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Accounting of plastic deformations in the calculation of frames using the displacement method

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Abstract. Method for calculating statically indeterminate frames taking into account plastic deformations, which is based on the use of a schematized diagram of material with hardening is proposed. Two types of standard beams with supports are used during the implementation of the displacement method (DM) like the elastic solution of the problem: "fixed" - "pinned" and "fixed" – "fixed", but unlike the elastic solution, standard beams contain special zones that besides elastic part include elasto-plastic zone (EPZ), plastic zone (PZ) and reinforcement zone (RZ). Therefore, as the stresses in these zones did not exceed the yield stress in the nonlinear frame calculation, we took measures to transform the PZs into equal strength plastic zones (ESPZ). The calculations were made for both types of beams for all unit and load impacts. The frame calculation consists of three stages (elastic, elasto-plastic and plastic). At the elastic and elasto-plastic stages, yield moment and plastic moment diagrams and the corresponding loads are determined. For a practical use of the DM in a nonlinear frame calculation, two simplifying prerequisites are introduced, with the help of which a stress-strain state is modeled in two zones: EPZ and PZ. According to the prerequisites, deformation of fibers occurs without hardening in EPZ and with hardening in PZ. The plastic stage of the calculation is performed at a given length of the PZ using the method of sequential loadings. At each iteration with small loading steps, incremental equations for DM are written, which establish relations between incremental moments and the incremental load, which allows us to build a resulting moment diagram. This diagram represents a sum of the moment diagram obtained at the elastic and elasto-plastic stages and the diagrams of incremental moments at all previous loading steps of plastic stage. According to the resulting diagram, the length of the PZ can be calculated, together with the limiting load. The calculation is considered complete if the length of the PZ does not exceed the specified value within the margin of error. Proposed algorithm is illustrated with an example of static calculation of 2-storey steel frame which perceives horizontal load actions that model a seismic impact.

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1. Introduction

Elastic-plastic deformations are generally accounted within the framework of the limiting equilibrium theory, which is based on the representation of an ideal elastic-plastic behavior of the material. The theory was developed by Soviet scientist A.A. Gvozdev, who in 1938 formulated three basic limiting equilibrium theorems (static, kinematic and duality theorems) [1]. The creation of this theory allowed to developed effective methods for calculating and designing many structures, especially reinforced concrete structures.

According to Prandtl diagram, the stresses of the construction material in the most loaded element cannot exceed the limit of yielding, and if the load is increased, the internal forces will be redistributed from more loaded elements to less loaded ones where the plastic state has not been reached yet. It is assumed that in a bended element that has reached the limiting equilibrium, the cross section is completely in the plastic state (a plastic hinge occurs), and the adjacent sections are in the elastic-plastic state, where the elastic core is retained.

In scientific literature, the concept of PZs is used mainly in seismic construction. For the first time, this concept was introduced by T. Paulay and I.N. Bull [2] in the calculation of reinforced concrete earthquake-resistant frames. Experts are long familiar with the fact that plastic deformations have the ability to absorb seismic energy, transforming it into thermal energy and then dissipating it into the environment. The article [3] proposes a method for evaluating the plastic design characteristics of beams and connections which may affect a seismic response of frame structures. The ability of loaded structural elements to absorb and dissipate energy generally ensures a decrease in the seismic impact on the frame. Thus, the structure, apart from its main designation, also works as an energy absorber. However, the operation of the structure beyond the limit of elasticity often leads to material degradation and destruction in these zones [4]. To overcome such weaknesses of concrete buildings as brittle fracture and lack of plasticity of the material, developments are underway to create new materials. In [5], the use of the reinforced fiber cement composite “HPFRCC” with increased of material ductility and high ability to absorb energy was shown. Experimental results showed that the use of HPFRCC layers in reinforced concrete beams allows to increase the ultimate load, the characteristics of the plastic hinge and the ability to redistribute the moment of these beams compared to the reference beam.

Developments related to the use of PZs aroused considerable interest among experts; they were consolidated in regulatory documents (codes) of the United States and other countries [6–8]. There appeared many papers covering a wide range of issues related to PZ parameters, such as the length of a zone, its location in the structure, the number of PZs, etc. Most of these studies deal with design features of PZs in reinforced concrete (RC) [9–17] and metal [18–22] structures. The problem of accounting for PZs is basically studied as applied to cyclic loadings of structures associated with seismic effects [9–11], [15–17], [19–22].

The articles [9–17] deal with the design features of PZs in reinforced concrete (RC) structures. In [9], a numerical analysis of the behavior of plastic hinges was carried out for bending structural elements, using the DIANA computational software. With the calibrated FEM model, the extent of the rebar yielding zone, concrete crush zone, curvature localization zone and the real plastic hinge length are studied.

In publications [10–15] discuss issues of studying the plastic hinge length of reinforced concrete columns. In [10], studies were carried out in the nonlinear version of the SAP2000 8 program for 4- and 7-story flat reinforced concrete (RC) frames, where the properties of plastic hinges are set by default according to ATC-40 [6]. PZs were determined at both ends of the beams and columns. It was shown by the example of a numerical experiment that the length of the PZ considerably influences relative horizontal displacements of the frame top. It was noted that this value differed by 30% when the plastic hinge length was modeled by different formulas: for the length $l_p = 0.5h$, where h is the height of the cross section of the element set by default [6], and for lengths l_p recommended in the works of R. Park, T. Paulay, M.J.N. Priestley et al. In [11] the plastic hinge behavior was studied for cyclic and monotonic loading using 3D FEM. Lengths of the plastic hinge zones include reinforcement yielding zone, curvature localization zone, concrete crushing zone and equivalent plastic hinge of RC column. It has been shown that for cyclically loaded columns this length is larger than that of monotonically loaded ones. Influence of the various parameters on this length was studied. It was noted that parameters such as the aspect ratio of the column and the hardening modulus of reinforcement loading and loading scheme defined by the number of cycles have a significant impact on it. Based on the numerical results under cyclic and monotonic loadings, a simple empirical model for the equivalent length of PZ under cyclic loading is proposed. This model takes into account the change in the length of PZ as far as the number of load cycles changes. However, dependence between the PZ length and the amplitude of the cyclic load has not been investigated. In study [12] considers similar problems as in [11], but taking into account the use of fiber reinforced polymer (FRP). Parametric studies of the plastic hinge length were first carried out for the calibrated FEM model, and then an improved model for FRP in RC columns was proposed. In [13] discusses the problem of assigning the PZ length in a RC column under the cyclic action of a lateral force and axial load. Behavior of plastic hinge under lateral and axial loading was studied. The influence of column size, physical and mechanical properties of reinforcement and concrete, the number of longitudinal bars, its diameters and other parameters on the length of PZ were taken into account. It notes the role of the principal reinforcement in a deformed member with particular emphasis on the part of the reinforcement that is strained beyond the yield stress in the hardening field. The article [14] proposes an expressions which allow to predict the equivalent plastic hinge length according to physical properties of HPFRCC material. On the basis of the probabilistic approach, [15] proposes a method for determining the length of the PZ in a RC column. A plastic hinge mechanism was constructed, in which a probabilistic model of the plastic zone length takes into account unknown parameters of the model using experimental data.

The article [16] studies the elastic-plastic response of the cylindrical composite sandwich panel under the action of lateral pulse pressure loading. Nonlinear differential equations of motion are solved benefiting from DQ–Newmark numerical method. It studies the development of elastic-plastic deformations in the facesheets of the core layers of the sandwich panel with different exposure modes. In particular, it is shown that plastic zones are first formed along the sides of the sandwich panel, and with the passage of time these zones progress towards the center of the panel.

The article [17] discusses a method for calculating reinforcement in earthquake-resistant RC buildings and structures which is based on theoretical basis of concept of nonlinear static analysis. The need for a justification of consistency of hinge zones design characteristics and its design parameters adopted at a conceptual design stage is explained. It is explained that length of an RC member end intended for arranging strengthened web reinforcement and a plasticity length of hinge zone may in fact have different values for the same element.

The articles [18–22] deal with the features of designing PZs in metal structures. In [18] developed a two-node super-element with generalized elasto-plastic hinges for static and cyclic analysis of frame structures. As opposed to the distributed plasticity analysis, the super-element uses a model with two generalized (concentrated) plastic hinges located at the ends of the elastic beam element. These hinges are modeled by a set of axial and rotational elastic-plastic springs and are used to reproduce plastic properties in the axial and angular direction of the element. This ensures elongation or shortening of the plastic hinge along the axial rod axis, as well as changes in the element rotation angle. Thus, the conditions for the interaction between the axial forces and bending moments in the plastic hinge zone are created. In the nonlinear calculation of beams and frames the dependence “force-displacement” was studied. The bearing load is estimated using plasticity models witch are related to the concept of a generalized plastic hinge. The same ideas were used in [19] only in the analysis of impact loadings. A model for nonlinear dynamic analysis of steel frame structures subjected to impact is presented. The generalized plasto-elastic hinges at the both ends of the rigid element, behavior of which is managed by the yield surfaces of super-elliptic shape was developed. An examples of impact calculation of the frames was considered. The graphs of cross-section displacement was shown. Articles [16, 17] does not include an analysis of relations between the amplitude values of loading and the length of PZ.

To analyze frame tubular structures, a number of plastic mechanisms have been developed that allow us to use of the same generic cyclic plasticity format [20]. In accordance with this format, each plastic mechanism is determined by an energy function, a yield surface, and a plastic flow potential. This allows us to create a set of functions regulating the elastic and plastic characteristics of the plastic mechanism’s cyclic model.

When analyzing steel frames fabricated according to the “strong columns - weak beams” design concept, after an earthquake, as noted in [21], major yielding zones and, as a result, fractures in the steel beam end are observed. In this regard, the authors of this article proposed a composite beam-to-column connection, including a friction damper. The composite beam consists of a steel base and an ultrahigh-performance concrete (UHPC) layer located in the upper part of the ultrahigh-strength concrete (UHPC). The designated plastic hinge length is limited by the level regulated by design features of the connection (l_p = 120, 240 mm). The force producing the yielding in UHPC layer was define in five experimental models at the moment of friction damper slip. Authors of [22] continued experimental and analytical studies of the seismic characteristics of the proposed device, in particular, they noted that the device was resistant to damage when the plastic zone length at the beam ends was l_p = 120 mm. For the noted length of PZ recommended thickness is 100 mm for UHPC layer and 20 mm for the steel layer.

It should be noted that the concept of PZ is considered as a zone of equal resistance or a zone of equal bearing capacity, since its construction is based on the limiting equilibrium theory. Therefore, the stresses inside these zones should not exceed the yield stress σ_y.

This article proposes a new approach to the calculation of statically indeterminate frames using the displacement method, based on a physically nonlinear material deformation according to a hardening diagram (Fig. 1). It is worth acknowledging that an initial attempt of this approach was undertaken in [23]. However, the method employed a less precise mathematical model. Consequently, an accurate evaluation of its performance is necessary once all the assumptions applied to the current model are implemented. According to the bilinear diagram with hardening, when a limiting state appears in any section of the structure, a further load increase will lead to an increase in internal forces and stresses exceeding the yield stress σ_y. As a result, a plastic zone (PZ) of some length l_p will appear. A fragment of an earthquake-resistant frame which includes PZ in edges of crossbar with length $2l$ is shown on Fig. 2. Red dotted line corresponds the level of stress σ_y and plastic moment $M_0 = W_0\sigma_y$, where W_0 is plastic section modulus. Since building codes do not allow the presence of plastic deformation in the joints of structures, PZ is designed at a distance ul from the column in the reinforcement zone (RZ), where load-bearing capacity of crossbar is provided on account of its increased stiffness. It should be noted that this is not the only way to relocate the plastic zone out of joints [24]. An elasto-plastic zone (EPZ) with length d and elastic moment $M_e = W_x\sigma_y$ (W_x is elastic section modulus) is located between PZ and elastic zone.

Since the stresses within the length l_p must not exceed the yield stress and correspond to the equal strength zone, measures should be taken to increase the element stiffness in the section with the stresses $\sigma_{max} > \sigma_y$. To this end, the element stiffness should vary according to the variable law and be consistent with the nature of the bending moment.

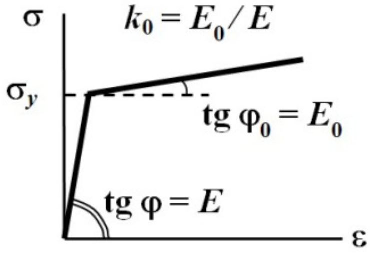

Figure 1. Diagram of the linearly hardening material deformation.

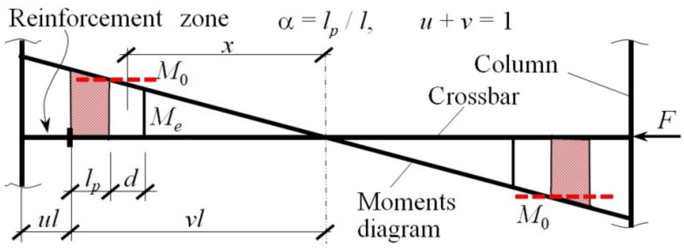

Figure 2. PZ with length l_p in a crossbar of a seismic-resistant frame.

2. Methods

To account for PZs in a statically indeterminate frame based on a bi-diagonal diagram, the displacement method (DM) is used as a calculation algorithm. The solution of the problem of transforming the area with nonlinear deformations into an equal strength plastic zone (ESPZ) should be integrated into the calculation algorithm of the method and performed in parallel with the nonlinear process of determining the limiting load F_0 for a given length of the PZ.

In case of a nonlinear calculation of frames by the DM, as well as in the classical version of this method, standard elements are used - beams with two types of supports: “fixed” - “pinned” and “fixed” – “fixed”, which should be designed for different types of unit and load impacts. However, unlike the classical version, the calculations of both types of beams should be performed taking into account the presence of ESPZs. These zones should contain the parameters determining their relative length $\alpha = l_p/l$, (l is beam length), location in the beam span, the law of variation of the area moment of inertia of the section within the length of the PZ and the physical and mechanical properties of the material. As a result, the calculated characteristics of standard beams will have the same coefficients as in the classical approach, but unlike them, they will contain additional dimensionless functions $f_j(\alpha)$, characterizing the nonlinear operation of the standard element.

In order to make non-linear calculation more accessible for the design engineer, two new premises are introduced that complement the well-known hypotheses of DM related to sequence of evolution of stress-strain state in EPZ and PZ.

First prerequisite: the deformation of the fibers of EPZ occurs by theory of idealized elasto-plastic operation of material (i.e. without hardening) with a variable modulus of elasticity which complying with the quadratic law.

Second prerequisite: the deformation of the fibers of PZ with length l_p occurs by bilinear diagram with constant hardening modulus E_0.

According to theory of idealized elasto-plastic body, a changing of size of elastic layer within EPZ length d complying with the quadratic law [25] $y = f(x_1) = \frac{h}{2}\sqrt{\frac{x_1}{d}}$ (Fig. 3). This allows assuming that the value of elasticity modulus in EPZ is proportional to the ratio of height of the elastic core to cross-sectional height.

$$E_x = E\sqrt{\frac{x_1}{d}}(x_1 \in [0;d]). \tag{1}$$

Stress in PZ: $\sigma \geq \sigma_y$, therefore all fibers deformed with constant hardening modulus E_0.

In the study [23], the segment addressing the non-linear deformation of the beam is also divided into two distinct sections. Moreover, there are no difference for the PZ segment compared to the current methodology. However, concerning the EPZ (referred to as the intermediate section in [23]), the elastic modulus is established using a constant value denoted as kE, which depends on the stiffness coefficient k. The coefficient k assumes values within the range $k \in [1, k_0]$. Since the coefficient k is unknown, estimating the stress-strain state within the EPZ becomes challenging. Additionally, determining the critical loads corresponding to the PZ since it is required to build a series of curves for different values of k. The uncertainty associated with the k coefficient serves as motivation for revising this mathematical model. In this article, this significant drawback of the previous model is overcome by introducing a quadratic dependence (1) for the quantity *E*.

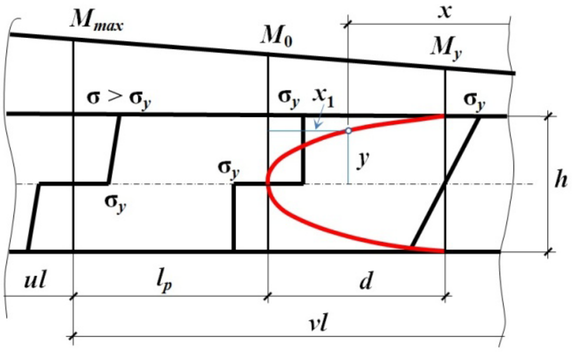

Figure 3. Plastic and elasto-plastic zones.

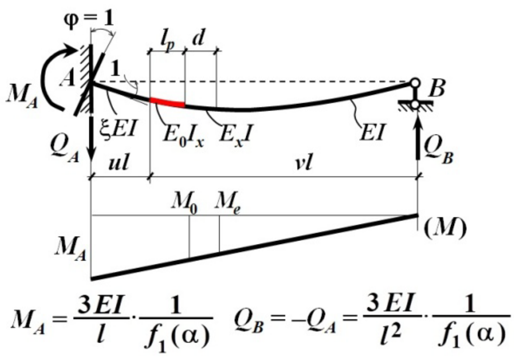

Figure 4. Design scheme of beam with ESPZ ("fixed" – "pinned") at a unit turn of fixed support.

Equal strength plastic zones. As it follows from the review, when the deformation of the frame caused by the seismic action, PZs can occur in the end parts of both horizontal frame elements — crossbars, and vertical elements — columns. The procedure of design an ESPZ for horizontal frame elements is shown below (Fig. 2).

In case of the linear law of the moment diagram, it is also convenient to take a linear dependence of the expression of the area moment of inertia for the creation of an ESPZ $\left(x \in \left[lv - l_{pi}, lv\right]\right)$:

$$I_x = I\frac{x}{l(v-\alpha)} \tag{2}$$

For standard beam supported according to the “fixed – pinned” concept, the rated forces for the rotating the fixed support by an angle φ=1 (Fig. 4) are shown. The correction nonlinear function $f_1(\alpha)$ has the form:

$$f_1(\alpha) = \left(1 - v^3\right) / \xi + 3\alpha\beta(v - \alpha / 2) / k_0 + m\beta^3(6 - 4m + \frac{6}{5}m^2) + \beta^3(1 - m)^3, \quad (3)$$

where

$$\beta = (v - \alpha), m = \left(1 - \frac{M_e}{M_0}\right). \quad (4)$$

$\alpha = l_p / l$, $k_0 = E_0 / E$, ξ is the stiffness coefficient of the support zone.

In the absence of PZ $(l_p = \alpha = 0)$ nonlinear function (3) takes more simple form:

$$f_1(0) = (1 - v^3) / \xi + v^3(1 + 3m - m^2 + \frac{1}{5}m^3). \quad (5)$$

The dimensionless function (3) contains four terms, each of which takes into account the contribution made to the overall ductility δ_{11} by the corresponding bar section, including the reinforcement zone (1st term), the plastic zone (2nd term), and the elasto-plastic zone (3rd term), elastic part (4rd term).

The 2nd term in (3) was obtained for a section of the length l_{pi} taking into account the variable stiffness (2):

$$\delta_{11}^{(PZ)} = \int_{vl-l_p}^{vl} \frac{x^2 dx}{E_0 I_x} = \int_{vl-l_p}^{vl} \frac{xl(v-\alpha)dx}{E_0 I} = \frac{l^3}{3EI} \cdot 3\alpha(v-\alpha)(v-\alpha/2)/k_0.$$

Third term for the part with length $d = m\beta l$ was obtained with the variable stiffness $E_x J$ taking into account (1):

$$\delta_{11}^{(EPZ)} = \int_{vl-l_p-d}^{vl-d} \frac{x^2 dx}{E_x I} = \frac{\sqrt{d}}{EI} \int_{vl-l_p-d}^{vl-d} \frac{x^2 dx}{\sqrt{vl - l_p - x}} = \frac{l^3}{3EI} \cdot m(v-\alpha)^3(6 - 4m + \frac{6}{5}m^2).$$

During nonlinear calculation of the frame taking into account PZ the correction function such as (3) and (5) are used for the various standard beams that form a basic structure of DM. Nonlinear calculation is aimed at determining the limiting values of bending moments (M_p diagram) and limiting load F_p for the certain length of PZ l_p.

Three stages of frame calculation (elastic, elasto-plastic and plastic).

Elastic stage. A definition of the yield moments (M_{el} diagram) and the yield load F_e occurs at this stage:

– plotting the bending moment M diagram arising under the action of given load F;
– determination the ratio $k_e = M_e / M_j$ for the critical cross-section *j* with the moment M_j;
– with the help of coefficient k_e the M diagram and the load F getting closer to the yield level:

$$M_{el} = k_e M, \; F_e = k_e F. \quad (6)$$

Elasto-plastic stage. According to 1st prerequisite a definition of the plastic moment M_{p0} diagram and the plastic load F_{p0} for the length of PZ l_p = 0:

– calculation of non-dimensional functions $f_j(0)$ such as (5);
– determination of the stiffness matrix based on function $f_j(0)$ in DM basic structure;

– solving the system of canonical equations of DM from unit load and plotting the diagram of bending moment $\bar{M}$;
– determination of the incremental load dF according to equality of incremental moment $dM_k = \bar{M}_k \cdot dF$ and ordinate $(M_0 - M_e)$ in cross-section k on the line between RZ and EPZ:

$$dF = \frac{M_e}{\bar{M}_k} \cdot \frac{m}{1-m}, \tag{7}$$

– determination of the limiting load F_{p0} and the moment diagram M_{p0} (for l_p = 0):

$$F_{p0} = F_e + dF, M_{p0} = M_{el} + \bar{M}dF. \tag{8}$$

The plastic stage of the calculation is performed at a given PZ length l_p using the method of sequential loadings [26]. For each loading stage dF, we use incremental ratios connecting the diagrams of incremental moments and incremental loads.

Incremental system of resolving equations of DM has the form:

$$K(\alpha_{i-1})dZ_i + R_{dF,i-1} = 0, \tag{9}$$

$$dZ_i = -\left[K(\alpha_{i-1})\right]^{-1} \cdot R_{dF,i-1}, \tag{10}$$

$$dM_{pi} = \bar{M}(\alpha_{i-1}) \cdot dZ_i + dM_{F,i-1}, \tag{11}$$

where $K(\alpha_{i-1})$ is the stiffness matrix of the frame including PZ at the 1st step of loading;

$R_{dF,i-1}$ is the response vector in DM basic structure caused by incremental load dF;

dZ_i is the incremental displacement vector;

$\bar{M}(\alpha_{i-1}), dM_{F,i-1}$ are the moment matrix caused by a unit loads and the incremental moment vector caused by incremental loads obtained in DM basic structure.

For the first loading stage, the initial PZ length l_{p1} can be taken based on the linear nature of the distribution of forces, for example, for the diagram M_{p0} multiplied by the coefficient $n = 1 + dF / F_e$. Based on the calculated correction (nonlinear) functions $f_j(\alpha_1)$, where $\alpha_1 = l_{p1}/l$, we form the coefficients (reactive forces) of the system of canonical equations (10) and the right sides of the equations from incremental loads dF. During the action of a horizontal seismic load the vector $dM_F^{(i-1)}$ in (12), is usually equals zero. After solving the system (11) and obtaining the diagram of incremental moments dM_{p1}, we build a resulting diagram: $M_{p1} = M_{p0} + dM_{p1}$, from which we calculate the PZ length l_{p2} for the next iteration step by the maximum value of the moment ($> M_0$). We simultaneously determine the current limiting load: $F_{p1} = F_{p0} + dF$. The obtained length is used to determine nonlinear functions $f_j(\alpha_2)$ for the second loading stage. In each i-iteration, we built: the incremental moment diagram dM_{pi}, the resulting diagram M_{pi}, the limiting load F_{pi}:

$$M_{pi} = M_{p,i-1} + dM_{pi}, F_{pi} = F_{p,i-1} + dF, \tag{12}$$

correction functions $f_j(\alpha_i)$ and the PZ length l_{pi}. The loading process continues until the obtained value does not reach the specified length l_p according to the inequality:

$$(l_p - l_{pi}) \le eps. \tag{13}$$

The proposed approach is illustrated by an example of a static calculation of a two-story frame on the action of horizontal forces simulating the seismic impact.

3. Results and Discussion

The design scheme of a two-story steel frame is shown in Fig. 5a (F = 40 kN, F_1 = –0.3F, $F_2 = F$, l = 300 cm, h_1 = 1.9l, h_2 = 1.6l). The crossbar of the lower story is made of a wended I-beam No. 26 (shelf - sheet 0.6×12.0 cm; wall - sheet 0.5×24.8 cm: I_x = 2958.5 cm^4; W_x = 227.58 cm^3); the crossbar of the top floor and the vertical elements are made of twin channels No. 20.

The strength and deformability characteristics are yield stress and ultimate strength, respectively: σ_y = 345 MPa, σ_u = 490 MPa, set after break ε_u = 0.21. The modulus of elasticity is E = 2.1·10^5 MPa, the modulus of hardening is $E_0 = (\sigma_u - \sigma_y)/(\varepsilon_u - \sigma_u / E) =$ 647.33 MPa. The elastic moment and the plastic moment are, respectively: $M_e - W_x \sigma_y = 78.51$ kN·m, $M_0 = W_0 \sigma_y = 91.08$ kN·m, where W_0 = 1.16W_x; flexural stiffness of the bars – EI = 6212.85 kN·m^2, $E_0 I$ = 20.59 kN·m^2; the coefficient k_0 = 0.0033.

The preliminary calculation shows that the maximum bending moments occur in the end parts of the crossbar of the 1st floor. The PZ is designed at u = 0.05 and ξ = 1.5 (Fig. 5a).

The example aims to show the method of nonlinear calculation of frame using DM with determination of limit loads F_p for the given ESPZ length l_p. The lengths of PZ from 2 cm to 14 cm, multiple of 2 cm, are considered.

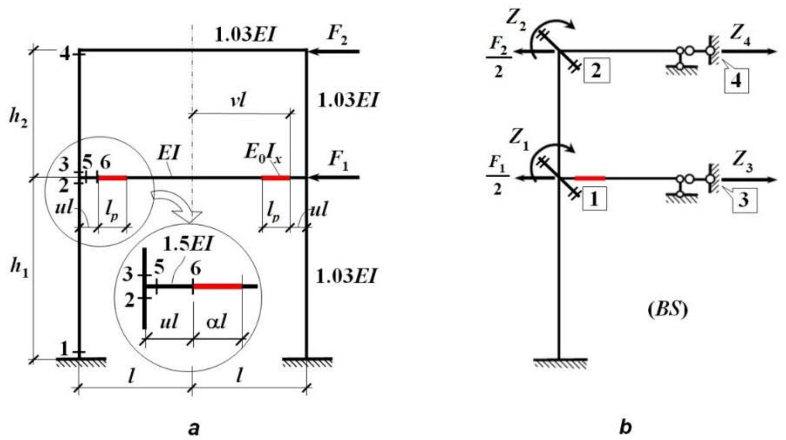

Figure 5. Design scheme of a two-story frame with plasticity zones in the crossbar of the 1st floor (a); b – basic structure of the DM taking into account the frame symmetry.

Due to the frame symmetry, the basic structure of the MD has four unknowns – two angular and two linear displacements Z_k (Fig. 5b). The numbering of additional bonds is shown by the numbers in small squares. For the plotting the diagram of unit moment M_1 we use the solution obtained for standard beam on Fig. 4 for the rotating the fixed support by an angle φ = 1 taking into account (5) within the length of PZ. The relative length α_i of the ESPZ is formed in a nonlinear process at each i-th loading stage.

The pattern of solving is shown below. From the preliminary frame calculation (at F = 40 kN), we obtain the highest stresses in section 6 (Fig. 5a). According to (6) (at k_e = 1.15), we will obtain a moment diagram M_{el} and the yield load F_e = 46.08 kN. As a result of elasto-plastic calculation (7), (8) we will obtain a values of limiting load F_{p0} = 54.1 kN and a moment diagram M_{p0} (at l_p = 0). The diagram M_{p0} is shown at right half of the frame (Fig. 6, blue, the values are given in brackets).

Coeffisients of the system of canonical equations of DM (9) at i th step of loading are:

- the elements of stiffness matrix $K(\alpha_i)$:

$$r_{11} = \left(4.732 + 3\frac{1}{f_1(\alpha_i)}\right)\frac{EI}{l}, r_{12} = 1.284\frac{EI}{l}, r_{13} = 0.701\frac{EI}{l^2}, r_{14} = -2.408\frac{EI}{l^2}, r_{22} = 5.652\frac{EI}{l},$$

$$r_{23} = -r_{14}, r_{24} = r_{14}, r_{33} = 4.808\frac{EI}{l^2}, r_{34} = -3.01\frac{EI}{l^3}, r_{44} = -r_{34};$$

– the elements of the vector R_{dFi} from incremental loading dF:

$$R_{1dF} = R_{2dF} = 0, R_{3dF} = -0.15dF, R_{4dF} = 0.5dF.$$

During the iterations when finding the limiting load for a given length l_p, we adjust the parameter α_i and the function (3) $f_1(\alpha_i)$.

A diagram of incremental bending moments for the 1st loading stage dF = 0.087 kN is shown on the left half of the frame (Fig. 6). At the initial stage, when we set the PZ length on the assumption of a linear nature of the distribution of moments: $l_{p0} = vl(1 - M_0/M_6)$ = 0.142 cm, where $M_6 = 1.0005M_0$, we form the correction function (3) f_1 = 1.614. After solving the system of the canonical equations of the DM and building the diagram of incremental moments dM_{p1} (shown on the left half of the frame, Fig. 6), we obtain the resulting diagram $M_{p1} = M_{p0} + dM_{p1}$ (the diagram M_{p1} is on the right half of the frame, Fig. 6). According to the results of the 1st iteration, the ESPZ length was l_{p2} = 0.392 cm, the load was $F_{p1} = F_{p0}+dF$ =54.2 kN. At the next loading steps for a given length l_p = 2 cm, the following results were obtained: l_{pi} = 2.001 cm, F_{pi} = 54.77 kN. The final nonlinear moment diagram M_p is shown on the left half of the frame (Fig. 7). The bending moment at the left end of the ESPZ (section 6) was M_6 = 91.72 kNm > M_0 = 91.08 kNm. The stresses in the supporting part of the frame were: σ_1 = –238 MPa, in the upper node σ_4 = 236.2 MPa; the stresses in the node of the 1st floor σ_5 = 282.8 MPa < σ_y = 345 MPa; $\sigma_6 = \sigma_7 = \sigma_y$.

The moment diagram M_p for the length l_p = 14 cm obtained at the limiting load F_p = 59.8 (65.18) kN is shown on the right half of the frame (Fig. 7). Stresses in support zone of frame: σ_1 = 344.6 MPa, in upper joint σ_4 = 341.6 MPa, in 5th joint σ_5 = 295.4 MPa, stresses $\sigma_6 = \sigma_7 = \sigma_y$.

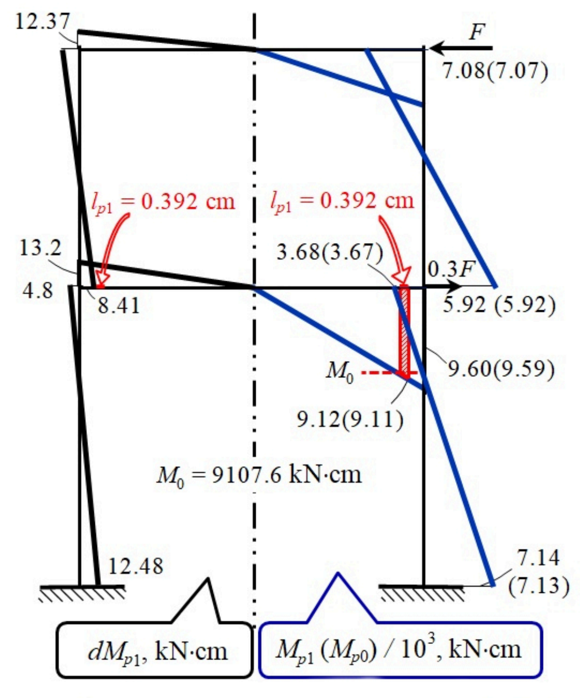

Figure 6. Bending moment diagrams at the first loading step at l_{p1} = 0.392 cm: to the left – incremental dM_{p1}.

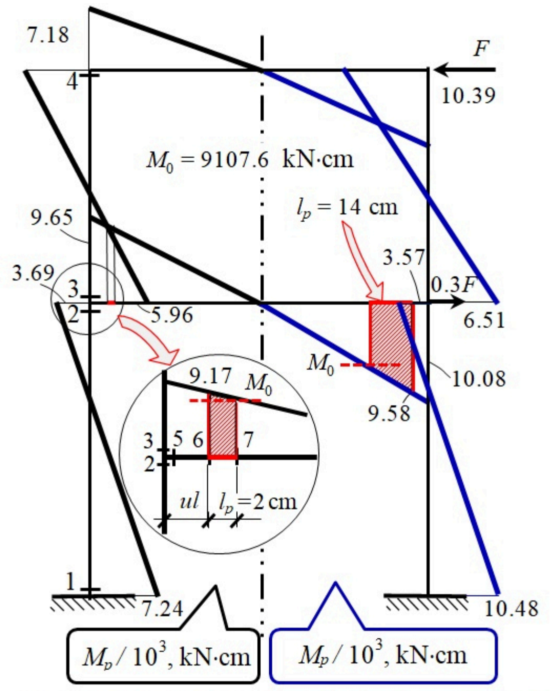

Figure 7. Bending moment M_p at the ESPZ length: l_p = 2 cm (to the left); l_p = 14 cm (to the right).

The nature of the change of bending moments depending on external load is shown at Fig. 8. Beginning with yield level when F_e = 46.075 kN the moments for each cross-section (shown in numbers) show weak nonlinearity at first (before PZ appears with F_{p0} = 54.11 KN). Then it begin to deviate significantly from the linear characteristics of the moments (dotted lines on the graph). Horizontal dashdotted lines show the level of load bearing capacity of the frame elements. For RZ (section 5) it is equals 1.34M_0.

Fig. 9 shows more common picture of the change of limiting loads depend on ESPZ length. Graphs show two limiting load curved lines for corresponding lengths l_p obtained when the plastic deformations in EPZ was taken into account (red line) and when it was not (blue line). The differences of values do not exceed 1%. Collapse load P_0 = 77.18 kN is calculated by limiting equilibrium method and shown by the black horizontal line.

For comparison consider the analysis of the steel frame performed in articles [19], [20]. One of the models of the frame (S3 model) considered by the authors of [19], [20] has a number of similar features with the model given in current study:

the lengths of plastic zones: the lengths l_p from 0 cm to 14 cm in steps of 2 cm were considered in current study; the length l_p = 12 cm was considered in S3 model;

the location of PZ is nodal zone of the I-beam and besides the beam in the S3 model is composite;

presence of RZ with the length of 15 cm in current study (Fig. 5) and 29 cm in S3 model;

construction material in the current study and in S3 model have the same mechanical characteristics;

the plastic zone is presented as ESPZ;

However, there are significant differences between these two schemes associated with the formulation of the research problem and the design features of PZ. The main difference of S3 model is that this zone includes a composite rectangular layer with the size of 120×300 mm (20 mm steel plate and 100 mm UHPC layer) and friction damper instead of I-beam. The length of PZ is constant (l_p = 12 cm). The yielding of fibers in the layer of this zone causes by a cyclic load acting the crossbar in the vertical direction. The amplitude of cyclic load is 13.5 kN. It is transmitted to the PZ as a longitudinal force N which creates tension-compression deformations. The longitudinal force cannot exceed the value of 100 kN because the friction damper will slip otherwise.

It should be noted that the limiting load for PZ with the length of l_p = 12 cm is 66.42 kN (Fig. 9). From the above it follows that these differences do not allow us to compare the limiting loads of both cases.

Thus, a method is proposed to the nonlinear calculation of statically indeterminate frames based on the DM which can be used in the design of structural systems in regions of an increased seismic activity in addition to the limiting equilibrium method.

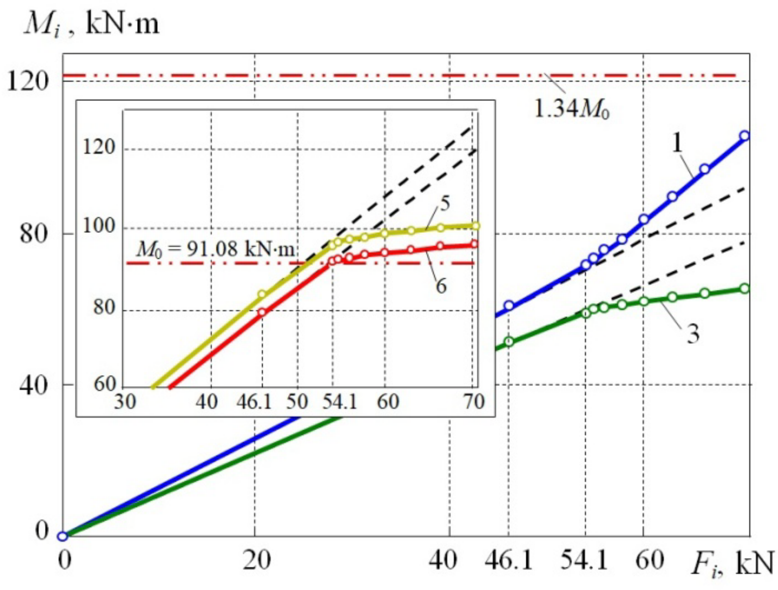

Figure 8. Bending moments in the frame cross-sections (numbers on graphs) depending on the load (dotted line is elastic response).

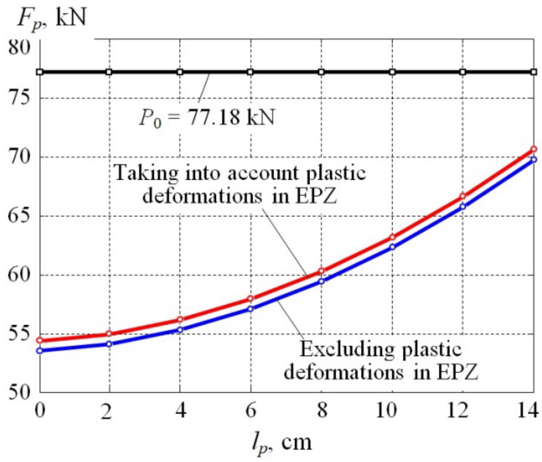

Figure 9. Limiting loads for the corresponding ESPZ.

Recommendations for future research. Further research is necessary to enhance the mathematical models of elastic-plastic calculation. Some important questions that need to be addressed are:

- simulation of the stress-strain state of the rod in the zone of non-linear deformations incorporating linear hardening of the material;
- one of the key areas of research is the development of a set of standard elements, such as statically indeterminate beams, in which calculations for single actions take into account the plastic zone. Specifically, it is important to address the issues related to calculating a rigidly fixed beam with two plastic zones located at its end parts;
- development and construction of a complete system of correction (nonlinear) functions, taking into account corrections to the linear calculation of the frame.

In addition, an important element of the study is the development of a methodology that provides the procedure for embedding the proposed scheme for the nonlinear calculation of frames into the algorithm of mathematical models for the calculation of seismic-resistant frames with PE.

4. Conclusions

This article proposes a new approach for conducting static analysis of bar frames, which incorporates plastic zones using the displacement method in conjunction with the sequential loading method. To implement this approach, two important theoretical problems had to be solved. The first involved developing a stress-strain model for a rod within the zone of elastic-plastic deformations using the linear theory of material hardening. The second involved calculating standard elements (statically indeterminate beams) for single actions, while taking into account special zones, such as plastic, elastoplastic, and reinforcement zones. Based on the research, the following conclusions were drawn:

1. To divide the zone of physically non-linear deformations of the beam into two areas (EPZ and PZ), two simplifying prerequisites were introduced. The simulation of stress-strain state of the rod in each of these areas was carried out. In EPZ, fibers' deformation occurs according to the theory of an ideal elastic-plastic body, without hardening and with a variable modulus of elasticity. In PZ, the deformation of all fibers occurs beyond the elastic limit, with a constant hardening modulus E_0.

2. In the EPZ section, the height of the elastic layer changes according to a quadratic law, therefore, for the variable modulus of elasticity, a quadratic dependence is also adopted, according to which the value of E_x is proportional to the ratio of the elastic core of the section $2y$ to the height of the section h.

3. In the plastic deformation zone, in order to ensure that the normal stresses do not exceed the yield strength and correspond to the zone of equal capacity, a linear dependence is adopted for the moment of inertia of the section, which is consistent with the linear character of the moment diagram.

4. The introduced dependences of the modulus of elasticity in the EPZ and the moment of inertia in the PZ are represented by convenient analytical functions. This allows to perform calculations of standard elements for single actions and construct correction (nonlinear) functions that take into account corrections to a linear calculation.

5. A computational scheme for a nonlinear analysis of statically indeterminate frames by the displacement method was created based on the calculation of standard elements and constructed correction functions. This was achieved using a step-by-step procedure of the method of successive

loadings with small steps. In this case, a complex nonlinear problem is divided into a sequence of linear problems, which are solved at each stage as elastic problems. The system of canonical equations of the displacement method is written in increments for fixed values of the correction functions $f_j(\alpha_i)$. During the transition from one loading stage to another, the length of the PZ l_{pi} increases, with subsequent adjustment of the correction functions.

6. An example of a nonlinear calculation of a 2-story steel frame for the action of horizontal forces was used to demonstrate the reliable operation of the nonlinear analysis algorithm. This included determining the limiting values of the load and internal forces (bending moment diagrams) for a given length of the PZ. All the main stages of the analysis were presented, including the limiting elastic, limiting plastic, intermediate, and final states of the structure model. The analysis showed that, in practical calculations of seismic-resistant frames, plastic deformations in the EPZ can be neglected.

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