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Clay soil stiffness under consolidated isotropic drained triaxial tests

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Abstract. Triaxial tests are common laboratory methods to study the mechanical properties of soils. According to international practice, it allows determining the reliable strength and stiffness properties. This research paper describes the results of statistical analysis of the deformation parameters for clay soils obtained from triaxial tests. The research focused on clay deposits of the Quaternary, Jurassic and Carboniferous periods of diverse genesis. The results of 992 consolidated isotropic drained triaxial tests of clay soils in Russia (Moscow) and Belarus (Minsk) were analysed. More than 50% of the tests were carried out under unloading/reloading conditions. As a result, empirical equations enabling evaluation of the effects of physical properties and stress state on stiffness of clay soils with different age and genesis were proposed. Comparison of accomplished tests of Quaternary and Jurassic soils from Thailand, Europe and the USA showed that stiffness for overconsolidated soils is in the same range as soils from Moscow and Minsk sites. The performed studies revealed the values of the Hardening soil model m-parameter depending on soil forming factors and its preconsolidation degree. In overconsolidated soils, values of the m-parameter are on average twice less than in normally consolidated or lightly overconsolidated soils. Proposed equations can be applied for preliminary estimation of the stiffness parameters for finite element method calculation, as well as used in geotechnical models that allow variability, horizontal and vertical distribution of stiffness to be taken into account. In general, geotechnical engineers may utilize the obtained results by applying them to design of complex soil models.

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1. Introduction

Triaxial compression is the most common method for characterizing mechanical properties of soil and rock [1, 2]. ASTM D2850, ASTM D4767, ASTM D7181, ISO 17892-9, BS 1377, Russian State Standard GOST 12248.3, regulating the triaxial test, mainly focus on the strength parameters of soil [3]. Practically, undrained triaxial tests are usually carried out, in order to obtain strength parameters with the least time consumption.

The soil stiffness is traditionally obtained from oedometer tests. However, for modern models, which allow predicting highly realistic soil behavior, the results of drained triaxial tests are used [4]. Soil behavior in consolidated isotropic drained triaxial compression (CID test) is well characterized by hyperbolic law [5, 6].

Janbu in [7] suggested a relationship between stiffness and stress state. Dependence of stiffness on stress state and shear deformation was suggested and developed by Duncan and Chang [8]. The tangent stiffness value in isotropic triaxial compression conditions for any stress states can be expressed as:

$$E_t = \left(1 - R_f S\right)^2 K p_a \left(\frac{\sigma_3}{p_a}\right)^n, \tag{1}$$

where σ_3 is the minor principal stress; p_a is the atmospheric pressure (100 kPa); K is the modulus number; n is the power for stress-level dependency of stiffness describing the rate of variation E_t and σ_3; $S = \frac{\left(\sigma_1 - \sigma_3\right)}{\left(\sigma_1 - \sigma_3\right)_f}$; $R_f = \frac{\left(\sigma_1 - \sigma_3\right)_f}{\left(\sigma_1 - \sigma_3\right)_{ult}}$ is the failure ratio that is always less than one and derived from test results; $\left(\sigma_1 - \sigma_3\right)_f = \frac{2c\cos\varphi + 2\sigma_3 \sin\varphi}{1 - \sin\varphi}$.

Equation (1) characterizes standard consolidated isotropic drained triaxial test performed at a constant minimum stress rate. For evaluating plane and volumetric strains, the specimen destruction and its stress state should be considered with respect to three principal stresses.

The Duncan and Chang variable stiffness model has been widely applied to FEM analyses of soil–structure interaction [9]. For example, the PLAXIS hardening soil model utilizes the dependence that considers cohesion pressure $c \cdot \cot\varphi$ [10]:

$$E_{50} = E_{50}^{ref} \left(\frac{c \cdot \cot\varphi + \sigma_3}{c \cdot \cot\varphi + \sigma_{3ref}}\right)^m, \tag{2}$$

where E_{50}^{ref} is the reference secant modulus at the mobilization of 50 % of the maximum shear strength, corresponding to a reference confining pressure σ_{3ref} = 0.1 MPa. The E_{50} parameter was considered due to unambiguity of its definition.

Similar general principles using E_{50} stiffness are used in hardening soil models implemented in Midas GTS NX, OPTUM, Z-Soil, etc. software.

At the same time, articles [10–13] proposed more complex variable stiffness models than Duncan and Chang. The most advantageous feature of the variable stiffness models is its simplicity. However, it precludes the coupling between the deviatoric and volumetric components, which is a significant property of dilatant materials such as stiff clay and dense sand. In addition, none of the proposed variable-stiffness models satisfies the continuity condition [14].

The main disadvantage of the described model is the use of isotropic stiffness. Clay soils (especially, dense and stiff) are known to show anisotropy [15–18]: stiffness is different in vertical and horizontal directions. Use of σ_{3ref} equal to effective mean stress is a rough attempt to compensate for this disadvantage. In many researches, triaxial stiffness is obtained from anisotropic triaxial tests [19, 20]. Nonetheless there is the shear creep in a sample which significantly lowers anisotropic stiffness in comparison with values gained during field tests. The shear creep problem is distinguished as independent and in real practice should be solved for particular conditions (slope stability, pile-clay interaction) [21–23]. Most of the models that are currently used do not consider this effect. This is one of the disadvantages of such models, which underestimates settlements in soft clay soils [24].

Another drawback of the currently used models is that the accepted relations do not consider the effect and variation of physical soil properties on stiffness. It is known, that physical (and consequently mechanical) properties could vary in three dimensions. During triaxial compression, the variation of porosity can reach significant values, which noticeably affect the current stiffness [25].

Another essential drawback is that the statistical variability of stiffness is not considered in both horizontal and vertical directions. For most computational problems, the overall results of calculations show good agreement with observational data. In some cases, for sensitive models, stiffness distribution significantly affects the settlement, particularly for those with heterogeneous inclusions [26]. In fact, Paice, Griffiths, and Fenton [27] showed that the average settlement could increase by 12 % with increasing non-uniformity of the soil under foundation. Furthermore, the bearing capacity could change by 20–30 % with the coefficient of variation of the parameters [28]. This significantly affects the design decisions.

Presently, statistical geotechnical models, which take into account the horizontal or vertical distribution of properties, are becoming increasingly widespread [27–29]. Simulations that consider the soil property distribution more accurately reflect soil behavior more realistically [30]. The use of regression relationships and machine learning methods for developing these calculation methods is very promising and it is the most powerful tool for studying the influence of factors on soil properties [31–33].

However, existing correlations seldom incorporate the required input parameters for FEM calculation and they are made for specific regions: a few publications [34–36] present the analysis of a limited number of tests and do not encompass most types of clay soils. In case of variability of physical properties, they do not provide the possibility to evaluate the stiffness with sufficient reliability. The published data do not consider the variability of the soil properties, and therefore, should be used cautiously.

Considering the widespread use of various correlational relationships in engineering practice (at least for preliminary calculations in Russia), a thorough analysis of the experimental data collected for a wide range of clay soils is conducted.

The aim of this study is to obtain empirical equations for stiffness parameters during consolidated isotropic drained triaxial tests depending on age, genesis, stress state and physical properties. The stiffness parameters include E_{50}, E_{ur}, the m-parameter included equation (2), Poisson's ratios in course of primary and unloading/reloading stress path. The data can be applied for estimation of the clay soil stiffness without any further testing. Proposed equations can be used in geotechnical models that allow taking into account variability, horizontal and vertical distribution of stiffness. In general, geotechnical engineers may utilize the obtained results by applying them to design of complex constitutive soil models. The following research is the continuation of the completed work on sands [25].

2. Methods and Materials

2.1. General description of investigated soil

Results of CID triaxial tests were used as the experimental data collected on 15 different construction sites in Moscow (Russia) and Minsk (Belarus) (Fig. 1).

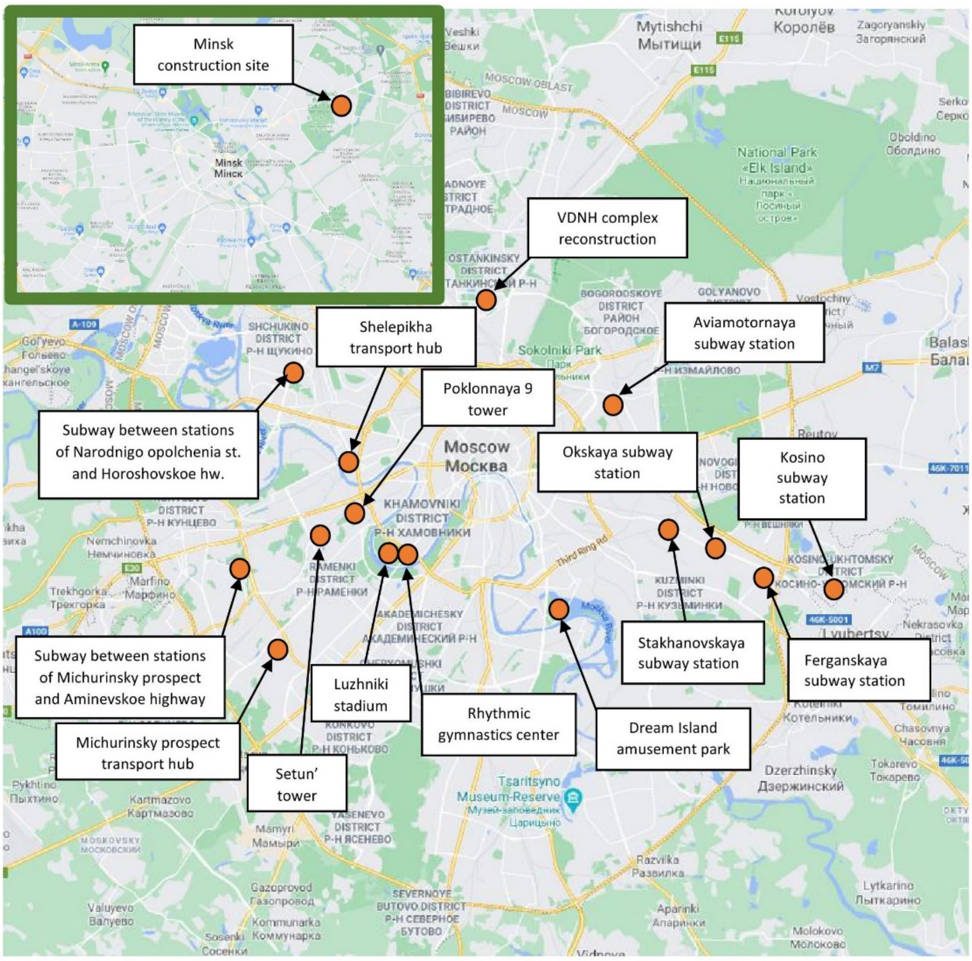


Figure 1. Layout of the construction site locations.

Very stiff and stiff clay soils were collected using single core tube core-barrel and thin wall sampler. Firm-stiff, firm and soft-firm soils were sampled using a thin wall sampler or a single tube core-barrel. The single core tube core-barrel and thin wall sampler equipped with a valve were used for sampling of soft-firm, soft and very soft clays. Sampling of soils was conducted in accordance with the technology described in Interstate Standard GOST 12071-2014 and ISO 22475-1:2006. Diameter of sampled monoliths was in the range of 75…120 mm; height – 100…400 mm.

Clay soils were classified by age and genesis. The physical properties were defined: density and Atterberg limits [37]:

– Plasticity index:

$$I_p = w_L - w_p, \tag{3}$$

where w_L is the liquid limit, defined using Soil Cone Penetrometer according to Interstate Standard GOST 5180; w_p is the plastic limit.

– Liquidity index:

$$I_L = \frac{w - w_p}{w_L - w_p}, \tag{4}$$

where w is the natural moisture.

Clay soils were categorized according to Interstate Standard GOST 25100 depending on plasticity index: at I_p < 1 soils are not clayey (and are not subject to study in this research). At 1 ≤ I_p ≤ 7 soils are classified as sandy loam; at 7 < I_p ≤ 17 as loam; and I_p > 17 as clay.

Liquid limit depends on the testing method. According to International Standard ISO 14688-2:2017 and ASTM D 2487—2017 liquid limit parameter is denoted as LL and calculate using Casagrande method. Transmission from LL to w_L is conducted using regional correlations and parallel test data. When there are no such correlations standard GOST 25100 allows the use of the following formula in order to juxtapose parameters of different classifications:

$$w_L = \left(LL + 8.3\right)/1.48. \tag{5}$$

Physical and mechanical properties of clay soils are presented in Table 1. The results of 967 consolidated isotropic drained triaxial tests were processed through statistical and regression analysis. Research was carried out for intact structure soils. Before the beginning of the tests, soil samples underwent the B-check procedure in accordance with ISO 17892-9, ASTM D7181 and Interstate Standard GOST 12248.3-2020.

Table 1. Physical and mechanical soil properties of different construction sites.

Construction site	Age and genesis	N	e, e.f.	I_p, e.f.	I_L, e.f.	c, kPa	φ, degree	σ_3, MPa	E_{50}, MPa	E_{ur}, MPa
Okskaya subway station[k]	fQ	24	0.45–0.76	0.05–0.14	0.05–0.62	27–65	17–31	0.05–0.6	10.8–32.6	–
	gQ	24	0.34–0.49	0.07–0.1	-0.14–0.38	33–98	23–30	0.05–0.25	13.1–39.2	–
	J	51	0.5–1.24	0.08–0.54	-0.29–0.15	40–117	18–26	0.05–0.75	14–52.3	–
Stakhanovskaya subway station[k]	fQ	6	0.55–0.81	0.08–0.14	0.39–0.43	28–45	19–22	0.1–0.55	11.8–22.4	29.4–30.8
	J	18	0.65–1.2	0.21–0.48	-0.22–0.29	43–87	18–23	0.1–0.95	13–36.7	24.6–84.3
	C	12	0.37–0.49	0.12–0.21	-0.4–(-0.3)	110–138	27–30	0.25–1.2	55.9–85.2	133.3–250.0
Luzhniki stadium [41] [k]	J	40	0.77–1.45	0.27–0.66	0.03–0.27	41–111	17–23	0.08–0.35	7.5–20.9	29.1–93.8
Subway between stations Michurinsky prospect and Aminevskoe highway[k]	fQ	13	0.44–0.62	0.08–0.26	0.11–0.64	20–81	16–33	0.03–0.33	5.8–23.2	–
	gQ	14	0.36–0.97	0.12–0.24	0.01–0.40	38–82	16–31	0.02–0.25	6.9–19.8	–
	prQ	5	0.63–0.76	0.18–0.25	0.07–0.34	0.52–63	19–23	0.01–0.035	2.7–13.8	–
	J	49	0.50–1.43	0.08–0.58	-0.26–0.55	25–170	13–31	0.38–1.25	18.4–52.3	167.6–270.4
	C	4	0.47–0.77	0.08–040	0.00–0.73	–	–	1.11–1.24	38.0–82.3	–
Subway between stations of Narodnigo opolchenia street and Khoroshovskoe highway[k]	fQ	12	0.45–2.01	0.07–0.44	0.00–0.72	17–28	32–34	0.14–0.18	5.6–18.5	31.7–107.7
	gQ	12	0.35–0.53	0.11–0.15	0.25–0.49	41–48	29	0.07–0.09	7–13.3	44.9–65.1
	K	6	0.66–0.74	0.11–0.17	0.27–0.70	43	27	0.167	13.4–24.5	91.1–128.6
	J	66	0.56–1.29	0.09–0.60	0.01–0.48	44–106	17–31	0.12–0.57	11.9–27.9	21.0–198.4
	C	12	0.44–0.86	0.04–0.26	-0.11–0.48	35–84	24–37	0.17–0.68	19.7–29.4	46.8–172.1
Kosino subway station[a,k]	J	17	0.59–1.31	0.17–0.47	-0.35–0.07	179	13	0.36–0.96	9.1–28.1	51.3–184.4
Minsk construction site[s]	fQ	22	0.16–0.69	0.04–0.09	-2.85–0.97	18–28	38–40	0.05–1.3	16.3–20.9	242.1–412.9
	gQ	73	0.14–0.48	0.02–0.08	-2.95–1.56	15–46	37–38	0.05–1.45	8.2–152.5	57.3–443.9
VDNH complex reconstruction[k]	aQ	17	0.33–0.74	0.06–0.11	0.25–0.98	4–32	35–41	0.24–0.37	18.9–38.5	121.2–262.9
Aviamotornaya subway station[k]	fQ	100	0.45–0.70	0.05–0.16	0.27–0.74	8–59	20–32	0.05–0.19	4.3–24.3	59.4–142.9
	gQ	7	0.49–0.51	0.12–0.14	0.51–0.57	24	28	0.03–0.04	2.5–8.5	29.1–45.5
	J	64	0.46–0.98	0.14–0.51	0.03–0.49	47–93	18–31	0.20–0.29	12–27.5	47.1–138
	C	73	0.43-0.39	0.06–0.25	-0.15–0.94	18–163	15–41	0.28–0.74	16.2–45.4	49.7–255.3
Shelepikha transport hub[k]	J	6	1.08–1.52	0.47–0.50	-0.03–0.31	54–96	16–21	0.07–0.1	8.1–10.6	20.1–34.3
	C	23	0.39–1.03	0.07–0.33	-0.22–0.93	20–82	23–36	0.16–0.69	19.7–46.5	59.9–166.3
Setun' tower [35] [s]	aQ	7	0.41–0.53	0.05–0.46	0.59–1.09	31	23–28	0.19–0.24	3.4–39.7	48.7–283.0
	J	24	0.48–1.0	0.07–0.35	-0.54–0.78	31–90	20–30	0.5–0.79	6.8–53.6	53.9–296.2
Michurinsky prospekt transport hub[k]	fQ	6	0.56–0.58	0.11–0.12	0.46–0.47	24–30	18–23	0.16–0.66	4.6–21.5	–
	gQ	27	0.45–0.58	0.11–0.12	0.34–0.50	28–39	20–24	0.18–0.92	8.3–24.7	–
	J	36	0.81–1.20	0.22–0.44	0.04–0.19	73–104	18–23	0.19–0.99	10.9–31.3	–
Luzhniki rhytmic gymnastic center[k]	J	18	0.69–1.4	0.21–0.58	0.02–0.50	44–117	17–22	0.2–0.4	12.6–18.3	–
Dream Island amusement park[s]	aQ	22	0.51–1.87	0.1–0.25	-0.19–1.03	12–57	6–26	0.08–0.85	1.9–31.2	–
	J	5	0.54–1.18	0.1–0.40	-0.34–0.17	37–72	20–28	0.16–0.76	16.8–42.2	–
Poklonnaya 9 tower[a, k]	J	4	0.93–1.18	0.24–0.58	-0.08–0.14	153	11	0.4–0.6	11.5–24.6	–
Ferganskaya subway station[s]	J	42	1.0–1.35	0.23–0.45	-0.29–0.05	48–94	18–24	0.2–1.0	16.8–42.1	–

a – tests processed by the author; k – strain-controlled loading mode; s – stress-controlled loading mode; Q – Quaternary age (*a* for alluvial deposits, *f* for fluvioglacial and limnoglacial, *g* for glacial), J – Jurassic age, C – Carboniferous

The initial diameter of the test specimen varied from 38 to 50 mm and H/D=2. Deviator stress was applied under the stress-controlled or strain-controlled loading mode. Though the loading type is pivotal, according to many studies [39, 40], it mainly affects the post-peak parameters which are not considered here.

The test procedure was performed according to Interstate Standard GOST 12248.3-2020 and conformed to ASTM D7181 and ISO 17892-9.

The research was carried out on the soil that comprised Quaternary (alluvial, fluvioglacial, limnoglacial and glacial), Jurassic (Volgian, Oxfordian and Callovian stages) and Carboniferous (Kasimovsky, Myachkovsky, Neverovsky, Proterozoic, Ratmirovsky, Suvorovsky and Voskresensky stages) deposits. Organic soils were not considered. The depth of sampling varied within 0.6–99.6 m.

The following parameters were determined from CID triaxial test results: secant modulus at 50 % strength E_{50}; unloading/reloading modulus E_{ur} (Fig. 2); Poisson's ratio ν; unloading/reloading Poisson's ratio ν_{ur}; effective angle of friction φ and effective cohesion c.

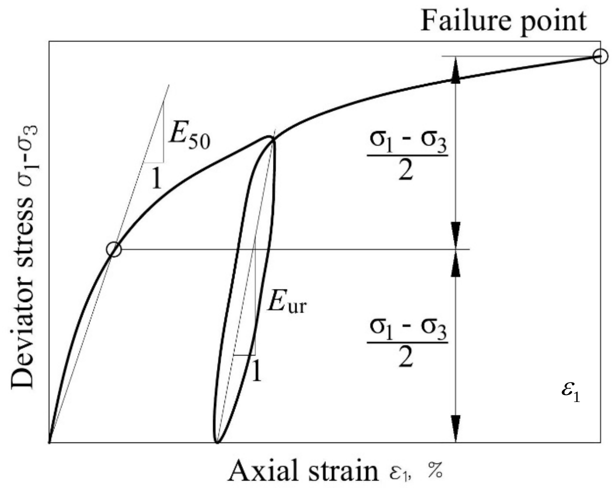


Figure 2. Definition of stiffness parameters.

2.2. Methods

The correlation and regression statistical data analysis technique was employed using MS Excel and IBM SPSS Statistics. The following stiffness parameters were analyzed: stiffness E_{50} and E_{ur}, Poisson's ratios ν and ν_{ur}, and the ratio of unloading/reloading modulus and secant modulus at 50 % strength:

$$k_E = \frac{E_{ur}}{E_{50}}. \tag{6}$$

At present, these parameters are used as input data for FEM computation for the hardening soil models and are of greatest interest for geotechnical engineers.

It should be noted that the ratio between E_{50}, E_{oed} and E_{ur} is not constant and depends on the soil type [10]. The k_E parameter was introduced for statistical analysis similarly to sand stiffness [25].

The experimental approach is as follows.

In the first stage, the experimental data were incorporated in a total sample. The strength of relationships was estimated via correlation analysis without considering the age and genesis. The Pearson's correlation coefficient ρ, significance level (using p-value), and sample correlation ratio η [42] were calculated via statistical analysis.

The Pearson's correlation coefficient ρ is widely used in statistical analysis. It evaluates the correlation relationship among the parameters and lies in the ranges from –1 to +1. The closer its value to +1 (or –1), the stronger the degree of linear relationship between the parameters is. If the ρ value is close to zero, it indicates a weak linear strength of relationship

The correlation parameter η is the ratio of a between-group dispersion to the total dispersion. It estimates the strength of the non-linear correlation relationship between the parameters and ranges from zero to one. If η is close to zero, the strength of the relationship is weak or does not exist; if it is close to one, the relationship is strong. The correlation ratio and the Pearson's correlation coefficient satisfy the condition $\eta \geq \rho$.

The correlation analysis revealed the most significant factors and nature of the relationship (linear or non-linear). Relationship in correlation interaction was analyzed at the significance level α = 0.05. This corresponds to the Interstate Standard GOST 20522-2012 requirements for calculating soil safety factor.

It is well known that stiffness depends on the soil density, stress state, and strength [7, 43–44]. Therefore, the following factors that are considered to affect the soil stiffness were analyzed:

– physical properties: initial void ratio e, plasticity index I_p and liquidity index I_L;

– stress state considered as radial stress σ_3 and relative radial stress [45] that is expressed as:

$$RRS = \frac{c \cdot \cot\varphi + \sigma_3}{c \cdot \cot\varphi + \sigma_{3ref}}; \tag{7}$$

– strength properties: the friction angle φ, cohesion c, and cohesion pressure $H = c \cdot \cot\varphi$;

– stiffness parameters: E_{50}, E_{ur}, and the Poisson's ratios ν and ν_{ur}.

Relative radial stress RRS reflects stress state in stiffness calculation in the Hardening soil model proposed by Schanz et al [45]. The advantage of the RRS parameter in comparison with σ_3 is in taking into account the cohesion pressure $H = c \cdot \cot\varphi$, which may lead to significant adjustments to stiffness in overconsolidated clay soils with high cohesion. The RRS parameter includes the reference radial stress and treats the dependency of stiffness properties on the soil stress state. Here, the reference pressure σ_{3ref} is considered as 0.1 MPa.

These stiffness parameters were chosen because they were used to characterize sand during engineering surveys at different construction sites (at least in Russia) and in FEM calculations.

In the second stage, the influence of stress state, density and Atterberg limits was studied in detail, based on the age and genesis. Clay deposits were divided into three groups according to age: Quaternary (Q), Jurassic (J) and Carboniferous (C). The Quaternary deposits were then divided into two subgroups depending on the deposition mode: alluvial, fluvioglacial and limnoglacial (further denoted as aQ), glacial (further denoted as gQ). It is worth mentioning that genesis alluvial, glacifluvial, limnoglacial deposits is different, however it was found out during the analysis that genesis had no statistically significant effect on the clay stiffness. Therefore, the mentioned soils were united into one subgroup. The Jurassic and Carboniferous clays were not divided by genesis.

3. Results and Discussion

3.1. Analysis of the total sample

The relationships between all parameters are highly non-linear (Table 2). The radial stress σ_3 and RRS considerably affect the clay soil stiffness parameters E_{50}, E_{ur}. The stiffness/RRS relationship is 25 % stronger than the stiffness/σ_3 relationship associated with the strength parameters considered in RRS, which is 2.5 times higher than in a similar research for sands [25]. At the same time, η between RRS and E_{50} or E_{ur} exceeds ρ by 1.5–2.3 times and η is close to one. This indicates a non-linear relationship. Therefore, it is preferable to use RRS for a more effective description of the relation between stiffness and stress state.

However, pressure factor exerts a lower degree of influence in clayey soils than in sands as shown in a similar research [25]. Clay particle content, density and moisture have additional influence, which is confirmed by the high values of correlation parameters η and ρ between stiffness and e_i, I_p and I_L.

Non-linear behavior prevails here, and η exceeds ρ by 1.3–3.5 times. Similar non-linear influence was reported for sands in [4, 7, 25]. Moreover, the same relationship can be observed during triaxial or oedometer tests. When analyzing the stiffness according to triaxial test results, the essential non-linear influence of the void ratio and I_L (due to moisture change during squeezing water out of sample during the test) should be taken into account.

The overconsolidation ratio (OCR) is another significant factor influencing the stiffness. Due to lack of the data, the OCR was not considered within this work. Though, the OCR can be interpreted indirectly through I_L: the more overconsolidated the soil, the lower I_L. High strength of the relationship between I_L and E_{50} indicates the vital influence of overconsolidation on stiffness during the primary loading. At the same time, the relationship between I_L and E_{ur} is not statistically considerable, which means that unloading of the sample in drained triaxial test is located in elastic region. Unloading of the sample takes place under average pressure lower than preconsolidation pressure for each sample.

Table 2. Estimation outcome for the correlated parameters for the total sample.

Stiffness parameter		Factor											
		e_i	I_p	I_L	σ_3	RRS	φ	c	H	E_{50}	E_{ur}	ν	ν_{ur}
E_{50}	ρ	-0.239	-0.162	-0.441	0.518	0.649	0.393	-0.03	-0.09	–			
	η	0.857	0.469	0.721	0.797	0.980	0.733	0.722	0.778	–			
	N	955	950	950	967	963	963	963	963	–			
E_{ur}	ρ	-0.361	-0.326	-0.053	0.336	0.398	0.310	-0.104	-0.131	0.676	–		
	η	0.789	0.556	0.782	0.769	0.930	0.720	0.846	0.931	0.954	–		
	N	371	371	371	375	371	371	371	371	362	–		
ν	ρ	-0.242	-0.274	-0.326	0.071	-0.043	0.009	-0.182	-0.156	-0.208	0.015	–	
	η	0.576	0.299	0.208	0.707	0.686	0.707	0.352	0.930	0.707	0.719	–	
	N	656	649	649	658	654	654	654	654	649	335	–	
ν_{ur}	ρ	0.053	0.086	-0.109	0.03	0.005	-0.021	0.035	0.037	0.157	-0.152	-0.067	
	η	0.681	0.496	0.648	0.699	0.742	0.450	0.607	0.686	0.853	0.877	1	
	N	265	265	265	267	267	267	267	267	267	266	267	
k_E	ρ	-0.245	-0.275	0.232	0.006	0.073	0.197	-0.247	-0.253	-0.177	0.475	0.006	0.103
	η	0.855	0.479	0.739	0.721	0.781	0.661	0.705	0.811	0.910	0.996	0.704	0.839
	N	349	349	349	353	353	353	353	353	353	353	334	265

– correlation relationship at significance level α = 0.05

Similar to sands [25], the direct relationship between stiffness and strength is statistically significant, but it is weak. Poisson's ratio ν exhibits a weak relationship with physical properties. Meanwhile, the ν_{ur} does not statistically relate to the analyzed factors. Boguz and Witowski had similar outcomes for glacial sediments in Poland [46].

The k_E ratio has a weak relationship with the parameters investigated. At the same time, the relationship between k_E ratio and physical parameters is relatively significant. However, the relationship between E_{ur} and E_{50} is non-linear. With an increase in E_{50} due to an increase in confining pressure and change of physical parameters *e*, I_p and I_L. The stiffness E_{ur} increases less intensively (Fig. 3).

Nevertheless, for the sake of convenience, a linear relationship is appropriate because ρ = 0.676 is close to η = 0.954. It is worth mentioning that the determined strength of the relationship is significantly higher than that described previously [35]. This depends on the volume of the data sample and on the wide range of measurements.

In general, the following conclusions can be drawn based on the performed analysis of the total data sample:

- stiffness of clay soil essentially depends on the radial stress, clay particle content, density and moisture and to a lesser extent, on the strength parameters;
- stiffness E_{ur} does not depend on the degree of overconsolidation;
- stiffness parameters E_{50} and E_{ur} are strongly related to each other;
- the Poisson's ratio ν slightly depends on the physical properties of clayey soil; ν_{ur} coefficient does not depend on physical and mechanical properties of soil.

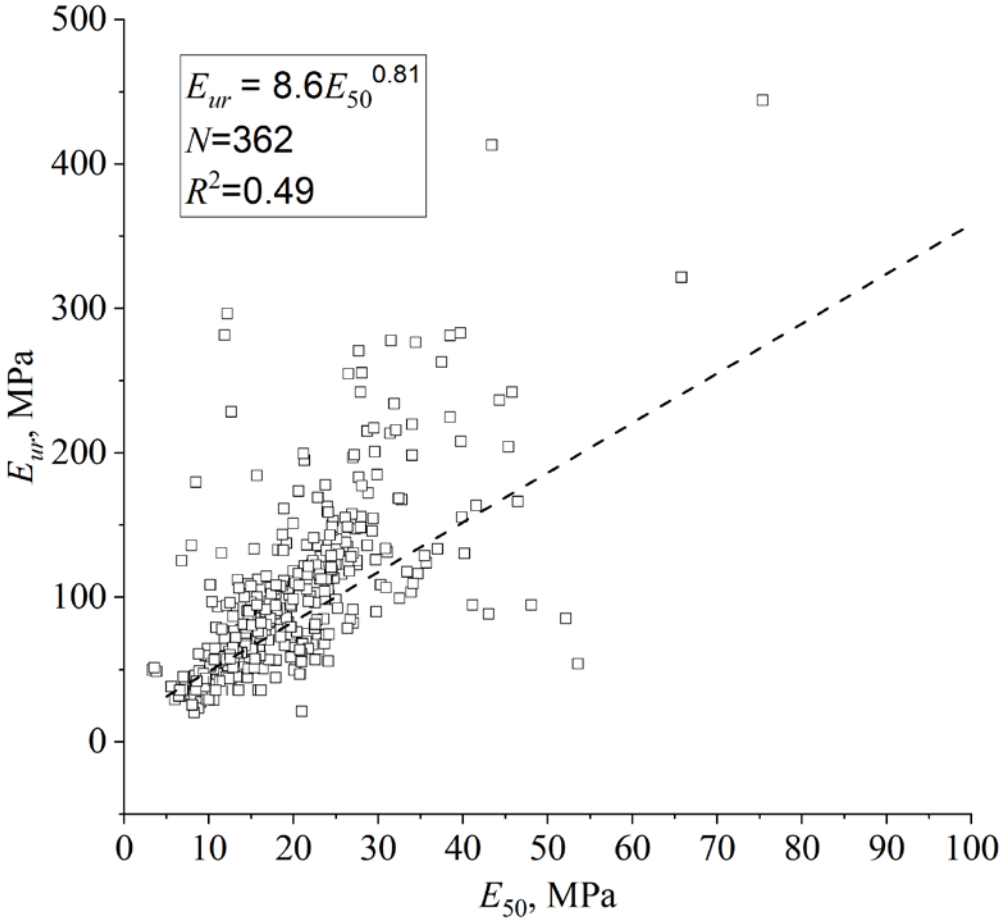
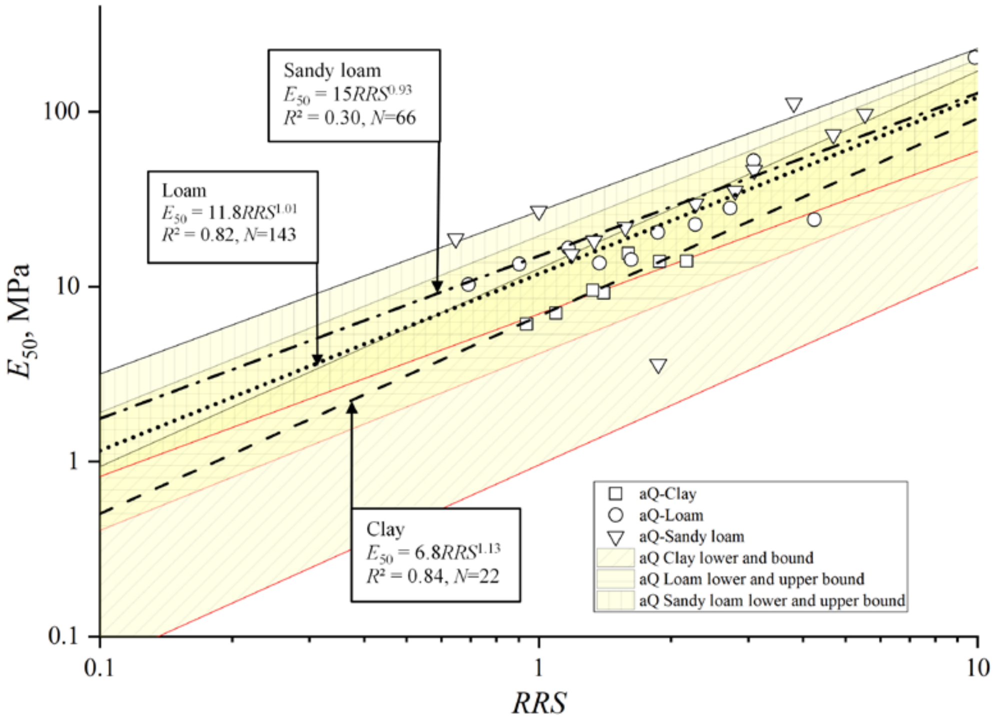


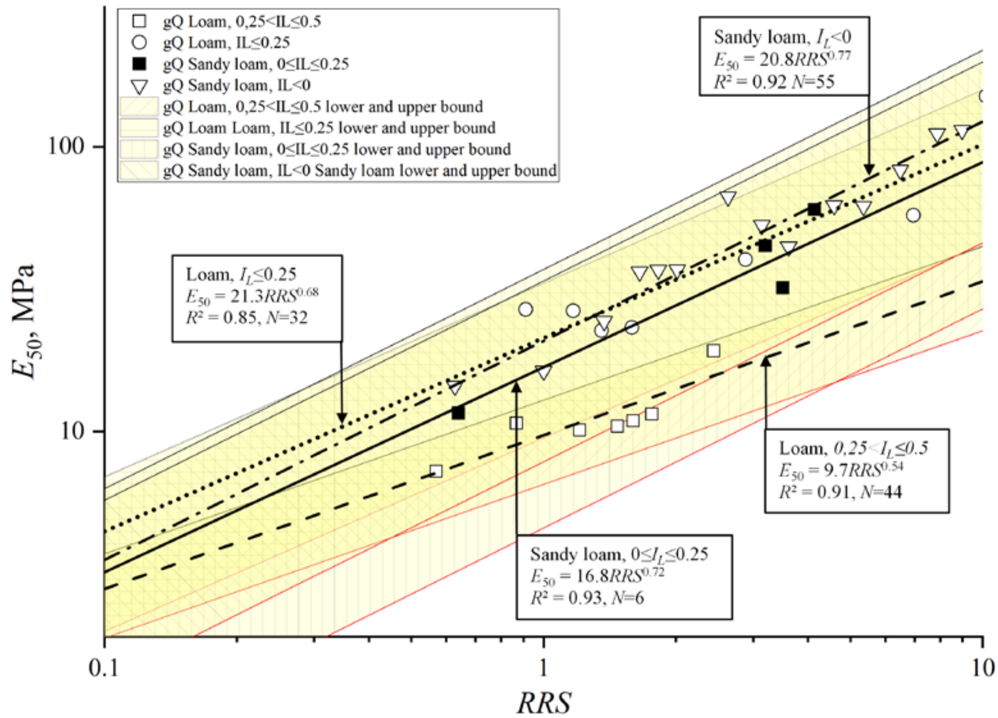
Figure 3. Relationship between stiffness moduli of the total sample $E_{50} - E_{ur}$.

3.2. Influence of the physical parameters and radial pressure on E_{50}



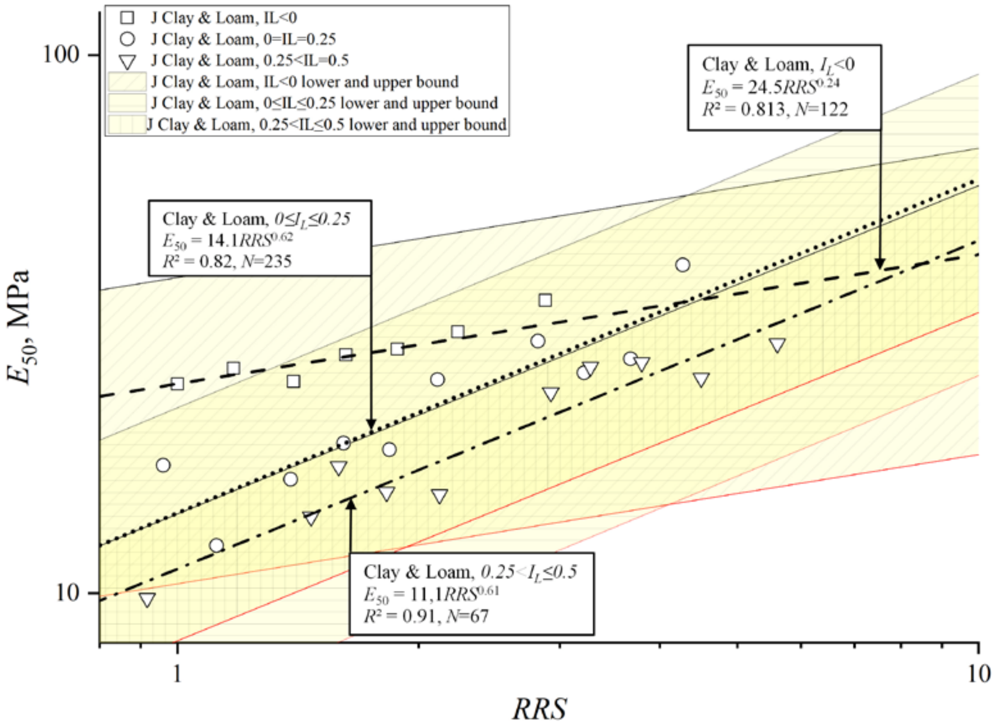
Sandy loam
$E_{50} = 15RRS^{0.93}$
$R^2 = 0.30, N=66$
Loam
$E_{50} = 11.8RRS^{1.01}$
$R^2 = 0.82, N=143$
Clay
$E_{50} = 6.8RRS^{1.13}$
$R^2 = 0.84, N=22$
aQ-Clay
aQ-Loam
aQ-Sandy loam
aQ Clay lower and bound
aQ Loam lower and upper bound
aQ Sandy loam lower and upper bound
E_{50}, MPa
RRS

a)

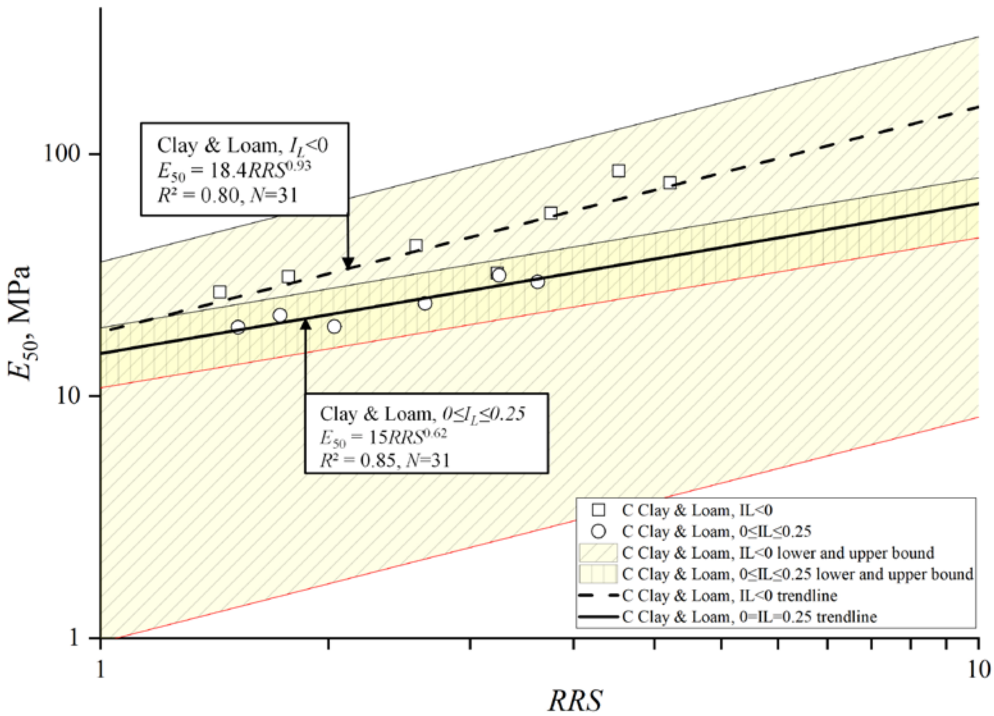


gQ Loam, 0,25<IL≤0.5
gQ Loam, IL≤0.25
gQ Sandy loam, 0≤IL≤0.25
gQ Sandy loam, IL<0
gQ Loam, 0,25<IL≤0.5 lower and upper bound
gQ Loam Loam, IL≤0.25 lower and upper bound
gQ Sandy loam, 0≤IL≤0.25 lower and upper bound
gQ Sandy loam, IL<0 Sandy loam lower and upper bound
Sandy loam, I_L<0
$E_{50} = 20.8RRS^{0.77}$
$R^2 = 0.92$ $N=55$
Loam, I_L≤0.25
$E_{50} = 21.3RRS^{0.68}$
$R^2 = 0.85, N=32$
Loam, 0,25<I_L≤0.5
$E_{50} = 9.7RRS^{0.54}$
$R^2 = 0.91, N=44$
Sandy loam, 0≤I_L≤0.25
$E_{50} = 16.8RRS^{0.72}$
$R^2 = 0.93, N=6$
E_{50}, MPa
RRS

b)



c)



d)

Figure 4. Influence of the relative radial stress on stiffness E_{50} for: alluvial, fluvioglacial and limnoglacial (a); glacial (b); Jurassic (c); Carboniferous (d) deposits.

Based on the empirical data, the diagram showing the dependency of RRS on stiffness E_{50} was drawn (Fig. 4). Lower and upper bounds correspond to the significance level α = 0.05. It is important to highlight relatively low values of E_{50} for Carboniferous clays. In natural stratification in Moscow region

Carboniferous clays alternate with limestones and their stiffness in-situ is near 70–200 MPa [47]. Therefore, the data for Carboniferous formations should be calibrated using results of in-situ plate-load and pressuremeter test.

Fig. 4 provides quantitative and qualitative assessment of the effect of RRS on the measured parameters, depending on clay particle content and moisture. In alluvial, glacifluvial and limnoglacial quaternary clay deposits, with increasing of the plasticity index I_p, the intensity of the RRS influence reduces. At the same time, there is no such phenomenon in overconsolidated glacial deposits. It is connected with historic hardening (overconsolidation) and structural strength. This can be observed in Jurassic clays (Fig. 4c) where for the most overconsolidated soils with I_L < 0 the intensity of E_{50} growth decreases by 2.6 times with the growth of RRS.

In case of sands, stiffness has strong correlation with stress state and density [25, 48], whereas in clay soils clay particle content and degree of overconsolidation (which is expressed through I_p and I_L) are more significant. In addition to mentioned parameters, density has essential impact on clay soil stiffness E_{50} (Fig. 5). The greatest influence was seen in quaternary deposits, where with growing void ratio stiffness parameter E_{50} decreases according to the power law (R^2 = 0.658–0.955) and shows weak dependency on soil consistency. For overconsolidated Jurassic and Carboniferous soils the relation between E_{50} and *e* is mainly characterized as weak and notable, respectively, and can be described with a linear relationship.

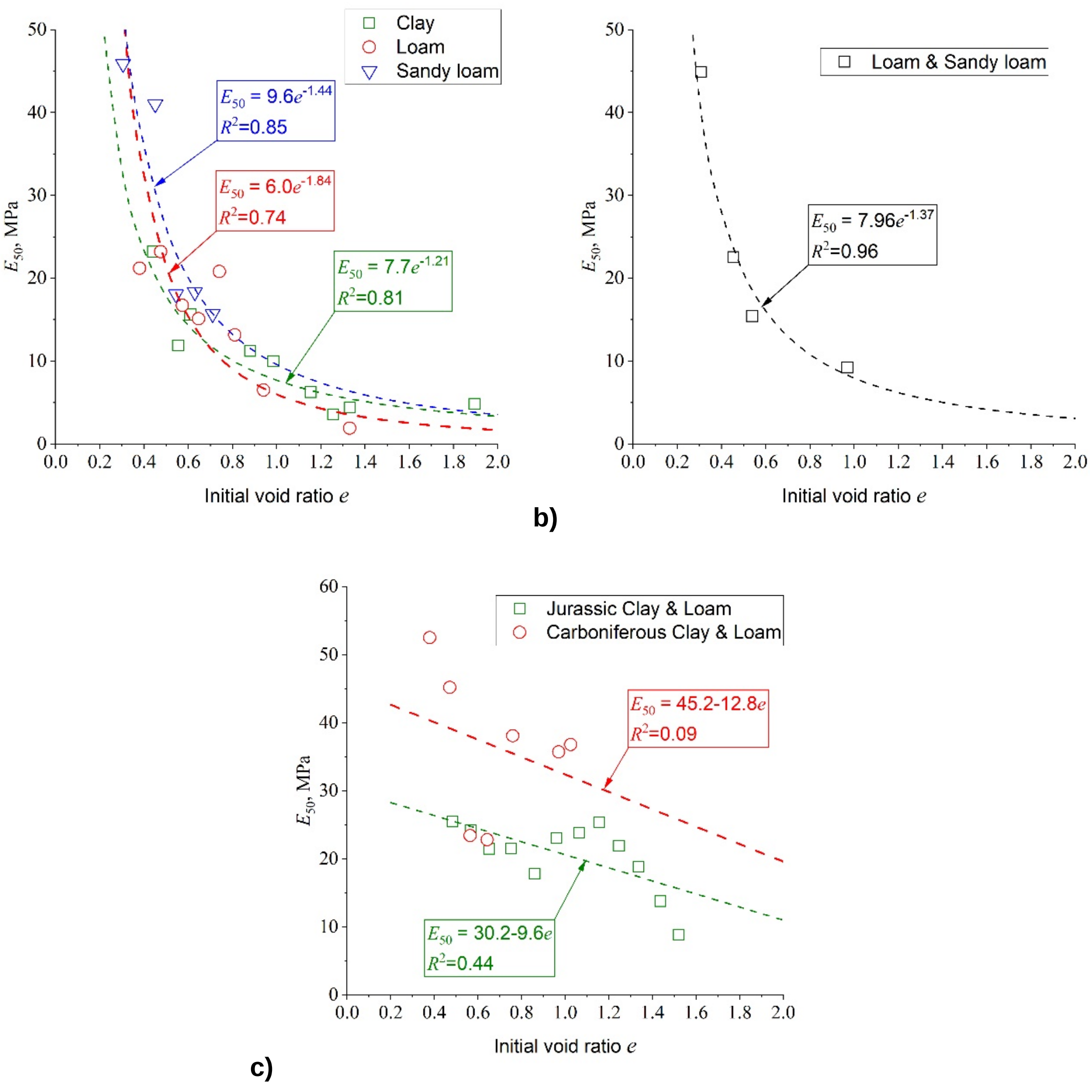


Figure 5. Influence of the void ratio on sand stiffness E_{50} for: alluvial, fluvioglacial, limnoglacial (a); glacial (b); Jurassic and Carboniferous (c) deposits.

The relationship incorporating the combined effect of the confining pressure and physical characteristics on the clay soils stiffness is shown above. Considering the processed data of a large number of triaxial tests performed on various types of clay soils, the following empirical dependence was proposed:

$$E_{50} = a_1 RRS + a_2 e + a_3 I_p + a_4 I_L + a_5. \tag{8}$$

The empirical coefficients a_1, a_2, a_3, a_4 and a_5 can be defined for all types of clay soils. Accordingly, Table 3 presents a_1, a_2, a_3, a_4 and a_5 values in dependence of soil age and genesis in the Moscow and Minsk regions. Fig. 6 shows the results of comparing the calculated and actual values of E_{50}. The use of the polynomial function is due to the large number of initial parameters. Other multifactor models (linear and non-linear) [49] produced similar or less significant results. The values observed and calculated, when compared using Fisher's test, do not reveal any statistically significant difference at the bilateral significance level α = 0.05.

Table 3. Empirical coefficients of regression equation for E_{50} in relation to age and genesis of the deposits.

Coefficients of regression equation for E_{50}	Quaternary		Jurassic	Carboniferous*
	Alluvial, fluvioglacial and limnoglacial	Glacial		
a_1	14.2	8.6	4.3	9.2
a_2	-6.2	-38.6	-3.5	44.8
a_3	-23.9	-144.6	-6.5	-162.2
a_4	-11	-3.5	-25.7	-76.1
a_5	7.8	38.7	20.7	19.4
Multiple ρ	0.851	0.858	0.687	0.874
Multiple R^2	0.724	0.736	0.472	0.764
Ranges of parameters				
RRS	0.57–9.85	0.33–12.27	0.76–7.68	1.3–4.08
e	0.18–2.01	0.14–0.58	0.45–1.52	0.37–1.03
I_p	0.04–0.46	0.03–0.16	0.07–0.66	0.12–0.33
I_L	-0.62–1.09	-2.95–1.56	-0.56–0.78	-0.43–0.23

* This estimation needs to be accounted for, since these clays have inclusions of semi-rocky carbonaceous soils and, in fact, the actual deformation modulus value for such soils is much higher than the one stated [47].

The existing scatter can be attributed to the large sample size, in-between laboratory error in stiffness measurement and influence of other factors [50]. Empirical correlation is limited by the range of physical properties of soils, as shown in Table 3.

In addition, E_{50}, defined according to Equation (8), was compared with other published test results (Table 4). Offshore, organic, artificially compacted deposits were not considered, due to the special features, which were not considered in these researches. The stiffness obtained during the undrained and anisotropic triaxial tests was not considered due to different stress-strain state.

The analysis of the open-source data showed that the volume of CID triaxial test results is quite insignificant. Nevertheless, the available volume of data allows us to assess the proposed equation.

Test results obtained by the other researchers are plotted in Fig. 6. The values of E_{50} in Table 4 fall within the same range as those corresponding to the Moscow and Minsk regions mentioned in Table 1. Results obtained using Equation (8) agree well with experimental data.

The best agreement of the suggested equation with the actual data was revealed for the overconsolidated glacial soils. Moreover, soils in Eastern Europe and the USA also agreed well. This confirms the dependence of soil genesis on its stiffness.

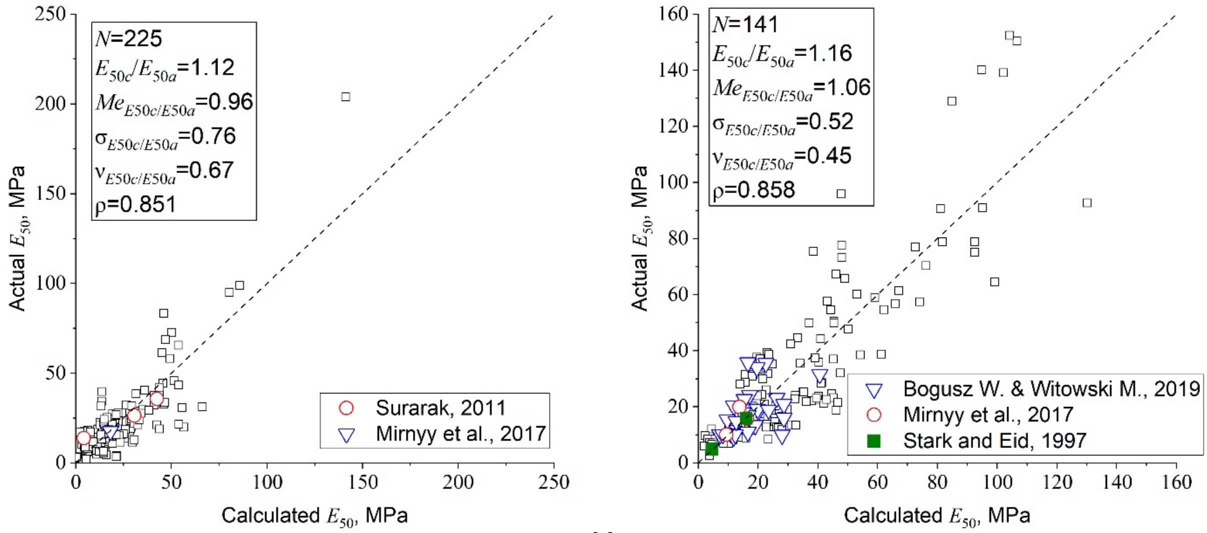
On the other hand, comparison of stiffness modulus of Jurassic Oxford clay at a site near Bedford [51] with the one calculated using Equation (8) showed that values are in the same range as Moscow ones; the difference between calculated and actual values of E_{50} is 1.9…2.6 times with similar physical properties. Both clays are highly overconsolidated as a result of great depths of overlying sediments in the past. At the same time, Jurassic Oxford clays of Moscow have greater stiffness. This might be the reason of different sliding surfaces development: specifically, during landslides in Bedford, the sliding surface was formed in Jurassic clay, near the ground surface; in Moscow, the sliding surface was formed on the border of Jurassic clay with quaternary deposits [52].

Equation (8) can be used for a generalized evaluation of soil stiffness with sufficient engineering accuracy. The comparison analysis of the foreign data confirms the conclusion that the stiffness of the Moscow and Minsk Quaternary glacial till insignificantly differs from the glacial till of the other regions.

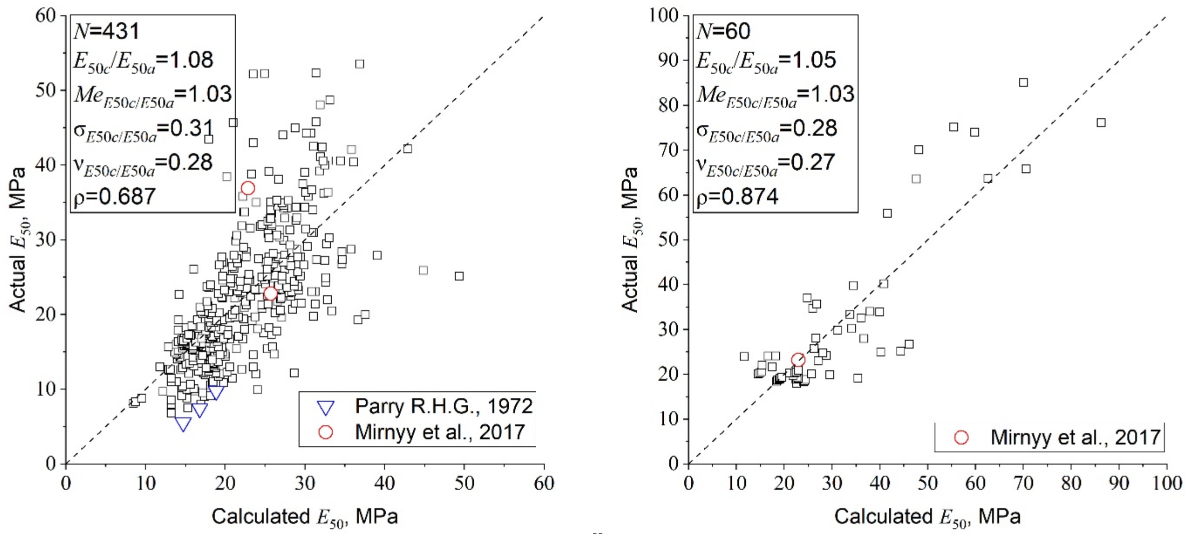
Table 4 Summary data used for comparison with the results obtained using Equation (8).

Reference	Soil region	Age and genesis	N	e, e.f.	I_p, e.f.	I_L, e.f.	σ_3, MPa	E_{50}, MPa	E_{ur}, MPa	m
Surarak et al [34]	Bangkok (Thailand)	aQ	3	1.2	0.41	0.07	0.1–0.55	13.8–35.7	–	0.48
Mirnyy et al [35]	Moscow (Russia)	aQ	18	0.85	0.12	0.625	0.223	17.6	135	0.35
		gQ	54	0.55	0.12	0.375	0.14–0.285	10.1–19.9	59.2–86.8	0.53–0.62
		J	18	0.77–1.2	0.15–0.42	0–0.375	0.565–0.633	22.8–36.9	73.9–132.0	0.30–0.31
		C	24	0.6	0.17	0.25	0.675	23.2	109.0	0.35
W. Bogusz and M. Witowski [36, 53]	Poland	gQ	27	0.37	0.151	0.19	0.045–0.800	9.3–35.7	–	0.54
T. Stark and H. Eid [54]	Urbana, Illinois (USA)	Glacial till gQ	2	0.85	0.08	-0.95	0.07–0.275	5–16*	–	–
R. Parry [51]	Bedford (UK)	Oxford clay J	3	0.68–1.07	0.45	0.11	0.07–0.21	5.6–9.7*	–	0.7*

* – stiffness calculated by the author



a) **b)**



c) d)

Figure 6. Comparison results for the calculated and actual E_{50} as regards: alluvial, fluvioglacial and limnoglacial (a); glacial (b); Jurassic (c) and Carboniferous (d) deposits.

3.3. Analysis of the power-law coefficient m

The state-of-art non-linear models with isotropic hardening treat the relation between the stiffness and stress state based on Equation (2). Parameter m is used in the models that consider the dependence between stiffness and stress state [7, 8, 45]. The m can be obtained both on the basis of oedometer tests (to identify the compression law) and on the basis of triaxial compression (to identify the dependence of stress state on the shear stiffness) [25].

Table 5 shows the values of m calculated using Equation (2) for the Quaternary and pre-Quaternary clay deposits. For the Quaternary deposits the value of m tends to increase with the increasing I_p and decreasing I_L. For lightly overconsolidated soils (aQ4), the m-values tend to 1. Similar values are obtained for Thailand soils [34].

For the glacial soils the m parameter varies slightly and lies in the range 0.54…0.77. These values are also close to the glacial soils, which were analyzed for Poland [36], where m values are equal to 0.49…0.516. Parameter m in overconsolidated glacial soils is approximately twice lower than in normally consolidated ones. In other words, the influence of stress state on stiffness in overconsolidated soils (2) is less than in lightly or normally consolidated clays. For example, for overconsolidated Jurassic clays m is equal to 0.24–0.62, for Carboniferous clays its range is 0.62–0.93. Similar findings were obtained for sands, when in loose soils m values were higher [25]. The processed CID triaxial test results [51] showed that for Jurassic clays m value is equal to 0.7.

Parameter m depends on formation conditions and degree of consolidation of clay soils. For overconsolidated soils, it is difficult to find any reliable correlations between m and moisture and density.

Nowadays, m parameter might be obtained based on both triaxial and oedometer tests. In case of sand, there are acknowledged data about decreasing of m with increasing void ratio [4]. If m values obtained by Mirniy [35] using an oedometer test are compared with triaxial test results, the latter are twice greater.

Table 5. Values of the power-law coefficient m.

Age	Soil type		N	m
Quaternary Alluvial, fluvioglacial and limnoglacial	Clay		22	1.13
	Loam		143	1.00
	Sandy loam		66	0.94
Glacial	Loam	I_L ≤0.25	32	0.68
		0.25< I_L ≤0.50	44	0.54
	Sandy loam	I_L <0	55	0.77
		0 ≤ I_L ≤0.25	7	0.72
Jurassic	Clay and loam	I_L <0	122	0.24
		0 ≤ I_L ≤0.25	235	0.61
		0.25< I_L ≤0.50	67	0.62
Carboniferous	Clay and loam	I_L <0	31	0.93
		0 ≤ I_L ≤0.25	31	0.62

3.4. Analysis of k_E ratio

Fig. 7 illustrates the correlation between k_E obtained using equation (6) and void ratio for different ages and I_p and I_L for clay soils. It can be observed that with the growth of void ratio k_E noticeably tends to decrease. The correlation has mainly linear character. The mentioned fact, on the one hand, contradicts the research results of Z.G. Ter-Martirosyan et al. [55], where the relation between unloading/reloading stiffness and primarily loading stiffness grows with the increase of void ratio. On the other hand, in [55] the research was carried out based on in-situ plate load test, during which compaction with minor lateral extension takes place. Moreover, there is a lack of data about the deformation interval during which the unloading was carried out, which might affect the results.

The k_E parameter takes into account the development of shear deformation with a slight change in volume (for clay soils the dilatancy can be disregarded). A decrease in k_E with an increase in the void ratio is associated with a difference in the intensity of changes in E_{ur} and E_{50} with a change in e.

For practical application k_E is useful for the E_{ur} stiffness estimation. Coefficient k_E might be determined based on the below given equation of regression, and depends on the physical parameters of e, I_p and I_L:

$$k_E = b_1 e + b_2 I_p + b_3 I_L + b_4 \quad (9)$$

The results of the empirical coefficients are shown in Table 6. It should be considered that E_{ur} is stress-dependent and should be evaluated based on RRS.

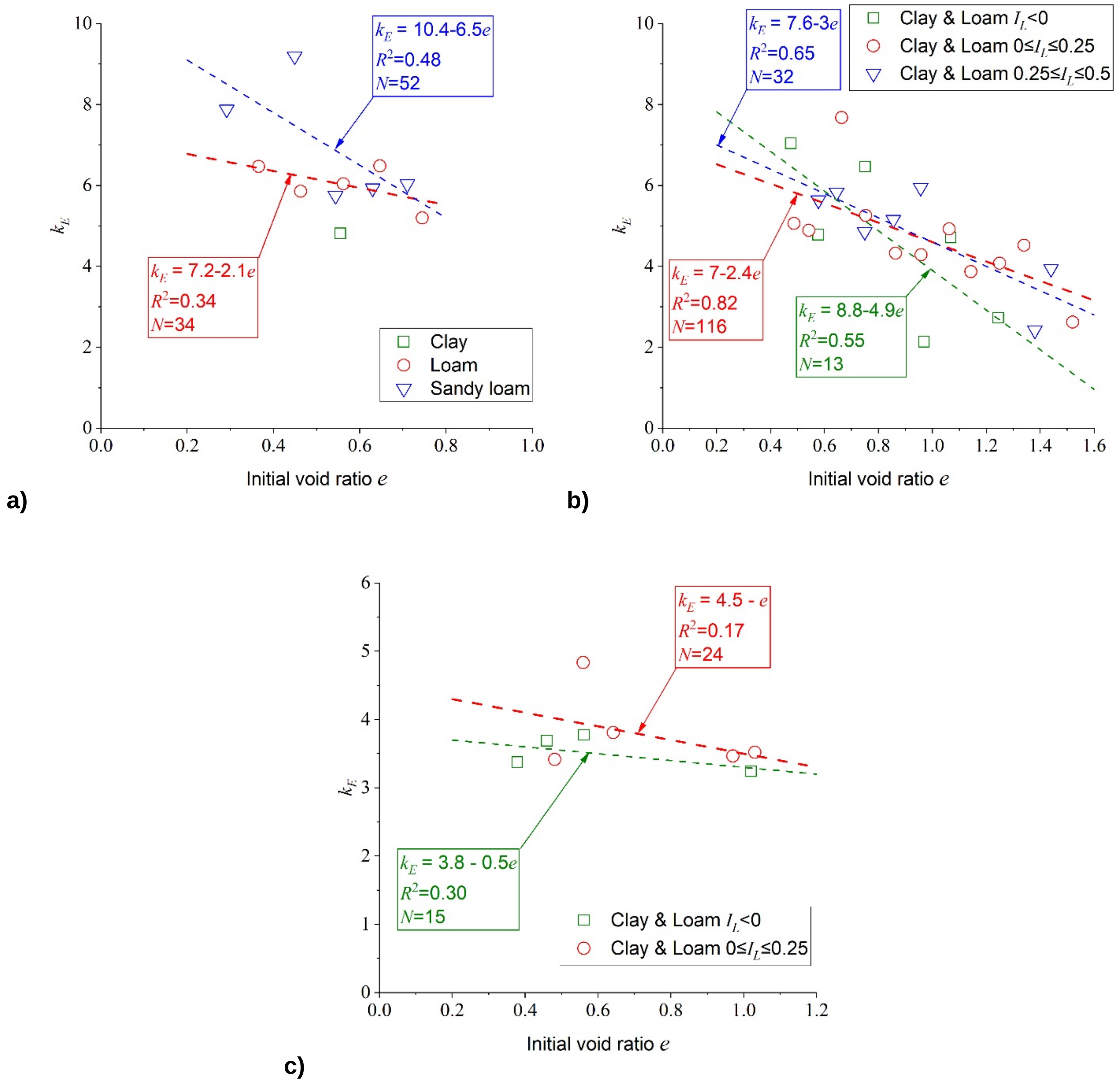


Figure 7. Diagrams of k_E dependency on void ratio e for Quaternary (a), Jurassic (b), and Carboniferous (c) soils.

Table 6. Empirically determined coefficients in the equation of regression for k_E in relation to the deposits age and genesis.

Coefficients of regression equation for k_E	Quaternary		Jurassic	Carboniferous
	Alluvial, fluvioglacial and limnoglacial	Glacial		
b_1	-2.5	-4.8	-4.5	1.9
b_2	-2.2	-52.4	2.5	-17.3
b_3	1.3	2.1	7.2	2.7
b_4	7.8	13.1	7	6.5
Multiple ρ	0.346	0.422	0.516	0.530
N	64	36	176	30

3.5. Analysis of Poisson's ratio

The outcome of the statistical analysis of Poisson's ratios ν and ν_{ur} is given in Table 7. The Poisson's ratio ν ranges from 0.09 to 0.45, and the average values are in the range of 0.29...0.36 and weakly depend on the age and type of clay soil. A similar feature is observed for the unloading/reloading Poisson's ratio ν_{ur}, which ranges from 0.11 to 0.41 with averages of 0.17…0.20. Besides, the variation coefficients at most do not exceed 0.3, which demonstrates the weak variability of these parameters. For preliminary calculations, the values of the Poisson's ratios given in Table 7 can be used.

Table 7. Results of the statistical analysis of Poisson's ratios.

Soil type	N	Average	Median	Standard deviation	ν_σ	Min.	Max.
			Poisson's ratio ν				
			Quaternary age				
Clay	30	0.35	0.35	0.04	0.11	0.23	0.40
Loam	132	0.33	0.35	0.05	0.14	0.09	0.45
Sandy loam	34	0.32	0.34	0.06	0.17	0.18	0.38
			Jurassic age				
Clay	288	0.29	0.29	0.06	0.21	0.11	0.42
Loam	51	0.32	0.34	0.05	0.14	0.21	0.40
			Carboniferous age				
Clay	38	0.33	0.35	0.06	0.17	0.20	0.40
Loam	6	0.36	0.36	0.02	0.05	0.34	0.37
			Unloading/reloading Poisson's ratio ν_{ur}				
			Quaternary age				
Clay	4	0.27	0.29	0.05	0.20	0.19	0.31
Loam	41	0.17	0.17	0.02	0.13	0.12	0.20
Sandy loam	18	0.19	0.16	0.09	0.45	0.12	0.39
			Jurassic age				
Clay	92	0.18	0.16	0.07	0.37	0.10	0.41
Loam	20	0.20	0.19	0.04	0.19	0.14	0.27
			Carboniferous age				
Clay	30	0.19	0.18	0.06	0.30	0.11	0.35

4. Practical application of the research results

The conducted analysis represents a degree of influence of age, physical properties (clay particle content, moisture and density) and initial stress rate on stiffness of clay soils. For geotechnical calculation purposes the subjects of greater interest are E_{50}, E_{ur} and m parameters.

In some cases, the cost of soil testing can be optimized for acceptance of stiffness characteristics when performing the preliminary calculations. To obtain the final stiffness characteristics for specific soils, it is necessary to confirm the characteristics by direct tests.

Moreover, obtained equations (8) and (9) might be used in geotechnical models, which consider statistical variation of stiffness in three dimensions due to the differences in physical properties and stress state. For example, soil moisture might vary in depth depending on the ground water level and pore pressure distribution. The closer soil is to gravitational water (for instance, in contact with saturated sand), the greater the moisture is. Therefore, I_L in this zone will be higher than normal which significantly reduces soil stiffness.

The suggested equations (8) and (9) can become the basis for constitutive models, where dependence of void ratio change, and, consequently, soil moisture during the volume change is realized. Such an approach was put into practice in sandy soils, where stiffness changed depending on density [25].

In general, geotechnical engineers may utilize the obtained results, applying them to the simulation of complex constitutive soil models.

5. Conclusions

1. The results of 967 consolidated isotropic drained triaxial tests performed on clay soil specimens from Moscow (Russia) and Minsk (Belarus) construction sites were processed using statistical and regression analyses. The empirical equations for determining E_{50} (eq. (8)) and E_{ur} (eq. (6) and (9)) which consider the mutual influence of the confining pressure and physical properties on clay soil stiffness were obtained.

Comparison of the completed tests of Quaternary and Jurassic soils from Thailand, Europe and the USA showed that stiffness for overconsolidated glacial soils is in the same range as soils from Moscow and Minsk sites. At the same time, Jurassic soils in Moscow region, which are located at great depth, have greater stiffness than soils in the UK, which are located closer to the surface. This confirms the influence of clay soil genesis on its stiffness. The degree of influence of genesis, conditions of sedimentation and stress state is higher than physical properties. Therefore, Equation (8) is applicable to samples obtained from places outside the Moscow and Minsk regions, with sufficient engineering accuracy.

The proposed Equations (8) and (9) can be used in geotechnical models that allow variability, horizontal and vertical distribution of stiffness to be taken into account. This facilitates more accurate modelling of the mechanical behavior in the computational model.

2. The m parameter describes the stress-state dependence on the stiffness in non-linear Hardening soil model [45]. The performed studies revealed the values of the m parameter depending on sedimentation conditions and degree of overconsolidation. In overconsolidated soils, values of the m parameter are on average twice less than in normally consolidated or lightly overconsolidated soils. The recommended values of the m parameter for preliminary calculations depending on age, I_p and I_L are presented in table 4. However, this parameter depends on the test method. For compression-type problems, it should be determined using oedometer tests, for shear-type problems, triaxial test data are more appropriate.

3. Numerical values of the ratio of the unloading/reloading stiffness to the secant stiffness at 50 % strength k_E were obtained. The tendency to the decrease of k_E with the increase of void ratio was found out for clay soils. The stated phenomenon differs from field tests obtained by Z.G. Ter-Martirosyan [55]. This phenomenon is explained by the prevalent influence of shear strain during triaxial compression in conditions of minor density change.

4. An obvious direction for further research is the study of the influence of samples quality on stiffness parameters. It is promising to introduce models with the dependence of stiffness on three-dimensional soil physical properties distribution into the FEM software (for example, PLAXIS, etc.). In addition, the influence of statistical variability of physical properties and stiffness on the structures' deformation should be researched.

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