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Optimum space frames with rectangular plans

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Abstract. In this article, the object of research is spatial framed systems, one of the most commonly used types of spatial structures. The main feature of the research is the expansion of proven design solutions to the area of large-span frames with rectangular plan with an aspect ratio of less than 1:2, which is an urgent research and practical task. In this regard, the main purpose of the research study is to establish a connection between the main parameters of the projected object (geometric characteristics, structural loads) and their metal intensity. The study was based on a number of research methods. We used the finite element method in the numerical study of the coating stability of rods loaded in the axial direction. The method of physical modeling helped in experimental studies of models and coatings of their elements. Finally, the method of optimal design, specifically the Nelder–Mead method, was used to find the basic shape of a structure with a long-span rectangular plan. Main results. First, the data from theoretical and experimental studies confirmed a decrease in the estimated length of the compressed elements by 5...25 % due to their partial pinching in the ball nodes-connectors. Secondly, we developed an optimal design algorithm of spatial frames with long-span rectangular plans with an aspect ratio of less than 1:2. It differs from the previously developed ones due to a clarification of the load-carrying ability of axially loaded rods from the stability condition and the project designer's advanced capabilities in terms of their shaping. It provides an opportunity to use clear correspondences at the trial design stage and to clarify the specific metal consumption to set the optimal geometric parameters of the projected structure. We found patterns that make it possible to design optimal material consumption flat and spatial structural forms of spatial frames on rectangular large-span plans with an aspect ratio of less than 1:2, while taking into account the refined bearing capacity of rods loaded in the axial direction. The results obtained make it possible to use a proven limited range of structural elements in the form of round-section rods and connecting elements (ball-and-socket plug-connectors).

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1. Introduction

Space frames (Fig. 1 a, b) are among the most commonly used types of spatial bar structures for coverings. Therefore, further research studies for improving structural forms are the basic improvement of the calculation, design, and construction procedures. The analysis of the stress-strain state of space frames is carried out in various software systems for calculation and design. The structural model contains multiple statically indeterminate hinge-rod systems. In such a case, the resulting stress in the selected section is the longitudinal stress that presents itself in a single element.

Figure 1a. Fragment of space frames (Donetsk, DPR, Russia)

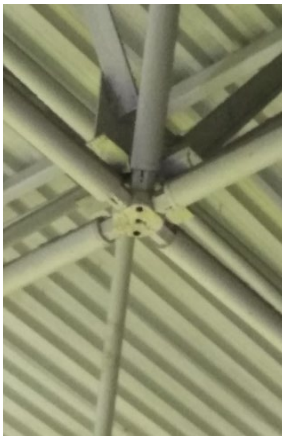


Figure 1b. Plug-connector node (pinned)

The space frame structures designed earlier for mass use have some typical limitations [1]:

- maximum span is 42 m;
- standard aspect ratio in the plan is 1:1…1:2;
- limited assortment of elements and nodes designed for spans up to 42 m and for the aspect ratio from 1:1 to 1:2 in the plan (Table 1).

Table 1. Geometric and strength characteristics of a limited range of elements used*.

No.	Calculated force in the rod, N	Mechanical characteristics of the material, MPa		Type of the element	Section, mm × mm
		σ_{yield}	$\sigma_{tensile\_strenght}$		
1	36800	240	360	Rod	48×3
2	50400				60×3.2
3	64600				76×3.2
4	83000				89×3.5
5	95600				102×3.5
6	122000				114×4
7	138300				127×4.5
8	201300				127×6
9	277300				133×8
10	377100				146×10
11	489100				159×12
12	-	785	980	Connector	∅120
13					∅150
14	-				∅170
15	-				∅200

* - the data given in Table 1 are taken from [1–3]

The approximating solutions as a discrete-continuous model, proposed by V.Z. Vlasov [4–6], are used for a multi-connected plate. Vlasov's concepts have been developed in [1–3]. However, the structural plates should be considered multiple statically indeterminate hinge-rod systems. In addition, in design, the forces in the rods of different zones of the plate are calculated depending on the structure's geometry, means of support, load applications, etc. Therefore, the use of such methods is possible at the stage of outline design of the grid.

Numerical approaches implemented in finite element analysis are the most common method of SSS analysis. The finite element analysis displays all the rod elements of the space frames that form the system. Such a system is a multiple statically indeterminate hinge-rod system. And the selection of sections is made for each individually modeled element. The authors performed a critical analysis of a number of scientific works, which can be divided into 3 large groups:

Papers in which the problem of stability of centrally compressed rods is considered from various positions. One of the most important indicators when choosing sections of rods with axial load is their stability. Since a huge number of works are devoted to this problem, we will consider only a few of them concerning the issues of studying stability using finite element models. Naturally, the matrices of shape functions used in finite element models gradually developed from the simplest, based on the hypotheses of L. Euler [7] (the initial deflection in the form of a half-wave of a sinusoid during elastic operation of the rod material is considered), to the more complex ones of F. Engesser [8] and F.S. Yasinsky [9] (the elasto-plastic work of the material in an approximate formulation is taken into account. By the way, the authors Y. Zheng, H. Zheng [10], who in their work substantiate application of a design solution that reduces the value of the calculated eccentricity in an eccentrically compressed rod). Finally, the classic papers of B.G. Galerkin [11] and N.S. Streletsky [12] made it possible to generalize the experience of numerous theoretical and experimental studies for centrally, eccentrically compressed and compressed-bent rods, loss of spatial stability of solid rods, and features of the operation of through rods. A.R. Rzhanitsyn supplemented these materials with the solution of a number of stability problems from the standpoint of probability theory, which served as the basis for assigning values of the safety factor, and the results of studies of the stability of rod systems [13]. Integration of the results into a unified system for describing the process of buckling, taking into account the influence of rod bending, is presented in the scientific papers of S.P. Timoshenko, J. Geer [14] and S.D. Leites [15]. A certain generalization of methods for calculating the stability of centrally and eccentrically compressed rods at the present stage was given by I.D. Anikeev, A.V. Golikov in article [16], within the framework of which a comparative analysis of methods for calculating the stability of rods, which form the basis of regulatory documents of the CIS countries, was carried out and Europe. The different methodological basis in determining the value of random eccentricity is emphasized.

Standing somewhat apart from this list are the results of a study of the stability of centrally and eccentrically compressed rods made of composite materials. Although composite materials range from traditional (B. Li, H. Luo, H. Wang, M. Bosco [17] defective steel bars reinforced with CFRP) to exotic (W. Zhao, Z. Chen, B. Yang [18] steel and bamboo), the authors use the above described methodological framework to conduct their research. And the most complete modern representation of the capabilities of the finite element method in solving stability problems, corresponding to the problems solved in this article, is presented in the papers:

- A.V. Perelmuter, V.I. Slivker, S.Yu. Fialko [19–22], which discuss the peculiarities of using universal calculation systems from the standpoint of assessing the complexity of systems and its components, the correctness of finite element models, the scale of the problems being solved, the heterogeneity of the finite elements and their relationships;
- F. Yunfeng, W. Li, T.E.E. Kong Fa [23], in which the implicit function apparatus was used to calculate the reliability index of steel coating structures in the presence of a large amount of data. The Monte Carlo method is used as a method for determining the numerical value of the probability of failure.

Papers devoted to the features of a refined analysis of the stress-strain state of spatial rod systems in the form of structural structures:

- in the research paper of A.B. Bondarev and A.M. Yugov [24], the influence of random imperfections acquired by a spatial structural structure at the installation stage, the method for calculating the accuracy of large-span metal rod systems and its mathematical model are considered. Very close to it in terms of the formulation of the problem and research methods is the study of M. Gordini and M. Habibi [25], in which the problem of the influence of installation imperfections is considered for two-layer lattice space trusses (DLGST). Differences in the length of elements were modeled by random variables generated in accordance with using the normal distribution law;

- Structural structures are considered from the same positions in the research of E. Gaylord, K. Gaylord and J. Stollmeyer [26], which presents calculation and design methods that make it possible to analytically assess the stress-strain state of a spatial frame. The authors went somewhat further in this direction: S. Liu, L. He, Z. Wu, J. Yuan [27], who take into account the influence of the deformability of the node, when it is considered as a deformable semi-rigid connection, on the change in the geometry of the elements converging in it when creating a stochastic element models;
- the problem of improving the design of a node for connecting elements of structural structures is considered in the paper of V. Hassani [28] (the possibilities of designing nodes focused on the technological capabilities of 3D printing for metal are presented. In this case, the procedure for optimal design of a node is performed using a genetic algorithm to minimize the maximum background stress von Mises as an objective function depending on the node mass as a constraint function). Close in meaning is the article of T. Sathish, S. Dinesh Kumar, S. Karthick [29], where the problem of improving the reliability of a unit is solved based on a comparative analysis of the results of using aluminum alloys AA2014, AA6061 and AA7075. In the same series are the papers of the authors J. Lange, T. Feucht, M. Erven [30] and K. Buchanan, L. Gardner [31], where the issues of forming units at the design and manufacturing stage are solved using additive manufacturing technologies. However, we immediately note that in all the analyzed works there is no feedback in the form of the influence of the selected node shape on the stress-strain state of the rod;
- very interesting and close in one of the final goals of our research is the paper of S. Lan, H. Tu, J. Xue and others [32], in which the problem of shaping a structural structure is solved using the proposed adaptive method for determining the shape. However, in contrast to the approach proposed in this research, here the problem of forming an effective structure is solved on the basis of minimizing the lengths of the rods;
- also, the problems of shaping spatial rod structures are solved in the article of S. Li, J. Xu, G. Feng, Z. Zhu [33] (the features of the stress-strain state of 2 structural systems of vertical and inverted hexagonal pyramids are analyzed), and the features changes in the dynamic characteristics of spatial rod structures due to the magnitude and nature of the application of snow load are discussed in detail in the research of H. Guan, H. Chen, J. He, H. Sun [34].

The majority of two factors in studying the buckling process using the FEM are noted:

- the difference between the actual fastenings of the rod from the idealized ones;
- different forms of stability loss and the ability to predict them.

The structural model simplifies in all the works studied. Namely, the detailed modeling of nodes and their influence on the load structure is not considered. Therefore, clarified modeling will allow the implementation of the followings:

- keeping the actual restraints due to the applied structural concept for nodes of space frames elements;
- the creation of such a grid of nodes in the structural model will allow describing most accurately the initial mistakes, which is characteristic of the subsequent deformation of the rod in the process of buckling.

It should be noted that all the researchers presented above dealt mainly with the issues of SSS of space frames' nodes. Less attention was paid to the stability of space frames' loaded elements. The influence of the initial geometric imperfections of the rods and rigidity of the nodes on the load-carrying ability of the diagonal member from the stability condition was not considered.

Papers devoted to the problem of optimization of structural structures: Works devoted to the problem of variant and optimal design represent a huge body of research, the analysis of which should be devoted to separate works. The standard criterion for the quality of the project (optimality criterion), namely, the objective function (effectiveness function), is the extreme value of the function with the required parameters. The most common quality criterion for a project is metal intensity. Therefore, without claiming to be a complete review on this issue, the authors would like to note that the practical application of variant and optimal design in the design of structural structures is associated, first of all, with researches of Ya.M. Likhtarnikov [35], in which the basis of the method was developed, within which the main approaches to the formulation of the goal function in the form of a minimum mass, labor intensity or cost of manufacturing and installation, cost in business or the present value of the structure were detailed. Optimization of standard and unique core slabs, shells and domes according to a complex economic criterion – reduced costs – was carried out in the paper of V.N. Shimanovsky, V.N. Gordeev, M.L. Grinberg [36]. Also, considerable attention is paid to the assessment of metal intensity, labor intensity of manufacturing and installation, and energy intensity of the structural form. The closest to the approach being developed is the paper of I.V. Romensky [37], in which the optimization of the goal function in the form of a minimum of mass, labor intensity or cost of manufacturing and installation, cost in the case of spatial long-span membrane coverings using the Nelder-Mead method.

Optimization based on the minimum mass criterion was used in the research of N. Petrović, N. Kostić, N. Marjanović, J. Živković, I.I. Cofaru [38], where optimization was performed by refining the geometric shape of four different topological variations of a typical trapezoidal roof truss, taking into account their effect on the total external surface area.

M. Kurniawan and A. Adha [39] present the results of a study of trusses that must satisfy an optimum of minimum cost while maximizing the use of load-bearing capacity. Due to the significant number of variables taken into account in the calculation, a feature of this study was the use of iterative procedures using genetic algorithms, in which the process proceeds in a stochastic manner.

The article by T. Zhang, K. Kawaguchi, M. Wu [40] presents a methodology for searching for an optimal strategy for folding frame structures, which is created on the basis of a generalized inverse theory and a genetic algorithm. A similar approach using a two-phase genetic optimization algorithm was used in the work of M. Kociecki, H. Adeli [41] for free-form steel space-frame roof structures. In [42] by the same authors, the algorithm was extended to optimize the topology and shape of free-form steel space-frame roof structures with complex geometry using evolutionary calculations. What is especially unusual is that in the proposed shape optimization algorithm, heuristic restrictions are introduced to achieve the goal, which make it possible not to distort the original architectural design during the optimization process.

As a brief conclusion based on the results of the critical analysis, it should be noted that in all the cases analyzed above, the rods that make up the structure are considered from the classical idealized positions of the hinge-rod model, when either the deviation of the length of the rod from the idealized value or the idealized (simplified) shape was considered in the form of an initial imperfection initial curvature, far from the geometry of the rod corresponding to the moment of loss of stability. And this factor has a huge impact on the final result of the calculation – the value of the critical force.

The approaches to optimizing the structural form of structural coverings discussed above do not take into account a number of important factors in the form of a refined load-bearing capacity of compressed rods from the stability condition, expanded possibilities for shaping structural coverings from a limited set of elements due to the transition from a flat shape to a shell of positive Gaussian curvature, and the influence of the length ratio sides of a rectangular covering plan. The above works disclose the optimization of long-span structures on traditional square plans or similar to them. These processes are possible per the current design standards with various variable parameters. However, the issue of optimizing structures on a rectangular plan with an aspect ratio less than 1:2, with the refined calculation models of space frame elements, was not addressed.

The results of the critical analysis made it possible to identify a number of key issues that were not properly reflected in the analyzed works and require further research. It is supposed to establish the relation between the main parameters of the designed object (geometric characteristics, structural design loads) and their metal intensity for large-span shells on a rectangular plan up to 126 meters in size and an aspect ratio up to 1:2. These conditions stand considering the current trend toward increasing spans and the transition to non-standard geometric shapes. The optimum design shape finding for overlapping such plans with space frames involves testing and subsequent use of two hypotheses:

- increasing the stability of axially loaded rods by taking into account their partial restraint in nodes (ball-and-socket connectors);
- lowering the maximum design forces in the chord elements of the space frames by changing their flat shape into deflected ones.

The combination of these two approaches in one design algorithm has a positive effect on:

- improvement of technical and economic indicators of the designed structures;
- the possibility of using a limited assortment of elements and nodes, designed for spans up to 42 m and for the aspect ratio from 1:1 to 1:2 in the plan.

This research aims to exploit the optimal space frame design shape on rectangular plans with an aspect ratio of less than 1:2. The significant design parameters and clarified load-carrying ability of axially loaded rods are considered. Thus, the possibility of using a limited range of structural elements provides.

Research objectives:

- to undertake the theoretical and experimental evaluation of the nodes' structural concept influence on the stability of axially loaded rods for space frames;

- to build an algorithm for the optimal design of space frames on rectangular plans, including those with an aspect ratio less than 1:2. The clarified load-carrying ability of axially loaded rods and extended approaches to shaping in the form of shallow shells of positive Gaussian curvature;
- to evaluate the influence of support compliance on the stress-strain state of the space frame structure on a non-standard plan.
- to give recommendations for the space frames' design of long-span rectangular plans with an aspect ratio of less than 1:2. Thus, the possibility of using a limited range of structural elements provides (rods, connecting elements).

2. Materials and Methods

2.1. Numerical methods

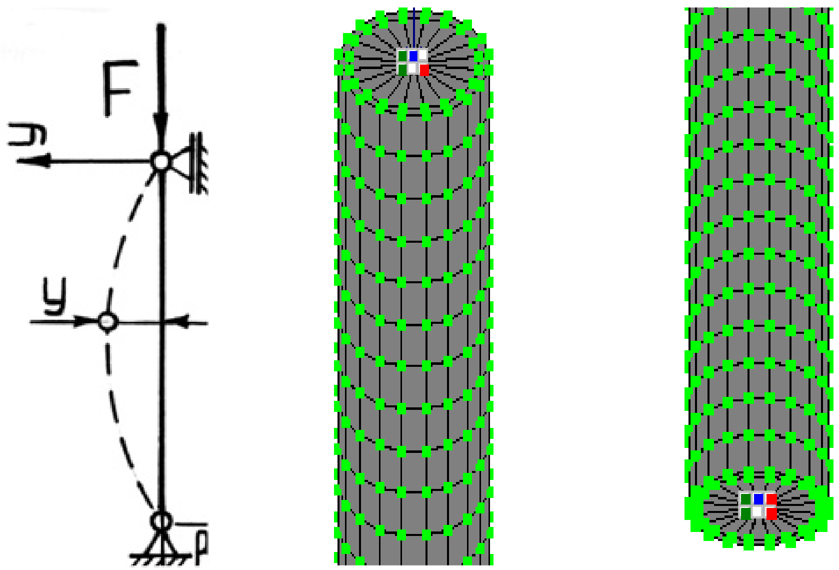
Clarifying the load-carrying ability of axially loaded rods from the condition of stability is carried out using the FEM through the displacement method. The process is possible by considering the structural design of the nodes and the spatial working of the rod elements. It should be noted that the structural model designed as a spatial shell-bar (Fig. 2) confirms its accuracy. Moreover, there is a possibility of further use in the numerical analysis of the rods' stability, taking into account the influence of nodes (Table 2). Also, the control of the stress-strain state makes it possible to more correctly take into account the change in the geometry of the rod in the process of its deformation. The process takes place during step-by-step load application in the calculation model.

The Nelder-Mead method is a preferable solution for the problem of optimizing the geometric shape of an initially flat space frame on a rectangular plan with an aspect ratio of 1:1 ... 1: 2.8 due to its main features:

- the zero-order method uses only the value of the objective function. It is easily applied to non-smooth and noisy functions. Thereby it does not apply restrictions on functions;
- the lack of the theory of convergence (the algorithm can diverge even on smooth functions);
- it does not depend on the number of control parameters;
- it allows control of the information at each iteration of optimum seeking.

2.2. Experimental methods

The method of physical modeling is chosen as an observation method. In this case, a space frame unit is taken as a model on a scale of 1:1 in relation to the full-scale structure. The full compliance with the material similarity is kept. This decision is justified by the possibility of observing the actual behavior of the structure on load and the strain capacity of nodes. Table 3 and Fig. 3 justify the accuracy of the used equipment.



Standart design scheme LIRA-SAPR 2019 R1 design scheme (FEM-program)

Figure 2. Design scheme for the model verification.

Table 2. Comparison of the results of verification calculation.

λ	Calculation results	Analystical solving (standart design scheme)	Finite element method solving (LIRA-SAPR 2019 R1)	Error between columns 3 and 4 (%)
80	σ_{cr} (Pa)	2.188e+8	2.175e+8	0.6
	N_{cr} (N)	92770	92250	
100	σ_{cr} (Pa)	2.042e+8	2.069e+8	1.3
	N_{cr} (N)	86810	87750	

λ is the flexibility of rod

σ_{cr} is the critical stress in the rod

N_{cr} is the critical longitudinal force in the rod

Table 3. Support the possibility of the measuring equipment.

Research rate	Type of equipment	Description of equipment			The graduation of a device	Measurement error
		Name	Country	Manufacturer		
Load	Jack Hydraulic (Fig. 3.a)	DG 100-200G	Russia	Consul Ltd.	1.0e+5 Pa	-
Relative deformation	Resistive strain gage (Fig 3.b)	KF5P1-20-200-A-12-S1	Russia		-	±1 Ohm
Displacement	Dial indicator (Fig 3.c)	ICH-10	Russia	JSC KP KRIN	0.01 mm	From 15 to 20 micron
Ohmic resistance change	Input module of resistive strain gage signals (Fig 3.d)	OWEN MV110-224.4.TD	Ukraine	OWEN	1.0e+4 Pa	±0.05 %

a) **b)** **c)** **d)**

Figure 3. Experimental equipment.

The physical model under analysis is created in line with geometric and physical similarity principles. At the same time, it has the same qualitative sense as the model object. There is a regulatory assortment for both rods and nodes. Thus, the indicators for scales (EI, EF geometric dimensions) are taken on a full scale of 1:1. Two experimental setups are used as experimental research for laboratory tests:

– the separate unit of the space frame in which diagonals lose stability in the elastic stage of material behavior (λ of diagonal is 120) (Fig. 4–5, a, b);

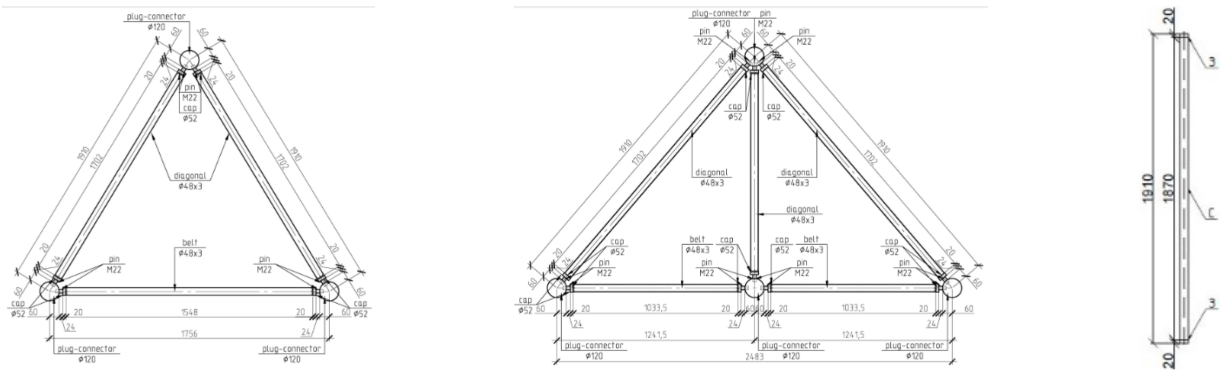
– a rod that loses stability in the elastic stage (λ = 120) (Fig. 4-5, c, d).

Based on the preliminary calculation results, the level of designed load on the tested model No. 1 is 22755 kg. The level of critical load in the inclined rods of the pyramid is $N_{cr(FEM)}$ = –79.545 kN. The level of designed load on the tested model No. 2 is 8800 kg and $N_{cr(FEM)}$ = –64.35 kN.

The load on the experimental models was applied using hydraulic jacks. For model No. 1, hydraulic jack DG-100 to the upper node is used. For model No. 2, hydraulic jack DG-20 to the lower node was used. The step of load applying for the first model is 550 kg, and for the second model is 100 kg (41 and 88 load steps, respectively).

a) b) c) d)

Figure 4. Experimental models.



a) b) c)

Figure 5. Model sizes.

The holding time of each stage is 35–55 seconds (for the load accommodation and distribution in the structure). The main reason is that the polling rate of all active strain gauges connected to the OVEN strain gauge station equals 7 polls per second. It makes it possible to average all the received data as clearly as possible. Moreover, it allows identifying the moment of distribution and stabilization in the structure of strain gauge readings. They were recorded using the MasterSCADA program as a text file. The dial gauge readings were recorded on video cameras and written down manually.

3. Results and Discussion

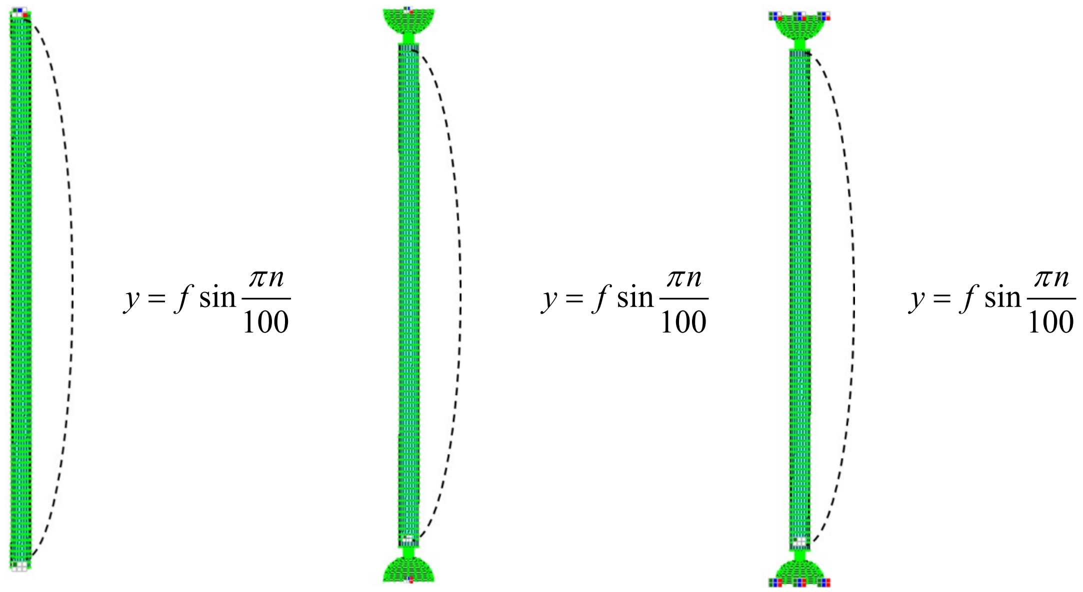
3.1. Results

3.1.1. Stability of centrally compressed rods

The axially loaded rods' buckling was studied using finite element analysis. Variable parameters are the sizes of the node structure and the flexibility of the rods.

The dimensions of the structural elements were taken per the limited assortment] (Table 1). The value of the flexibility of the elements varied in the range of 50–140. The models of shell finite elements were developed in the LIRA-SAPR 2019 R1 software package. Such elements make it possible to consider the nonlinear relation of both deformations and material properties on the load. In models, all rods have the following properties:

- initial imperfection in the form of rod's axis deviation along with the half sine wave f/l = 1/700 (Fig. 6);
- tensile stress-deformation diagram for steel S245 (R_y = 240 MPa) (for rod and plug) and 40X "Select" (R_y = 785 MPa) (for bolt and connector).



a) – scheme without node modeling **b) – scheme with are hingedly fixed node** **c) – scheme with are rigidly fixed node**

Figure 6. Design schemes with initial geometric imperfection in the form of a deviation from the rod axis along a half-wave of a sinusoid.

The methodological framework of the research work was based on the comparison of critical stresses with different modeling of rods:

- a rod without node modeling (Fig. 6, a);
- a hinged rod (Fig. 6, b);
- a rod with rigid fixing, reflecting the work of the rod in the space frame (Fig. 6, c).

Fig. 7 represents the influence of nodal connection modeling on the rod's bearing capacity as load-displacement relations.

A comparison of the data array of critical loads and stresses with the corresponding solutions is held to analyze the results of numerical studies. The results were obtained using the classical Euler method [7] and Engesser-Yasinskiy [8, 9] method, adopted in the latest scientific research over the past five years [16, 18, 43]. The results of numerical studies presented in Fig. 7 allow us to assert the need to consider the influence of pinching of the rod in the nodes when analyzing its stability. The main differences in the calculation results for the schemes presented in Fig. 6 are expressed by the following:

- there is a significant difference in the form of rod axis curvature at the moment of buckling from the deformation scheme in the form of a half sine wave used in the classical solutions of the stability problem;
- there is a pinching of a part of the length of the rod's support section in the plug-connector nodes. This pinching causes a reduction in its effective length. Therefore, the flexibility lowers, and the load-carrying ability rises from the condition of stability. The fact of lowering flexibility is confirmed by the lowering in the values of the maximum rod's deviations, corresponding to the moment of buckling ($W_{\max}$ by 3–4 times), due to the influence of the rigidity of the node (Fig. 7 a–d).

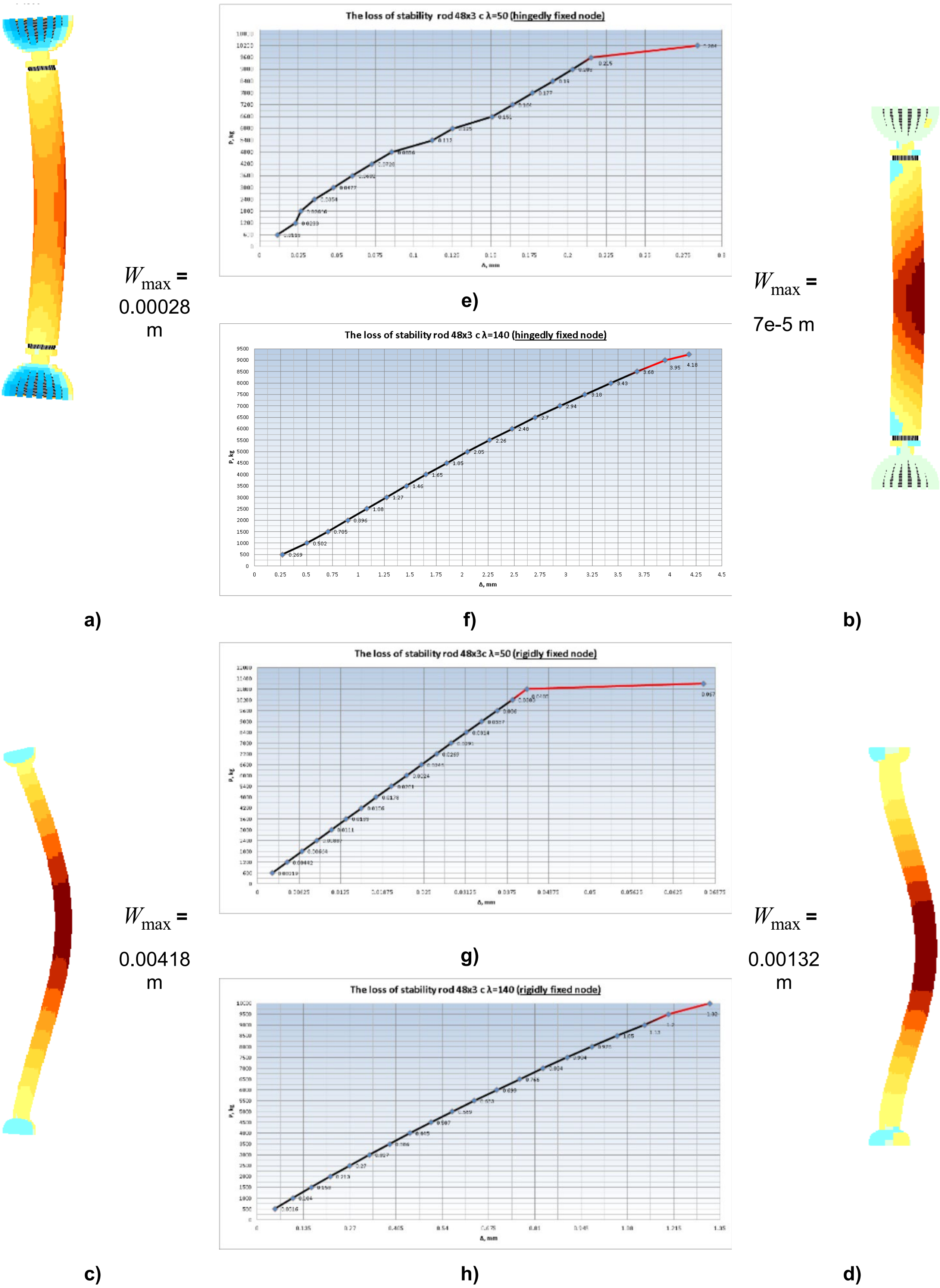


Figure 7. View of a deformed steel rod in a moment of loss of stability.

As a result, the load-carrying ability of the axially loaded rods of space frames is refined from the stability condition (1). Stability condition (1) considers the influence of nodes on the form of the rod's deformed axis, and the spatial work of the rod-shell was taken into account. The correspondence σ can be described by stability condition (1) with sufficient accuracy for practical calculations $\sigma_{cr} - \lambda$:

$$\sigma_{cr} = 0.0004\lambda^2 - 0.1536\lambda + 26.681. \quad (1)$$

Moreover, this formula (1) is the basis for the values of the conversion factor from geometrical length to calculation one (μ) and the factor of the longitudinal bending φ (for steel R_y = 240 MPa), which take into account the flexibility and pinching of the rod in the joints of structures with ball-and-socket plug-connectors. (Table 4).

Table 4. Dependences $\mu - \lambda,$ ***,*** $\varphi - \lambda$

λ	50	60	70	80	90	100	110	120
μ	0.83	0.88	0.91	0.93	0.94	0.95	0.95	0.95
φ	0.933	0.892	0.854	0.820	0.789	0.760	0.739	0.721

μ is the conversion factor from geometrical length to the calculation length of rod; φ is the factor of the longitudinal bending of rod; λ is the flexibility of rod;

3.1.2. Experimental verification of theoretical studies

Experimental studies of the influence of space frames' nodes on the stability of axially loaded rods were carried out on the basis of numerical studies [44]. Some test results are presented in tables 5 and 6 (critical longitudinal forces and their comparison with numerical studies). Figure 8 a–c shows buckled elements in experimental models.

a) Rod 1 (first model, structural covering cell) **b) Rod 2 (first model, structural covering cell)** **c) Rod 3 (second model, single rod)**

Figure 8. View of the unstable rod.

Table 5. Massive of critical loads (the first tests).

Massive of critical loads					Difference	
$N_{cr(FEM)}$, N	$N_{cr(UCC)}$, N	$N_{cr(rod1)}$, N	$N_{cr(rod.2)}$, N	$N_{cr(aver.)}$, N	Δ1–5, %	Δ2–5, %
-79545	-47000	-65801	-80245	-73023	8.2	35.64

$N_{cr(FEM)}$ is the critical longitudinal force on the finite element method.

$N_{cr(UCC)}$ is the critical longitudinal force on the Ukrainian Construction Code. Steel Structures design code..

$N_{cr(rod1)}$ is the critical longitudinal force in the first rod on the experimental model.

$N_{cr(rod.2)}$ is the critical longitudinal force in the second rod on the experimental model.

$N_{cr(aver.)}$ is the average critical longitudinal force on the experimental model.

Table 6. Massive of critical loads (the second tests).

Massive of critical loads				Difference		
$N_{cr(FEM)}$, N	$N_{cr(UCC)}$, N	$N_{cr(aver.)}$, N	$N_{cr(s.rod)}$, N	*Δ1–4*, %	*Δ2–4*, %	*Δ3–4*, %
-64350	-47200	-73023	-57410	10.78	17.78	21.038

$N_{cr(FEM)}$ is the critical longitudinal force on the finite element method.

$N_{cr(UCC)}$ is the critical longitudinal force on the Ukrainian Construction Code.

$N_{cr(aver.)}$ is the average critical longitudinal force on the experimental model.

$N_{cr(s.rod)}$ is the critical longitudinal force in the single rod (second experimental model).

The experimental studies confirm the correctness of the numerical studies' results. The hypothesis of an increase in the load-carrying ability of the axially loaded rods of the space frames due to their pinching in ball-and-socket plug-connectors is proved (the divergence of results is within 8–11 %).

3.1.3. Research results on optimization of design concepts

The search for the optimal constructive solution for space frames on rectangular plans with an aspect ratio up to 1:1 ... 1:2.8 was based on minimizing the theoretical mass of the structure. The mass of the structure is calculated from the final geometric parameters of the structural elements that make up the structure

$$G \to \min\left[\rho\left(\sum_{i=1}^{n} A_i l_i + \sum_{j=1}^{m} V_i\right)\right], \qquad (2)$$

where G is the optimized mass of the structure, ρ is the steel density, i = 1…n is the number of nod elements of the space frame, A_i is the cross-sectional area of the i-th element, l_i is the theoretical length of the i-th element (by the centers of nodes), j = 1…m is the number of connecting nodes, V_i is the true volume (excluding slots) of the j-th connector.

The space framing optimization algorithm on a rectangular plan with an aspect ratio up to 1:1…1:2.8 was based on the objective function (2). This algorithm considers the possibility of selecting sections for axially loaded rods, both per the requirements of regulatory documents and using dependence (1). According to the MATHLAB algorithm, a program optimizes the structural form of the space frame on a rectangular plan with an aspect ratio up to 1:1…1:2.8 (Fig. 9).

The optimization of the initial design solutions is carried out in two versions, using the capabilities of the algorithm:

- it is a preservation of the flat shape of the original design solution and searching for the optimal solution by varying a single parameter which is the relative height of the covering (h/b).
- the original flat shape of the design solution bending and transformation into a flat rod shell with the search for the optimal solution by varying two parameters: the relative height of the covering (h/b) and the relative camber (f/b) [45].

The following limitations are set when developing an optimal design algorithm:

- the studies are conducted for structural elements made of steel with a design resistance of 240 MPa. In the optimization process, a limited range of rod elements and ball-and-socket plug-connectors, shown in Table 1, was used;
- the relative height varies within (1/10…1/30) b here b is the short side of the plan. The reason for this is recommendations for the design of structures;
- the relative camber varies within (0…1/4.5) b. It is due to the possibility of preservation of a uniformly distributed snow load in the structural models for all optimal designs;

- the change in structural design loads is in the range of 40 to 240 kg/m^2 (the lower limit corresponds to the maximum value of the calculated constant load in the absence of snow load. The upper limit corresponds to the sum of the constant and maximum snow load);
- the aspect ratio in the plan is taken in the range from 1/1 to 1/2.8, which is explained by the thin plates theory (maximum efficiency at a ratio of 1/1 almost there is no spatial redistribution an aspect ratio of less than 1/2.5 ... 1/3);
- all rod elements in the structural model are tested for:
  - tension elements are tested for the strength conditions $\left(\sigma \le R_y\right)$;
  - axially loaded elements are tested for the strength and stability condition in one of two options specified by the project designer: in accordance with the requirements of current regulatory documents or per correspondence (1).

Statistical processing of the data array is performed using the LINEST function (multiple linear regression) in the Microsoft 365. Searching for the required function for the value of the specific density of the covering (G, calculated), the relative height of the covering $\left(h/b\right)$, and the camber $\left(f/b\right)$ was performed for multiple linear regression.

Table 7 presents the results of the optimization of structural models for the constructive structural form of coverings on a non-standard plan. Table 8 shows all the resulting formulas that can be the basis for calculating the optimal design parameters of the structure due to the setting of variable design parameters.

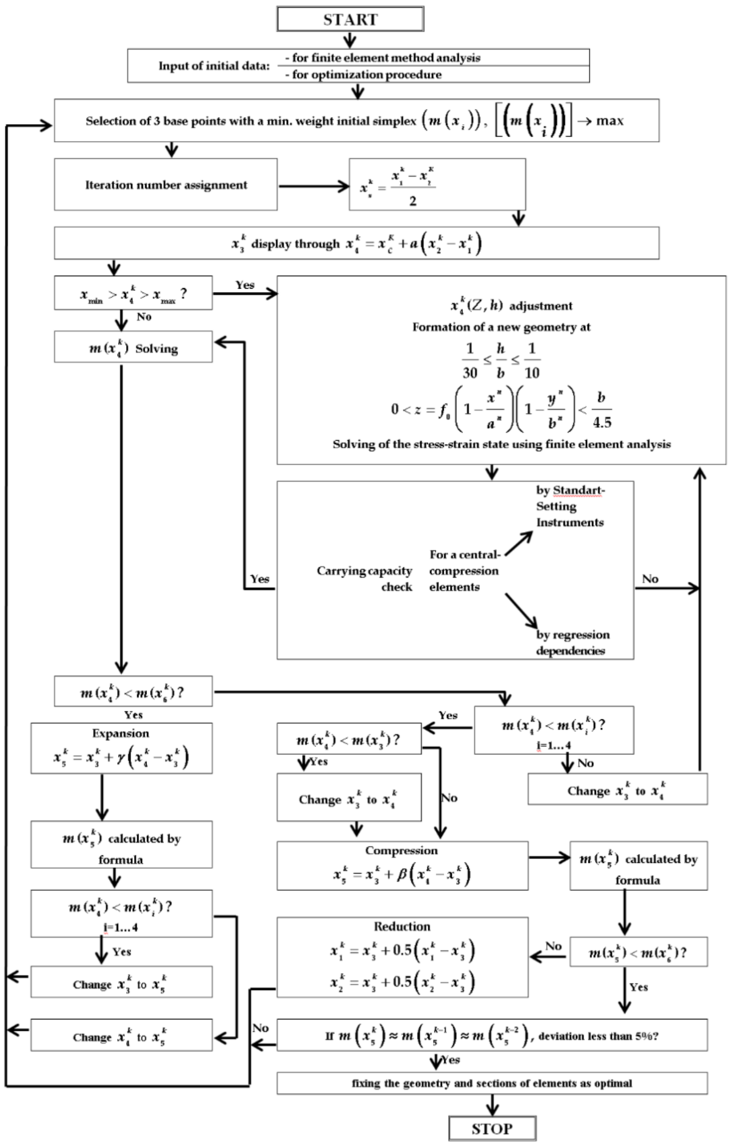


Figure 9. Implementation of the optimization algorithm block-diagram.

Table 7. Results of optimization of design solutions for the constructive form of coverings on a non-standard plan.

Structure covering in plan	Calculation load (N/m^2)	Optimization of geometric parameters			Mass of the structure (kg)	
		One parameter	Two parameters		Before optimization	After optimization (two param.)
		h (m)	h (m)	f (m)		
45×45	400	1.523	0.7031	7.856	22674.10	19610.10
	1000	1.934	0.7031	6.071	25155.60	21775.70
	1600	2.508	0.7031	5.174	28600.30	23384.00
	2400	2.836	0.7031	9.999	33275.80	25806.20
45×68	400	1.523	0.7031	4.285	30834.70	27360.60
	1000	2.344	0.7031	7.142	35591.70	30921.40
	1600	2.672	0.7031	7.856	41305.80	33750.50
	2400	3.000	0.7031	9.999	49.89960	37.10420
45×90	400	1.523	0.7031	4.107	40.82280	36.59150
	1000	2.508	0.7031	7.142	47.85630	40.68090
	1600	2.672	0.7031	9.999	56.22520	43.97220
	2400	3.000	0.7031	8.571	68.27860	48.92690
45×108	400	1.523	0.7031	4.285	50.16080	45.18870
	1000	2.508	0.7031	7.856	59.24530	49.39490
	1600	2.672	0.7031	8.928	69.07500	52.60220
	2400	3.492	0.7031	9.999	85.05950	59.66300
45×126	400	1.605	0.7031	5.000	57.59710	51.77130
	1000	2.508	0.7031	9.999	69.58510	58.43290
	1600	2.672	0.7031	9.999	82.64190	63.10020
	2400	3.328	0.7031	9.999	100.48210	70.88510

Table 8. Resulting regression formulas.

$$y = b_0 \pm b_1 x_1 \pm b_2 x_2, \tag{3}$$

y is the required indicator; x_1 is the covering calculation load; x_2 is the aspect ratio in plan; b_0, b_1, b_2 is the support factors.

Formula	Description	Correl. factor
$G_{ucc} = 8.56004+0.03523\times$ $\times q\text{-}0.038389\times a/b$	Specific gravity calculated from the height of the covering. The application of the formula for the selection of axially loaded rods is possible according to the current regulatory documents	0.99007
$G_{meth} = 8.52905 + 0.03384\cdot$ $\cdot q - 0.2448 \cdot a / b$	Specific gravity calculated from the height of the covering. The application of the formula for the selection of axially loaded rods is possible according to the proposed technique	0.99004
$h / b_{ucc} = 0.03809 + 0.0019\cdot$ $\cdot q - 0.0138 \cdot a / b$	Relative height calculated from the height of the covering. The application of the formula for the selection of axially loaded rods is possible according to the current regulatory documents	0.91866
$h / b_{meth} = 0.0393 + 0.0018\cdot$ $\cdot q - 0.0161 \cdot a / b$	Relative height calculated from the height of the covering. The application of the formula for the selection of axially loaded rods is possible according to the proposed technique	0.91912
$G_{ucc} = 8.24686 + 0.01873\cdot$ $\cdot q + 0.47052 \cdot a / b$	Specific gravity calculated from the height of the covering and camber. The application of the formula for the selection of axially loaded rods is possible according to the current regulatory documents	0.98861
$G_{meth} = 8.20306 + 0.01541\cdot$ $\cdot q + 0.73692 \cdot a / b$	Specific gravity calculated from the height of the covering and camber. The application of the formula for the selection of axially loaded rods is possible according to the proposed technique	0.98379

$$y = b_0 \pm b_1 x_1 \pm b_2 x_2, \quad (3)$$

y is the required indicator; x_1 is the covering calculation load; x_2 is the aspect ratio in plan; b_0, b_1, b_2 is the support factors.

Formula	Description	Correl. factor
$h / b_{ucc} = 0.22617 + 0.000003\cdot$ $\cdot q - 0.0234 \cdot a / b$	Relative height calculated from the height of the covering and camber. The application of the formula for the selection of axially loaded rods is possible according to the current regulatory documents	0.54018
$h/b_{Meth}=0.23334+0.0000046$ $*q-0.0429*a/b$	Relative height calculated from the height of the covering and camber. The application of the formula for the selection of axially loaded rods is possible according to the proposed technique	0.76918

The flexibility of supports affects the metal intensity of the covering and its optimal geometric parameters [46]. It should be considered when developing a design solution. Table 9 shows the results for the plan of 45×90 m with different ratios of the flexural rigidity of the supports and the span of the covering.

Table 9. Results of optimization of design solutions for the constructive form of coverings on a 45×90 plan with the account of the pliability of the supports.

Stiffness ratio k_1 *	Calculation load (N/m^2)	Optimization for two parameters		Optimal structural covering weight (kg)	
		h, m	f, m	On the fixed supports	On the pliability supports
∞	400	0.7031	10	36591.50	-
	1000	0.7031	10	40680.09	-
	1600	0.7031	10	43972.20	-
	2400	0.7031	10	48926.90	-
10:1	400	2.344	10	-	41189.10
	1000	3.574	9.582	-	44460.70
	1600	3.574	10	-	47992.30
	2400	2.139	10	-	52885.80
1:1	400	3.574	10	-	35478.00
	1000	3.369	10	-	39401.60
	1600	3.164	10	-	44270.90
	2400	2.344	10	-	48124.40
1:10	400	3.369	9.791	-	35801.50
	1000	2.344	6.666	-	42738.10
	1600	2.344	10	-	45771.00
	2400	2.344	10	-	50983.60

* - Note: $k_1 = EI / D \cdot b$, here k_1 is a coefficient considering the influence of the flexural rigidity of the span and supports on the optimal design parameters ratio; EI is the rigidity of supports (columns); D is the bending stiffness of the slab, b is the distance between the supports (columns) in the cross direction.

Table 10 gives the calculated regressional dependencies. It is possible to specify the main geometric parameters of the designed structure. These specifications can provide the optimal steel consumption considering the flexibility of the supports.

We conclude a significant reduction in the metal intensity of the space frame according to the analysis. This approach, based on the appointment of the change in geometry and the usage of a refined assessment to the load-carrying ability of axially loaded rods, fact confirms the correctness of the hypotheses underlying the research work.

Table 10. Resulting regression formulas with account the pliability of the supports

Formula	Description	Correl. factor
$G_{calc} = 8.63805 - 0.00034 \cdot k1 + 0.01923 \cdot q_{calc}$	Specific gravity calculated from the height of the covering and camber. The application of the formula for the selection of axially loaded rods is possible according to the current regulatory documents	0.958859
$G_{calc} = 8.74417 - 0.00037 \cdot k1 + 0.01587 \cdot q_{calc}$	Specific gravity calculated from the height of the covering and camber. The application of the formula for the selection of axially loaded rods is possible according to the proposed technique	0.933453
$h / b_{calc} = 0.064894 - 0.000046 \cdot k1 - 0.000021 \cdot q_{calc}$	Relative height calculated from the height of the covering and camber. The application of the formula for the selection of axially loaded rods is possible according to the current regulatory documents	0.784184
$h / b_{calc} = 0.072853 - 0.000048 \cdot k1 - 0.000066 \cdot q_{calc}$	Relative height calculated from the height of the covering and camber. The application of the formula for the selection of axially loaded rods is possible according to the proposed technique	0.907314

The final form of the covering structure and its geometric parameters based on the results of optimization is presented in Fig. 10 a–d.

3.2. Discussion

In this paper, the authors improved the process of designing space frames on rectangular large-span plans with an aspect ratio up to 1:1…1:2.8. The following step-by-step improvements are made:

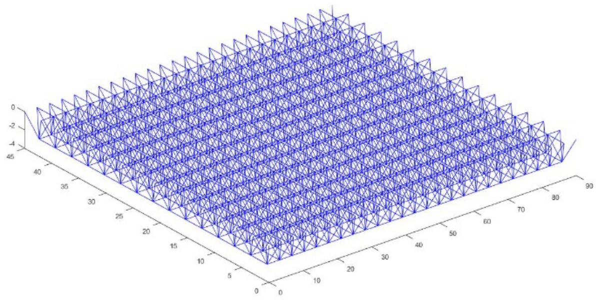
1. clarification of the traditional structural model;
2. clarification of the bearing load-carrying ability of rod elements;
3. optimization of the design shape.

The hypothesis on the influence of the node's design on the load-carrying ability of loaded rods from the condition of stability is confirmed. The results correlate with the data of A.V. Perelmuter, V.I. Slivker, S.Y. Fialko [19–22], I.D. Anikeev, A.V. Golikov [16], F. Yongfeng, W. Li, T.E.E. Kong Fah [23]. The experimental data by Central Research Institute of Building Structures named after Kucherenko (Moscow, Russia) [1] correlates in cases of coincidence of slenderness and conditions of jointing of the studied rods.

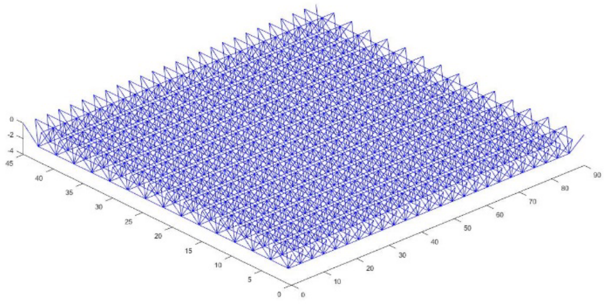
An important practical outcome of the research results is recommendations for project designers on using certain factors. These are the factor of the longitudinal bending φ and the conversion factor from geometrical length to calculation one (μ). The values of these factors for the rods jointed by nodes are given. Practical recommendations for the optimal parameters of the designed structure, which can be used at the initial design stage, are presented in the paper too.

At the same time, there are several topics for further research:

- expanding the research area for the nodal joint design and the load-carrying ability of the loaded rods of space frames by considering other types of nodes and steel classes;
- conducting additional research related to the type and parameters of the specified initial geometric imperfection while studying the stability of axially loaded rods. An additional issue of this line may be a connection between the design of the node and the given initial geometric imperfection of the rod element shape;
- developing the constructive form optimization algorithm associated with the expansion of the number of variable parameters and the specified structural forms.

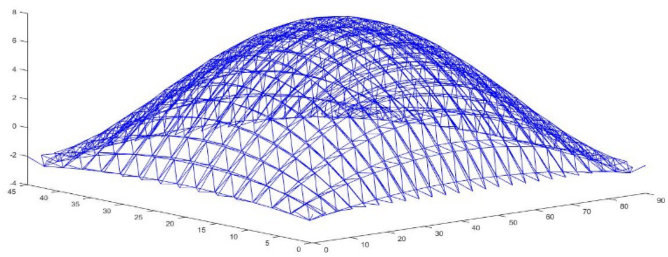


a) – the original scheme



$$\left(\frac{1}{12} < h_{opt}/b < \frac{1}{22}\right)$$

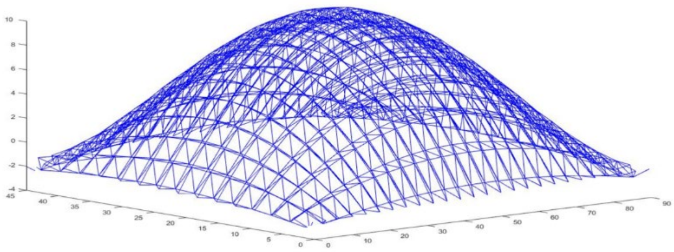
b) – optimization for one parameter



$$\left(h_{opt}/b \approx \frac{1}{64}\right);$$

$$\left(\frac{1}{4.5} < f_{opt}/b < \frac{1}{6}\right)$$

c) – optimization for two parameters



$$\left(\frac{1}{12} < h_{opt}/b < \frac{1}{19}\right);$$

$$(f_{opt}/b \approx \frac{1}{4.5})$$

d) – optimization for two parameters taking into account the pliability of the supports

Figure 10. A grapghic representaion of geomerty optimization of structural covering.

4. Conclusions

1. A correspondence of refining the load-carrying ability of axially loaded rods and the stability condition is presented. This correspondence was based on the approximation of multiple linear regression and has the stability condition (1). It fits for axially loaded rods of space frames connected at the nodes with ball-and-socket plug-connectors. The values of μ and φ factors considering the rod's flexibility and degree of pinching are presented.

We proposed an algorithm for the optimal design of space frames with rectangular plans and with an aspect ratio up to 1:1…1:2.8 presented as a block-diagram in Fig. 9. It differs from the previously developed ones by the possibility of:

- clarification of the load-carrying ability of axially loaded rods from the stability condition;
- considering the possibilities in the manufacturing and installation of the development of an optimal design solution in a traditional flat form, or in the form of a two-layer rod shell of positive Gaussian curvature;

- considering the flexibility of the supports when assigning the optimal geometric parameters of the designed structure.

2. The influence of support flexibility on the optimal parameters of the designed structure is presented in the paper. The analysis of changes in design parameters for a shell on a rectangular plan is represented by an example. It is found that:

- an increase in the supporting structures' flexibility using two control parameters (h/b and f/b) necessitates an increase of the camber f/b = 1/4.5≈0.222. If it is necessary to increase the height of the covering to h/b = 1/16… 1/20, due to the need to increase the rigidity of the span of the covering;
- the average increase of the metal intensity of the system due to the increase of the support's flexibility reaches 8 ... 12 %. There is a slight lowering in the metal intensity of the covering (up to 3 %) in some cases with a high level of support's flexibility and a low level of the design loads.

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