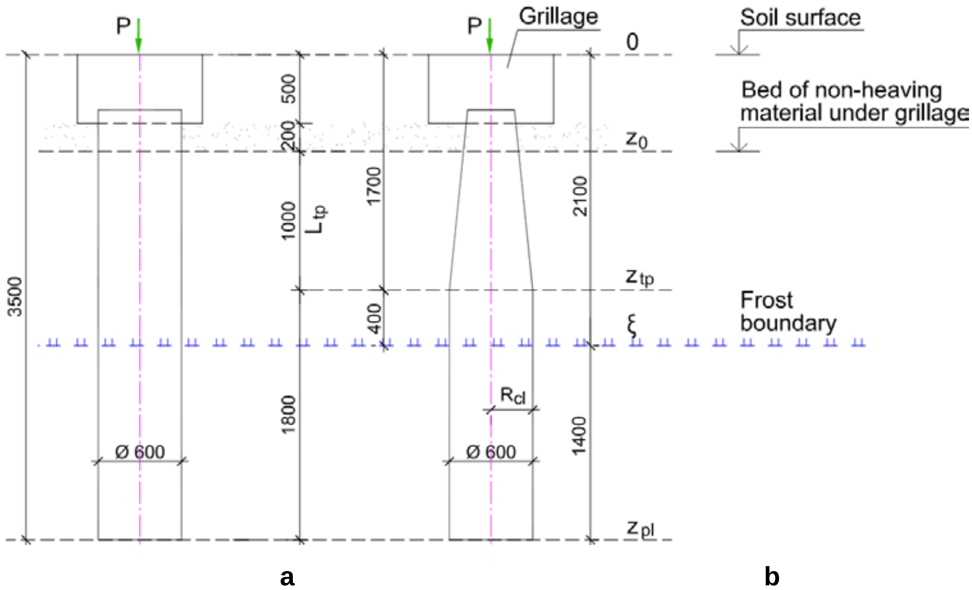

Figure 2. Piles with (a) constant section and (b) upper reverse taper, mm.

The equilibrium equation for the pile under the action of frost heaving forces [37–38] is illustrated in Fig. 1b and 2b.

$$0.5\sigma_f\left(z_{tp}-z_0\right)^2\left(\sin\alpha\right)^2+\left[-R_{cl}\sigma_f\left(z_{tp}-z_0\right)-0.5\tau_{f1}\left(z_{tp}-z_0\right)^2\right]\left(\sin\alpha\right)+$$
$$+R_{cl}\left[\tau_{f1}\left(z_{tp}-z_0\right)+\tau_{f2}\left(\xi-z_{tp}\right)-f_2\left(z_{pl}-\xi\right)\right]-0.5\pi^{-1}P=0. \quad (1)$$

The equation will make it possible to determine the taper angle required to ensure stability of the pile in the soil under frost heaving.

2.2. Prismatic square pile

A prismatic pile with a square cross section having sloping faces (hereinafter, prismatic square pile) is considered in the same soil conditions as the cylindrical taper pile discussed above, and their lengths are equal. Fig. 3 shows the pile and coordinates of its inherent sections. In Fig. 3 L_{pr} is a part of the pyramid subject to the influence of frost heaving forces; a_{pr} is the side of the upper base of the pyramid exposed to frost heaving forces; $a_{pr}(z)$ is the variable face width; a_{pl} is the side of the bottom base of the pyramid.

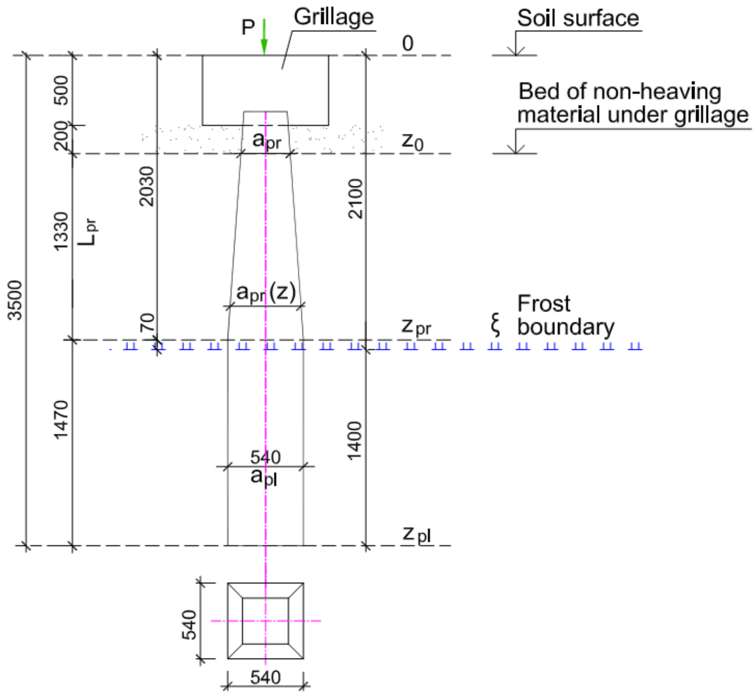

Figure 3. Prismatic square pile with sloping faces, mm.

Fig. 1b shows the forces acting on sloping and vertical faces of piles. The upper part of the prismatic pile is designed as a truncated pyramid.

The normal frost heaving force acting on the sloping faces of the pyramid (Fig. 1b) is presented as follows:

$$S_f=\int_{z_0}^{z_{pr}}\sigma_f dF_{pr}, \quad (2)$$

where F_{pr} is the side surface area (hereinafter surface area) of the pyramidal part of the pile.

The tangential frost heaving force acting on the sloping faces of the pyramid is presented by the equation

$$T_{f1}=\int_{z_0}^{z_{pr}}\tau_{f1}dF_{pr}. \quad (3)$$

And equation (4) presents tangential frost heaving force acting on vertical faces of the constant-section pile in the frozen zone.

$$T_{f2} = \int_{z_{pr}}^{\xi} \tau_{f2} dF_{pl}, \tag{4}$$

where F_{pl} is the side surface area (hereinafter surface area) of the pile part with constant cross-section.

The frictional force on the side of constant section pile in the thawed soil is as follows:

$$F_2 = \int_{\xi}^{z_{pl}} f_2 dF_{pl}. \tag{5}$$

Equation for the face slope angle of the square pile is based on equilibrium of acting forces. The equilibrium equation at frost boundary position $z_0 < z_{pr} < \xi$ (see Fig. 1b) is as follows:

$$-P - S_f \sin\alpha + T_{f1}\cos\alpha + T_{f2} - F_2 = 0. \tag{6}$$

Supposing that at small angles of face slope $\cos\alpha \approx 1,$ we rewrite the equation (6) as follows:

$$-P - S_f \sin\alpha + T_{f1} + T_{f2} - F_2 = 0. \tag{7}$$

Taking into account inherent sections in Fig. 1b and 3 and expressions (2–5), equation (7) will look like

$$-P - \left(\int_{z_0}^{z_{pr}} \sigma_f dF_{pr} \right) \sin\alpha + \int_{z_0}^{z_{pr}} \tau_{f_1} dF_{pr} + \int_{z_{pr}}^{\xi} \tau_{f_2} dF_{pl} - \int_{\xi}^{z_{pl}} f_2 dF_{pl} = 0. \tag{8}$$

The upper part of the square pile is designed as a truncated pyramid. Fig. 4 shows the projection of the pyramid face to the vertical plane, where z is an arbitrary coordinate; dF_{pr} is a variable area of the elementary stripe of the pyramid face. For small angles, it is assumed that the face area is equal to the area of its projection.

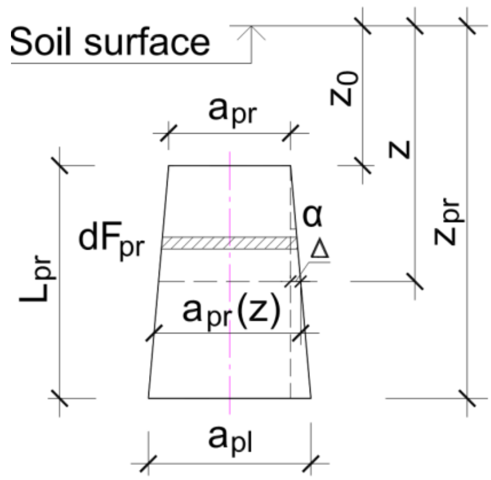

Figure 4. Pyramid face.

The variable area of the elementary stripe of the pyramid face, according to Fig. 4 is as follows:

$$dF_{pr} = 4a_{pr}(z)dz. \tag{9}$$

Its variable width is as follows:

$$a_{pr}(z) = a_{pr} + 2\Delta. \tag{10}$$

Taking into account $\Delta = (z - z_0) tg\alpha,$ and at small angles $tg\alpha \approx \sin\alpha,$ we obtain

$$a_{pr}(z) = a_{pr} + 2(z - z_0)\sin\alpha. \tag{11}$$

From the condition

$$\frac{(a_{pl}/2) - (a_{pr}/2)}{z_{pr} - z_0} = tg\alpha \approx \sin\alpha, \tag{12}$$

we have

$$a_{pr} = a_{pl} - 2(z_{pr} - z_0)\sin\alpha. \tag{13}$$

After substituting (13) in expression (11), we obtain

$$a_{pr}(z) = (a_{pl} - 2(z_{pr} - z_0)\sin\alpha) + 2(z - z_0)\sin\alpha. \tag{14}$$

After simplifying (14), the variable width of the elementary stripe will be written as the expression

$$a_{pr}(z) = a_{pl} - 2\sin\alpha(z_{pr} - z). \tag{15}$$

Taking into account (15), the variable area of the elementary stripe of the pyramid surface along its perimeter (9) will be

$$dF_{pr} = 4\left[a_{pl} - 2\sin\alpha(z_{pr} - z)\right]dz. \tag{16}$$

The elementary stripe area of the constant section pile along its perimeter is

$$dF_{pl} = 4a_{pl}dz. \tag{17}$$

Equation (8) after substituting (16) and (17) will be written as

$$\begin{gathered} -P - \left(\int_{z_0}^{z_{pr}} \sigma_f 4\left[a_{pl} - 2\sin\alpha(z_{pr} - z)\right]dz\right)\sin\alpha + \\ + \int_{z_0}^{z_{pr}} \tau_{f_1} 4\left[a_{pl} - 2\sin\alpha(z_{pr} - z)\right]dz + \int_{z_{pr}}^{\xi} \tau_{f_2} 4a_{pl}dz - \int_{\xi}^{z_{pl}} f_2 4a_{pl}dz = 0. \end{gathered} \tag{18}$$

After transformations, the equation (18) will take the form of a quadratic equation with respect to the sine of the face slope angle of the pyramidal part of the pile.

$$\begin{gathered} \sigma_f (z_{pr} - z_0)^2 (\sin\alpha)^2 + \left[-a_{pl}\sigma - \tau_{f_1}(z_{pr} - z_0)^2\right](\sin\alpha) + \\ + a_{pl}\left[\tau_{f_1}(z_{pr} - z_0) + \tau_{f_2}(\xi - z_{pr}) - f_2(z_{pr} - \xi)\right] - 0.25P = 0. \end{gathered} \tag{19}$$

The equation will make it possible to determine the taper angle required to ensure stability of the pile in the soil under frost heaving.

2.3. Prismatic hexagonal pile

A prismatic hexagonal pile with sloping faces (hereinafter prismatic hexagonal pile) is considered under the same soil conditions as the cylindrical taper pile discussed above and their lengths are equal. Fig. 5 shows the pile with the coordinates of the inherent sections. In Fig. 5 R_{cl} is radius of the circle inscribed in the bottom base of the pyramid, i.e. radius of the cylinder inscribed in the hexagonal constant-section of the pile.

Equation for the face slope angle of the hexagonal pile is based on equilibrium of acting forces. The equilibrium equation at frost boundary position $z_0 < z_{pr} < \xi$ (see Fig. 1b and 5) with regard to the inherent sections and expressions (2-5) is as follows:

$$-P-\left(\int_{z_0}^{z_{pr}} \sigma_f dF_{pr}\right)\sin\alpha+\int_{z_0}^{z_{pr}} \tau_{f_1} dF_{pr}+\int_{z_{pr}}^{\xi} \tau_{f_2} dF_{pl}-\int_{\xi}^{z_{pl}} f_2 dF_{pl}=0. \qquad (8)$$

The upper part of the pile is designed as a truncated hexagonal pyramid. The bottom base of the pyramid corresponds to a cross-section of a part of the pile with constant size. The angle of face slope for the truncated pyramid is calculated through the variable radius of the taper inscribed in the pyramid (Fig. 6). Fig. 6 shows: dF_{tp} is the variable surface area of the elementary strip of the inscribed taper; R_{tp} is the radius of the upper base of the inscribed taper subjected to soil frost heaving; $R_{tp}(z)$ is the variable radius of the taper; R_{cl} is the radius of the bottom base of the inscribed taper.

Radius of the circle inscribed in the bottom base of the pyramid, i.e. radius of the cylinder inscribed in the hexagonal section of the pile, is calculated as follows:

$$R_{cl}=\frac{a_{pl}}{2tg(\pi/6)}=285.8\text{ mm}, \qquad (20)$$

where a_{pl} is the face width of the hexagonal constant section pile (Fig. 5).

The variable area of the elementary stripe around the perimeter of the truncated hexagonal pyramid will be

$$dF_{pr}=P(z)dz, \qquad (21)$$

where $P(z)$ is the perimeter of the elementary pile stripe at its pyramidal part.

The perimeter of the elementary stripe of the pyramid surface is calculated through the radius of the inscribed circle.

$$P=2Rntg(\pi/n), \qquad (22)$$

where n is the number of faces of the pyramid; R is the radius of the inscribed circle.

The perimeter of the hexagonal pyramid is

$$P=6.928R. \qquad (23)$$

If the radius of the inscribed circle is assumed to be the variable radius of the taper inscribed in the pyramid, the area of the elementary stripe on the pyramidal part of the pile (21) will be as follows:

$$dF_{pr}=P(z)dz=6.928R_{tp}(z)dz, \qquad (24)$$

where $R_{tp}(z)$ is the variable radius of the taper inscribed in the pyramid (Fig. 6).

According to Fig. 6, the expression for the variable radius of the inscribed taper is presented as the equation

$$R_{tp}(z)=R_{tp}+\Delta. \qquad (25)$$

Taking into account $\Delta=(z-z_0)tg\alpha,$ we obtain the equation

$$R_{tp}(z)=R_{tp}+(z-z_0)tg\alpha. \qquad (26)$$

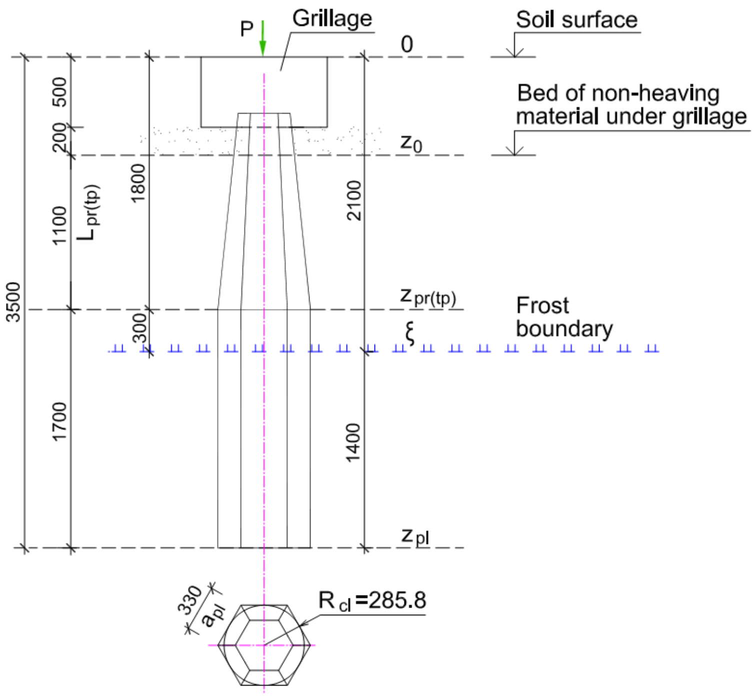

Figure 5. Prismatic hexagonal pile with sloping faces, mm.

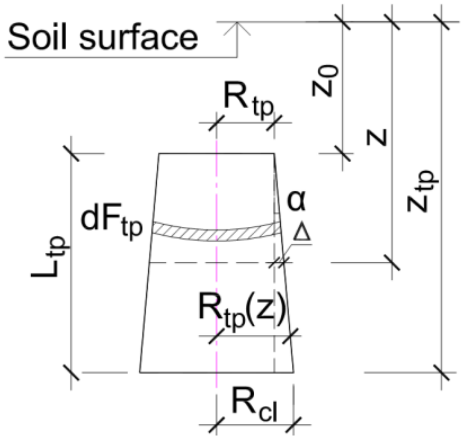

Figure 6. Taper inscribed in the pyramid.

Based on the condition

$$\left(R_{cl}-R_{tp}\right)/\left(z_{tp}-z_{0}\right)=tg\alpha, \tag{27}$$

the expression for radius of the inscribed taper will be as follows:

$$R_{tp}=R_{cl}-\left(z_{tp}-z_{0}\right)tg\alpha. \tag{28}$$

Then the expression for the variable radius of the taper (26) in view of (28) will be

$$R_{tp}\left(z\right)=\left[R_{cl}-\left(z_{tp}-z_{0}\right)tg\alpha\right]+\left(z-z_{0}\right)tg\alpha=R_{cl}-tg\alpha\left(z_{tp}-z\right). \tag{29}$$

Taking into account the variable radius of the inscribed taper (29), the expression for the area of the elementary stripe on the hexagonal pyramidal part of the pile (24) is written as

$$dF_{pr}=6.928\left(R_{cl}-tg\alpha\left(z_{tp}-z\right)\right)dz. \tag{30}$$

Given $tg\alpha\approx\sin\alpha$ for small angles, we obtain an expression for the area of the elementary stripe of the pyramid along its perimeter

$$dF_{pr} = 6.928\left(R_{cl} - \sin\alpha\left(z_{tp} - z\right)\right)dz. \tag{31}$$

where α is the angle of formatrix slope of the inscribed taper, R_{cl} is the radius of the circle inscribed in the hexagonal base of the pyramid.

As for the elementary surface stripe of the constant section pile, its area along the perimeter will be

$$dF_{pl} = 6a_{pl}dz. \tag{32}$$

Then equation (8) with account of (31) and (32) will be written as a quadratic equation with respect to the sine of the face slope angle of the pyramidal part of the pile.

$$\begin{gathered} 3.464\sigma_f\left(z_{tp} - z_0\right)^2\left(\sin\alpha\right)^2 + \\ +\left[-6.928R_{cl}\sigma_f\left(z_{tp} - z_0\right) - 3.464\tau_{f_1}\left(z_{tp} - z_0\right)^2\right]\left(\sin\alpha\right) + \\ + 6.928R_{cl}\tau_{f_1}\left(z_{tp} - z_0\right) + 6a_{pl}\left[\tau_{f_2}\left(\xi - z_{tp}\right) - f_2\left(z_{cl} - \xi\right)\right] - P = 0. \end{gathered} \tag{33}$$

The equation will make it possible to determine the taper angle required to ensure stability of the pile in the soil under frost heaving.

2.4. Prismatic octagonal pile

A prismatic pile with an octagonal cross section and sloping faces (hereinafter a prismatic octagonal pile) is considered in the same soil conditions as the cylindrical pile with a taper discussed above. The lengths of the prismatic pile and cylindrical one discussed above are the same. Fig. 7 shows the pile with coordinates of the inherent sections.

Equation for the face slope angle of an octagonal pile is based on equilibrium of acting forces. Taking into account inherent sections and expressions (2–5), the equilibrium equation at frost boundary position $z_0 < z_{pr} < \xi$ (see Fig. 1b) is as follows:

$$-P - \left(\int_{z_0}^{z_{pr}} \sigma_f dF_{pr}\right)\sin\alpha + \int_{z_0}^{z_{pr}} \tau_{f_1} dF_{pr} + \int_{z_{pr}}^{\xi} \tau_{f_2} dF_{pl} - \int_{\xi}^{z_{pl}} f_2 dF_{pl} = 0. \tag{8}$$

The upper part of the pile is designed as a truncated octagonal pyramid. The bottom base of the pyramid corresponds to a cross-section of the part with constant size. The angle of face slope of the pyramid is calculated through the variable radius of the taper inscribed in the pyramid (Fig. 6). Radius of the circle inscribed in the bottom base of the pyramid (Fig. 7) is calculated as follows:

$$R_{cl} = \frac{a_{pl}}{2tg\left(\pi/8\right)} = 298.7\ \text{mm}. \tag{34}$$

Variable surface area of the elementary stripe of the truncated pyramid is

$$dF_{pr} = P\left(z\right)dz. \tag{21}$$

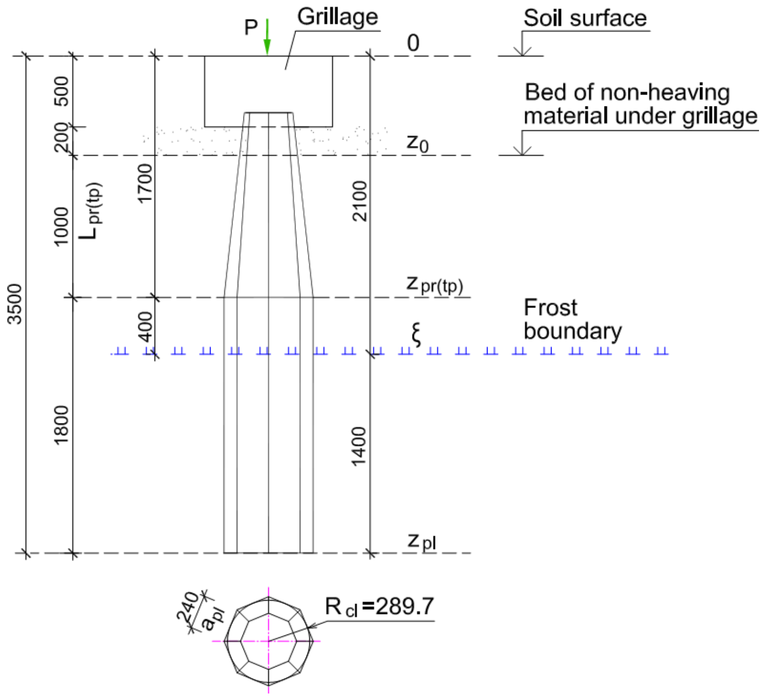

Figure 7. Prismatic octagonal pile with sloping faces, mm.

The perimeter of the elementary stripe of an octagonal pyramid is

$$P = 6.627R, \tag{35}$$

where R is the radius of the inscribed circle.

The radius of the circle inscribed will be considered as a variable radius of the taper inscribed in the pyramid. Then, taking into account the variable radius of the taper (21), the area of the elementary stripe on the pyramidal part of the pile is written as

$$dF_{pr} = P(z)dz = 6.627R_{tp}(z)dz. \tag{36}$$

When substituting the equation (29) in the expression (36) we obtain the equation

$$dF_{pr} = 6.627\left(R_{cl} - tg\alpha\left(z_{tp} - z\right)\right)dz. \tag{37}$$

Given $tg\,\alpha \approx \sin\alpha$ for small angles, the expression for the area of the elementary stripe of the pyramid along its perimeter can be written as

$$dF_{pr} = 6.627\left(R_{cl} - \sin\alpha\left(z_{tp} - z\right)\right)dz. \tag{38}$$

The area of the elementary surface stripe of the constant section pile along its perimeter is as follows:

$$dF_{pl} = 8a_{pl}dz. \tag{39}$$

where a_{pl} is the width of the face of the octagonal constant section pile.

Equation (8) with regard to (38) and (39) will be written as a quadratic equation with respect to the sine of the face slope angle of the pyramidal part pile.

$$\begin{gathered} 3.314\sigma_f\left(z_{tp} - z_0\right)^2(\sin\alpha)^2 + \\ + \left[-6.627R_{cl}\sigma_f\left(z_{tp} - z_0\right) - 3.314\tau_{f_1}\left(z_{tp} - z_0\right)^2\right](\sin\alpha) + \\ + 6.627R_{cl}\tau_{f_1}\left(z_{tp} - z_0\right) + 8a_{pl}\left[\tau_{f_2}\left(\xi - z_{tp}\right) - f_2\left(z_{cl} - \xi\right)\right] - P = 0. \end{gathered} \tag{40}$$

The equation will make it possible to determine the taper angle required to ensure stability of the pile in the soil under frost heaving.

2.5. Equation of face slope piles in general form

The equation for prismatic pile with sloping faces in general form is based on the equilibrium of all forces acting on the piles in the freezing soil.

$$-P-\left(\int_{z_0}^{z_{pr}} \sigma_f dF_{pr}\right)\sin\alpha+\int_{z_0}^{z_{pr}} \tau_{f_1} dF_{pr}+\int_{z_{pr}}^{\xi} \tau_{f_2} dF_{pl}-\int_{\xi}^{z_{pl}} f_2 dF_{pl}=0. \quad (8)$$

The upper part of the piles is designed as a truncated pyramid. The variable elementary surface area of the pyramid is

$$dF_{pr}=P(z)\,dz. \quad (21)$$

The perimeter of the elementary stripe of the pyramid surface through the radius of the inscribed circle is

$$P=2Rntg(\pi/n), \quad (22)$$

where n is the number of faces of the pyramid, R is the radius of the inscribed circle.

If the radius of the incircle is assumed to be the variable radius of the taper inscribed in the pyramid, then the expression (21) considering (22) will be as follows:

$$dF_{pr}=P(z)\,dz=2n\,tg(\pi/n)\,R_{tp}(z)\,dz, \quad (41)$$

where $R_{tp}(z)$ is the variable radius of the taper inscribed in the pyramid.

Then the area of the elementary stripe of the pyramidal part of the pile (41) with account of the variable radius of the taper (29) will be written as

$$dF_{pr}=2n\,tg(\pi/n)\left(R_{cl}-tg\alpha\left(z_{tp}-z\right)\right)dz. \quad (42)$$

where R_{cl} is the radius of the circle inscribed in the base of the pyramid.

Given $tg\alpha \approx sin\alpha$ for small angles, the expression (42) will be as follows:

$$dF_{pr}=2n\,tg(\pi/n)\left(R_{cl}-\sin\alpha\left(z_{tp}-z\right)\right)dz. \quad (43)$$

The elementary stripe area of a constant section pile along its perimeter will be

$$dF_{pl}=na_{pl}dz. \quad (44)$$

where a_{pl} is the face width of the constant section pile.

Equation (8) with (43) and (44) will be written as a quadratic equation with respect to the sine of the face slope angle of the pyramidal part of the pile.

$$\begin{gathered}ntg(\pi/n)\sigma_f\left(z_{tp}-z_0\right)^2(\sin\alpha)^2+\\+\left[-2ntg(\pi/n)R_{cl}\sigma_f\left(z_{tp}-z_0\right)-ntg(\pi/n)\tau_{f_1}\left(z_{tp}-z_0\right)^2\right](\sin\alpha)+\\+2ntg(\pi/n)R_{cl}\tau_{f_1}\left(z_{tp}-z_0\right)+\\+na_{pl}\left[\tau_{f_2}\left(\xi-z_{tp}\right)-f_2\left(z_{cl}-\xi\right)\right]-P=0.\end{gathered} \quad (45)$$

The equation will make it possible to determine the taper angle required to ensure stability of the pile in the soil under frost heaving.

3. Results and Discussion

The paper considers a cylindrical pile with an upper reverse taper and prismatic pile with sloping faces in the upper part, operating under frost heaving conditions. The cylindrical pile was developed by the author earlier. Prismatic piles with four, six and eight faces are modeled in this study. The upper part of the cylindrical pile is a truncated taper while that of the prismatic pile looks like a truncated pyramid with a different number of faces.

The author provides equations of equilibrium for prismatic piles with sloping faces, the equations being presented in integral form. These equations establish a relationship between the geometric parameters of the piles and the magnitudes and ratios of these forces, which makes it possible to determine the parameters of the piles. Considering the soil in the framework of elasticity theory, the integral values of forces within the given intervals are taken equal to their average values. In view of this and also sectional variability, the equilibrium equations are transformed into second-order equations with respect to geometric parameters of piles. To derive the second-order equations, the area of a truncated pyramid is used, the area being represented through the variable width of the faces and the variable radius of the taper inscribed in the pyramid. The calculation method developed by the author can be used for prismatic piles with any number of faces. The equations are given in Table 2.

Table 2. Pile equations.

Pile type	Equation regarding the geometric parameters of the pile	Equation number
Cylindrical pile with upper reverse taper	$0.5\sigma_f(z_{tp}-z_0)^2(\sin\alpha)^2 + +[-R_{cl}\sigma_f(z_{tp}-z_0)-0.5\tau_{f1}(z_{tp}-z_0)^2](\sin\alpha)+ +R_{cl}[\tau_{f1}(z_{tp}-z_0)+\tau_{f2}(\xi-z_{tp})-f_2(z_{pl}-\xi)]-0.5\pi^{-1}P=0.$	(1)
Prismatic octagonal pile	$3.314\sigma_f(z_{tp}-z_0)^2(\sin\alpha)^2 + +[-6.627R_{cl}\sigma_f(z_{tp}-z_0)-3.314\tau_{f1}(z_{tp}-z_0)^2](\sin\alpha)+ +6.627R_{cl}\tau_{f1}(z_{tp}-z_0)+8a_{pl}[\tau_{f2}(\xi-z_{tp})-f_2(z_{cl}-\xi)]-P=0.$	(40)
Prismatic hexagonal pile	$3.464\sigma_f(z_{tp}-z_0)^2(\sin\alpha)^2 + +[-6.928R_{cl}\sigma_f(z_{tp}-z_0)-3.464\tau_{f1}(z_{tp}-z_0)^2](\sin\alpha)+ +6.928R_{cl}\tau_{f1}(z_{tp}-z_0)+6a_{pl}[\tau_{f2}(\xi-z_{tp})-f_2(z_{cl}-\xi)]-P=0.$	(33)
Prismatic square pile	$\sigma_f(z_{pr}-z_0)^2(\sin\alpha)^2+[-a_{pl}\sigma-\tau_{f1}(z_{pr}-z_0)^2](\sin\alpha)+ +a_{pl}[\tau_{f1}(z_{pr}-z_0)+\tau_{f2}(\xi-z_{pr})-f_2(z_{pl}-\xi)]-0.25P=0$	(19)
Equation of a prismatic pile in general form	$n\,tg\,(\pi/n)\,\sigma_f(z_{tp}-z_0)^2(\sin\alpha)^2 + +[-2n\,tg(\pi/n)\,R_{cl}\sigma_f(z_{tp}-z_0)-n\,tg(\pi/n)\,\tau_{f1}(z_{tp}-z_0)^2](\sin\alpha)+ +2n\,tg(\pi/n)\,R_{cl}\tau_{f1}(z_{tp}-z_0)+na_{pl}[\tau_{f2}(\xi-z_{tp})-f_2(z_{cl}-\xi)]-P=0.$	(45)

For the conditions, given in Table 1, the geometric parameters of the piles such as cylindrical piles with upper reverse taper and prismatic piles with sloping four, six and eight faces have been obtained. The piles operate in the same soil and climatic conditions and are loaded with the same vertical load. All piles meet the design requirements for the section size at the point of embedding in the pile grillage. The length of all the piles is 3.0 m. The cross-section of the lower part of the piles is constant. The length and cross-section of the lower part is calculated from the condition of the vertical compressive load in the thawed soil, i.e. in summer. The cross-section of the upper part of the pile is variable with a sloping surface. The corresponding top length and angle are limited by the cross-section of the pile at the embedment point in the pile grillage. The surface slope angle is obtained from the equilibrium condition of the pile in the soil under the action of frost heaving forces. The parameters of the pile are shown in Table 3.

Table 3. Parameters of piles.

Piles type \ Parameters of piles	Bearing capacity of piles in thawed soil, кN	Pile (inscribed circle) radius, m	Taper (pyramid) length, m	Taper (inscribed taper) angle, degree	Pile volume, m^3
Cylindrical pile with taper	134.92	0.3	1.0	5.96	0.73
Prismatic octagonal pile	134.29	0.2897	1.0	6.13	0.71
Prismatic hexagonal pile	134.72	0.2858	1.1	5.57	0.71
Prismatic square pile	135.0	0.54, face width	1.33	4.6, face angle	0.70

A comparison of the piles (Table 3) for the geological conditions given in the article has shown that under equal geological conditions with the same vertical load, the square pile with sloping faces has the smallest volume. Consequently, the minimum material capacity has been shown by the prismatic square pile. As the pile spacing in the group is determined by the cross-section, the square piles with sloping faces require the smallest space between them.

Surface slope piles have been the subject of attention of many scientists. Piles with expanded base were investigated by V.F. Zhukov [2], B.S. Yushkov [7–8], S.V. Feshchenko [9], L. Domaschuk [10–11], M. Schafer and S.P. Madabhushi [14] and others. Huang and Sheng [12–13] tested bell-shaped piles. However, in existing studies the range of configurations is mostly limited to cylindrical piles, although prismatic piles also occur in design and construction practice. The author of the paper has considered the performance of both cylindrical and prismatic piles in heaving soil and developed a method for calculating the geometric parameters of piles under the set conditions.

Another important aspect is applicability and practical relevance of the calculation factors. V.S. Sazhin [39] investigated the behavior of the soil under a strip foundation and derived an equation for the soil uplifting and internal forces of the foundation. Relying on the equality of works [26] performed by the forces that contribute to the pile uplift and prevent it, V.S. Sazhin [26] also determined the displacement of a pile in the swelling soil. He pointed out that, despite different nature of soil swelling and heaving, the deformations of foundations caused by these processes were due to similar laws. Therefore, in his calculations he used the same methods both for swelling and frost heaving soils. V.S. Sazhin obtained a universal formula for the vertical displacement of piles with any cross-section under heaving forces.

Later on B.S. Yushkov and D.S. Repetsky [40] proposed a formula for calculating the displacements of a ‘double cone pile’ [40] under the action of tangential frost heaving forces. These equations can be successfully used to verify the displacements of piles operated under frost heaving. However, based on the personal experience, the author considers the geometric parameters of foundations that ensure zero displacement under specified conditions more important than displacements themselves. So, in construction design, the author emphasizes the importance of using the equations for calculating geometrical parameters of foundations, which is especially true for piles of complex shapes, in particular those with a surface slope.

In the paper the author presents solutions to determine the required geometric parameters of cylindrical piles with upper reverse taper and prismatic piles with sloping faces, where the equilibrium is zero displacement of piles under the action of frost heaving forces. The author follows the system approach to the solution of the problem, i.e. takes into account the frost boundary position, values of soil frost heaving forces, etc.

When deriving equations for calculating prismatic piles with a pyramidal part, the author uses the expression for the variable radius of the inscribed taper and the same experimental approaches [38] as for the pile with taper. It is due to the fact that the cylindrical pile with the upper reverse taper is effective in frost heaving conditions due to the taper part, while the prismatic pile with sloping faces provides such efficiency due to the pyramidal part. Taking into account that a taper is a pyramid with an infinite number of faces, the latter may be considered as a special case of a taper.

Modern methods of calculating piles under frost heaving conditions do not distinguish between cylindrical and prismatic piles. Only the surface area of the pile subject to frost heaving forces is taken into account. However, the author's experience reveals that in seasonally freezing soils the ribs of a prismatic pile affect the size and speed of its lifting by frost heaving forces. The rib effect and the difference in the operation of prismatic and cylindrical piles under frost heaving have become the subject of the author’s further research.

4. Conclusion

With large areas affected by frost heaving, protection methods based on the properties of constructions, without use of additional elements and measures, are of particular interest. Such constructions are piles with reversed surface slope, whose configuration makes it possible to reduce the negative effect of frost heaving.

In this study, the calculation method for cylindrical piles with an upper reverse taper is extended to prismatic piles with sloping faces. Equations for calculating geometric parameters of piles with sloping faces are derived and conclusions on the material capacity of the piles are drawn. The equations make it possible to determine geometric parameters of piles ensuring their stability in soil under the action of tangential frost heaving forces.

Further research will be aimed at performance of face slope piles in a group, as well as technological provisions for manufacturing piles at the construction site and in the factory. Attention will be paid to the automation of the developed calculation method for surface slope piles.

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