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Bending of multilayer beam slabs lying on an elastic half-space

***M.M. Mirsaidov* [1], *N.I. Vatin* [2], *K. Mamasoliev* [3]**

[1] *Tashkent Institute of Irrigation and Agricultural Mechanization Engineers, Tashkent, Uzbekistan*

[2] *Peter the Great St. Petersburg Polytechnic University, St. Petersburg, Russian Federation*

[3] *Samarkand State Institute of Architecture and Civil Engineering, Samarkand, Uzbekistan*

vatin@mail.ru

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Abstract. Mathematical models and analytical methods for solving contact problems of multilayer beam slabs lying on an elastic base are developed, considering the reactive normal and shear pressures of the base. In this case, an elastic filler is inserted between each pair of beam slabs. The rigidity of the filler placed between the slabs can differ in each layer. Each slab beam is subject to external loads and pressure of the filler. The stiffness coefficients of beam slabs are discrete and variable. The lower beam slab, which has a two-way connection with the elastic base, is under the influence (except for external loads) of reactive normal and shear pressure of the base. The mathematical model of the problem includes closed systems of integro-differential equations with corresponding boundary conditions. To solve the problem, an analytical method based on the approximation of Chebyshev orthogonal polynomials was used. The solution to the problem is reduced to the study of infinite systems of algebraic equations. The regularity of the resulting infinite system of equations is proven. To solve it, the reduction method was used. A test example is considered and a numerical solution to algebraic equations is obtained. The internal force factors arising in the beam slab are also investigated. Based on the analysis of numerical results, some new results were identified, i.e., a significant influence of the filler and the reactive pressure of the base on the internal force factors of the beam slab, etc.

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1. Introduction

Numerous scientific studies are devoted to the influence of contacting structural elements. Various physical models describe these relationships. Recently, the number of scientific articles in this sphere has increased dramatically; various aspects of modeling and analysis of contacting elements by mechanical parameters are presented in these articles.

Plate Behavior under Various Conditions were studied in [1–3]. The article [1] presents an effective theoretical solution for studying the time-dependent characteristics of elastic plates on layered soil. The study compares numerical results with the finite element method (FEM). It investigates the influence of load shape, plate rigidity, material anisotropy, and soft soil layers on the plate-soil system's dynamic behavior. The findings emphasize the significant impact of shallow weak layers and the increasing flexibility of the plate with decreasing burial depth.

Research in [2] proposes a strain recovery method based on surface strain measurements for analyzing large deflections in thin-plate structures. The authors develop an algorithm for strain field reconstruction and create an experimental platform to control strain under different loading conditions. Comparisons between theoretical and experimental data yield valuable insights into surface strain behavior under various loads.

A comparative study in [3] explores the nonlinear vibrations of plates fabricated from polymeric materials. The research derives governing equations for plates made of polymers and polymer composites, considering first-order shear deformations. Verification results are presented to validate the approach's accuracy.

Plate Design and Optimization were studied in [4, 5]. In [4], it introduces a novel Moving Morphable Component (MMC)-based approach for the topological design of rigid plate structures. This approach utilizes stiffeners as optimization building blocks. The research demonstrates the efficiency and effectiveness of the proposed method through numerical examples. Notably, a three-dimensional model for rigid bodies is presented, contrasting with traditional approaches. The study in [5] combines experimental and numerical analysis to assess the ultimate strength and fracture behavior of reinforced plates under combined biaxial compression and lateral loads. The results reveal that lateral pressure significantly enhances the ultimate bearing capacity.

Foundation Modeling and Analysis were done in [6–8]. In [6] and [7], the problem of determining the internal stresses of multilayer slabs was considered. In these studies, the slabs of buildings and structures foundations are considered beam slabs, and the effect of the beam on the internal tension forces in beam slabs is evaluated.

Building upon uniaxial load testing of orthotropic plates, [8] proposes an interaction function based on axial loads and shear forces. The research establishes interaction formulas for bending and failure under combined loading scenarios. The proposed formulas demonstrate excellent agreement with test results for bending interaction curves and fracture envelopes.

Dynamic Modeling and Optimization were proposed in [9, 10]. The research [9] presents a dynamic modeling approach for multi-plate structures connected by nonlinear hinges. The method utilizes Chebyshev polynomials to create dynamic models for each plate. The Rayleigh–Ritz method is then employed to derive the characteristic equation for determining the eigenfrequencies of multi-plate structures. In [10], the research focuses on optimizing the placement of longitudinal stiffeners in steel plates under pure bending. The obtained results are validated against established findings to ensure accuracy.

Simplified Models and Contact Mechanics were studied in [11, 12]. The study [11] explores one-dimensional linearly elastic models for composite layered beams. The results are compared with a two-dimensional finite element model under plane stress conditions. The findings demonstrate good agreement between the simplified multilayer sandwich model and experimental data. A mathematical model was constructed in [12] for the contact interaction between two plates with distinct elastic moduli. This model accounts for physical and structural nonlinearities. The method of variational iteration is employed to solve the governing partial differential equations, leading to a set of ordinary differential equations.

References [13, 14] provide comprehensive reviews on the development of mathematical models, methods for elastic analysis of non-homogeneous rigid bodies, and models for elastic and viscoelastic foundations in oscillating systems.

A broader range of studies, encompassing references [15–31], delve into the behavior of non-homogeneous elastic and viscoelastic systems, particularly their interaction with soil under various loading conditions. These studies offer valuable analyses of internal force factors and stress states within various systems under environmental interaction.

Along with these studies, a sufficient number of scientific works have been conducted to date, devoted to various structures interacting with media under dynamic impacts.

In [32], radial vibrations of cylindrical panels of finite length were considered using the concept of wave propagation in periodic structures. Several new results were obtained and it was shown that with the correct choice of a periodic element, it is possible to determine the boundary frequencies and corresponding modes in all propagation bands.

In [33], an assessment of large displacements of prismatic beams of variable cross-sections subjected to concentrated impacts was done. The resulting large amplitudes at the first frequency were estimated by the FEM and approximately by using polynomial functions.

The study in [34] describes the free waves propagation in a two-dimensional periodic plate using the FEM. In this case, infinite plates are considered a combination of periodic plates on an orthogonal array of simple, evenly-spaced linear supports. The eigenfrequency of the infinite plate was obtained for different wave propagation constants in two directions of the plate.

In [35], based on the theory of thin and thick plates, the vibration and energy flow of a fixed reinforced plate are studied using the finite integral transform method. It is found that including the rotational inertia of the beam and plate in the model affects only the component of the energy flow controlled by the moment coupling but not the component controlled by the shear force coupling.

In reference [36], the propagation of one-dimensional axial waves in an infinitely long periodically supported cylindrically curved panel exposed to a supersonic air flow was investigated. A line of instability was identified. The limiting frequency values and flutter pressure parameters are compared with the critical flutter state of a single curved panel by two methods – the exact method and the FEM.

Currently, many different methods have been developed for assessing the strength parameters of structures in contact interaction with elastic half-spaces under static and dynamic loads.

Despite this, today many questions related to the assessment of internal force factors arising in a structure working together with the soil base under the impact of static loads remain open.

Therefore, this study is devoted to an urgent problem of the development of mathematical models and analytical methods for assessing the internal force factors of multilayer beams lying on an elastic base.

2. Methods

2.1. The Beamslab Configuration

A n-layered beam slab interacting with a linearly deformable half-space is considered. Unlike [28], a more general model is presented here, which assumes the normal and shear pressure of the base on the strip slab. It is assumed that between each pair of beam slabs, there is an elastic filler. At that, normal external loads q_i and pressure of the filler $q_{z,i+1}$, $q_{z,1}$ act on each i-th beam slab. The first (lower) beam slab fits snugly to the base, i.e., a tear-free contact hold. It is assumed that vertical q_i and horizontal $T1$ external loads and reactive normal p and tangential τ pressures of the base act on the lower beam slab. Each beam slab has different mechanical characteristics of the material, i.e., the stiffness coefficients of the beam slabs D_i are discretely variable. It is also assumed that the length of each beam slab is the same, i.e., $2l$, the height is different, i.e., h_i, and the width equals one (Fig. 1).

Then, the problem of bending of n-layered beam slabs interacting with an elastic base is considered, an n-layered structure works with the base, considering the normal and shear pressure of the base on the strip slab under the impacts of static loads. It is required to develop a mathematical model and analytical methods for assessing the internal force factors, considering the pressure of the base and filler arising on strip slabs under various static loads.

2.2. Mathematical Model of the Problem

In the mathematical model, the origin of the system of rectangular Cartesian coordinates *xOy* is located at the center of symmetry of the lower beam slab (Fig. 1). If the deflection of the i-th beam slab is denoted by y_i, then the following functional dependencies $y_i = y_i(x)$; $q_i = q_i(x)$; $p = p(x)$; $\tau = \tau(x)$; $i = 1, 2, ..., n$ hold on the segment $[-l; l]$.

The pressure of the filler is assumed to be proportional to the differences in deflections connecting the beams of the slab $q_{z,i} = k_i(y_{i+1} - y_i)$.

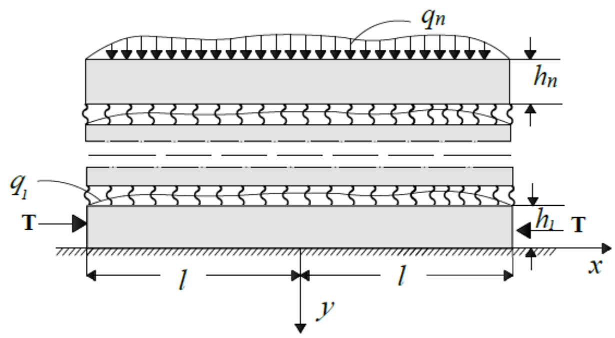

Figure 1. Calculation scheme of *n*-layer beam slabs interacting with a linearly deformable half-space.

Here, the coefficients of proportionality k_i refer to the stiffness coefficients of the filler.

Based on the above assumptions and notation, the following system of differential equations can be written for the beam slab deflection

$$\left.\begin{array}{l} D_n y_n^{IV} = q_n - k_{n-1}\left(y_n - y_{n-1}\right) \\ D_{n-1} y_{n-1}^{IV} = q_{n-1} + k_{n-2}\left(y_n - y_{n-1}\right) - k_{n-2}\left(y_{n-1} - y_{n-2}\right) \\ \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\ D_1 y_1^{IV} = q_1 + k_1\left(y_2 - y_1\right) - p - 0.5 h_1 \tau' \end{array}\right\}. \tag{1}$$

The following formula is used to determine horizontal displacements u_τ of the foot point of the first beam slab:

$$u_\tau = \frac{1-\nu_1^2}{h_1 E_1} \int_0^x \int_0^x \tau(x) dx dx - 0.5 h_1 y_1' + B_{1,\tau} x + B_{2,\tau}. \tag{2}$$

Here, E_1 and ν_1 are the modulus of elasticity and Poisson's ratio of the material of the first beam slab, respectively; $B_{1,\tau}, B_{2,\tau}$ are the unknown constants determined from the boundary conditions of the problem.

Vertical V and horizontal U displacements of the surface points of an elastic half-space are determined by the following formulas:

$$(V,U) = \int_{-l}^{l} \left[\alpha_1 (p(s), \tau(s)) \ln \frac{1}{|x-s|} + (\tau(s), -p(s)) \alpha_2 sign(x-s) \right] ds, \tag{3}$$

where:

$$\alpha_1 = 2\left(1-\nu_0^2\right) / \left(\pi E_0\right); \qquad \alpha_2 = \left(1+\nu_0\right)\left(1-2\nu_0\right) / 2E_0.$$

Equilibrium equations are written as:

$$\int_{-l}^{l} p(x) dx = R, \quad \int_{-l}^{l} \tau(x) dx = T, \quad \int_{-l}^{l} x p(x) dx = M. \tag{4}$$

Here, R, T, M are the sums of vertical and horizontal forces and their moments, respectively, from all external loads on the middle of the beam slab.

According to the assumptions, the first beam slab fits snugly into the foundation. The relationship in the form of contact conditions of the beam slab and foundation is written as:

$$y_1(x) = V(x), \quad u_\tau(x) = U(x), \quad -l \le x \le l. \tag{5}$$

The set of equations (1), (2), (3), (4), and (5) are the resolving equations of the problem, and they form a system of closed equations. The closedness of the system of resolving equations confirms the correctness of the problem under consideration.

3. Results and Discussion

3.1. Solution Method

The dimensionless coordinates equal to the ratio of absolute coordinates to the half-length of the beam slab were used and expressed in similar equations in matrix form for convenience.

A series in orthogonal $T_n(x)$ Chebyshev polynomials of the first kind approximates the normal and shear stresses of the elastic half-space:

$$(p,\tau)=(1-x^2)^{-\frac{1}{2}}\sum_{n=0}^{\infty}(A_n,B_n)T_n(x). \tag{6}$$

Here, A_n, B_n are the unknown constants to be determined.

The substitute (6) into the equilibrium equation (4), and considering the orthogonality of the polynomials, gives the relations:

$$A_0=R/(\pi l);\quad A_1=2M/\left(\pi l^2\right);\quad B_0=T/(\pi l). \tag{7}$$

Here, the Chebyshev norm was used defined by the following formulas:

$$\left\|T_n(x)\right\|=\pi,\, for\, n=0;\quad \left\|T_n(x)\right\|=\frac{\pi}{2},\, for\, n\neq 0. \tag{8}$$

Formulas (3), which determine the vertical and horizontal displacements of the surface points of the elastic base, considering (6), are written as:

$$(V,U)=\pi\alpha_1\left[(-A_0,-B_0)\ln 2+\sum_{n=1}^{\infty}(A_n,B_n)T_n(x)/n\right]+2\alpha_1\left[(B_0,-A_0)\arcsin x-\sum_{n=1}^{\infty}(B_n,-A_n)U_n(x)/n\right]. \tag{9}$$

Here, $U_n(x)$ are the Chebyshev polynomials of the second kind.

For simplicity, a two-layer beam slab interacting with an elastic base is considered. The system of differential equations (1) takes the following form:

$$\frac{D_2}{l^4}y_2^{IV}=q_2-k_1(y_2-y_1);\qquad \frac{D_1}{l^4}y_1^{IV}=q_1+k_1(y_2-y_1)-p-\frac{h_1}{2l}\tau'. \tag{10}$$

The general solution of the system of differential equations (10), considering (6), is represented in the following form:

$$(y_1,y_2)=\frac{l^4}{D_1+D_2}\Bigg\{(1,1)F_{1,q}(x)+(-D_2,D_1)\frac{1}{l^4}F_{2,q}(x)-(1,1)\sum_{n=0}^{\infty}\left[A_nf_{p,n}(x)+B_nf_{\tau,n}(x)\right]+$$
$$+(-D_2,D_1)\frac{1}{l^4}\sum_{n=0}^{\infty}\left[A_n\varphi_{p,n}(x)+B_n\varphi_{\tau,n}(x)\right]\Bigg\}. \tag{11}$$

Here:

$$F_{1,q}(x)=\sum_{i=1}^{4}C_i\frac{x^{4-i}}{(4-i)!}+f_q(x);\qquad F_{2q}(x)=\sum_{i=1}^{4}C_{i+4}u_i(\alpha x)+\varphi_q(x);$$

are the integration constants; $u_i(x)$ are the known functions [6]:

$$f_q^{IV}(x) = q_1(x) + q_2(x);\ \varphi_q(x) = \frac{l^4}{4\alpha^3}\int_0^x u_4\left[\alpha(x-5)\right]\left[\frac{q_2(5)}{D_2} - \frac{q_1(3)}{D_1}\right]ds; \tag{12}$$

$$f_{p,n}^{IV}(x) = (1-x^2)^{-1/2}T_n(x);\quad f_{\tau,n}^{'''}(x) = \frac{h_1}{2l}f_{p,n}^{IV}(x); \tag{13}$$

$$\left(\varphi_{p,n}(x), \varphi_{\tau,n}(x)\right) = \frac{l^4}{4\alpha^3}\frac{1}{D_1}\int_0^x u_4\left[\alpha(x-s)\right]\left(f_{p,n}^{IV}(s), f_{\tau,n}^{IV}(s)\right)ds. \tag{14}$$

Expression (2), which determines the horizontal displacements of the points of the slab foot, considering (6) and (11), takes the following form:

$$u_\tau = \frac{(1-\nu_1^2)e^2}{h_1E_1}\left[B_{1,n}x + B_{2,i} + \sum_{n=0}^{\infty} B_n f_{\tau,n}^{'}(x)\right] - \frac{h_1}{2l}y_1^{'}. \tag{15}$$

The contact conditions (5) are used to determine the unknown coefficients A_n, B_n. In this case, expressions (9), (11), and (15) are put into equations (5), respectively, and both equations are multiplied by the expression $(1-x^2)^{-1/2}T_\kappa(x)$. After that, integrate the equations on the variable x in [–1; 1]. In the integration, the orthogonality of polynomials is used, and it is possible to get the following results:

$$\begin{aligned} &a_{1,k} + \sum_{n=0}^{\infty}(a_{1,n,k}A_n + b_{1,n,k}B_n) = \pi^2\alpha_1\frac{A_k}{2k}; \\ &a_{2,k} + \sum_{n=0}^{\infty}(a_{2,n,k}A_n + b_{2,n,k}B_n) = \pi^2\alpha_1\frac{B_k}{2k}; \\ &k = 2,3,4,... \end{aligned} \tag{16}$$

The following notation is used:

$$a_{1,k} = \frac{l^4}{D_1 + D_2}\int_{-1}^{1}\left\{F_{1,q}^{'}(x) + f_q(x) - \frac{D_2}{e^4}F_{2,q}^{'}(x)\right\}(1-x^2)^{-1/2}T_k(x)dx; \tag{17}$$

$$a_{2,k} = \int_{-1}^{1}\left\{\frac{(1-\nu_1^2)l^2}{h_1E_1}(B_{2,\tau}x + B_{2,\tau}) + \frac{l^4}{2h_1(D_1+D_2)}F_{1,q}^{'}(x) - \frac{D_2}{e^4}F_{2,q}^{'}(x)\right\}(1-x^2)^{-1/2}T_k(x)dx; \tag{18}$$

$$a_{1,n,k} = -\frac{e^4}{D_1+D_2}\int_{-1}^{1}\left[f_{p,n}(x) + \frac{D_2}{e^4}\varphi_{p,n}(x)\right](1-x^2)^{-1/2}T_k(x)dx; \tag{19}$$

$$a_{2,n,k} = -\frac{e^4}{2h_1(D_1+D_2)}\int_{-1}^{1}\left[f_{p,n}^{'}(x) + \frac{D_2}{e^4}\varphi_{p,n}^{'}(x)\right](1-x^2)^{-1/2}T_k(x)dx; \tag{20}$$

$$b_{1,n,k} = -\frac{e^4}{D_1+D_2}\int_{-1}^{1}\left[f_{\tau,n}(x) + \frac{D_2}{e^4}\varphi_{\tau,n}(x)\right](1-x^2)^{-1/2}T_k(x)dx; \tag{21}$$

$$b_{2,n,k} = \int_{-1}^{1}\left\{\frac{(1-\nu_1^2)e^2}{h_1E_1}f_{\tau,n}^{'}(x) + \frac{e^4}{2h_1(D_1+D_2)}\left[f_{\tau,n}^{'}(x) - \frac{D_2}{e^4}\varphi_{\tau,n}^{'}(x)\right]\right\}(1-x^2)^{-1/2}T_k(x)dx. \tag{22}$$

The integration by parts can eliminate the singularity in integrals (17), (18), (19), (20), (21), and (22) and bring the integrals to a convenient form for calculation and evaluation.

System (16) is an infinite system of algebraic equations with respect to unknown coefficients A_n, B_n. For solving system (16) the reduction method is proposed.

The patterns of pressure distribution along the base are determined by substituting the obtained values of coefficients A_n, B_n into (6). Expression (6) is put into formulas (9) and (11) to find displacements of beam slabs and base points. Based on these formulas, the internal stress factors in beam slabs are determined according to certain rules of the elasticity theory. Thus, the solution to the problem is reduced to the study of a system of infinite algebraic equations of the form (21). The existence of a bounded solution to the system (21) is equivalent to the existence of solutions for the problem under consideration.

3.2. The Existence of a Bounded Solution of the System of Equations

Since a regular system has a unique bounded solution, the regularity of the system (16) is studied. The convergence of the following sequences consisting of free coefficients $a_{1,k}$, $a_{2,k}$ of the system (16) is studied. That is, the meeting of the following conditions is analyzed to study the regularity:

$$\left|a_{1,k}\right| < \infty \ , \ \left|a_{2,k}\right| < \infty \ , \ k = 2,3,4,... \tag{23}$$

Integrating integrals (17) and (18) by parts, applying the Cauchy–Bunyakhovsky inequalities, and considering the continuity of the integrands gives the following estimates:

$$(a_{1,k} \ , \ a_{2,k}) < (a_1, a_2)\frac{1}{2k}\left\|P_{k-1}^{(1/2,1,2)}(x)\right\|^{1/2}. \tag{24}$$

Here, $\left\|P_{k-1}^{(1/2,1/2)}(x)\right\|^{1/2}$ is the norm of Jacobi polynomials. The following designations are introduced:

$$a_1 = \left\{\int_{-1}^{1}\left|\frac{e^4}{D_1+D_2}\left[F_{1,q}'(x)-\frac{D_2}{e^4}F_{2,q}'(x)\right](1-x^2)^{1/2}\right|^2 dx\right\}^{1/2}, \tag{25}$$

$$a_2 = \left\{\int_{-1}^{1}\left|\frac{(1-\nu_1^2)l}{h_1E_1}\left(B_{2,\tau}+B_{1,\tau}x\right)-\frac{h_1l^3}{2(D_1+D_2)}\left[F_{1,q}''(x)-\frac{D_2}{l^4}F_{2,q}''(x)\right](1-x^2)^{1/2}\right|^2 dx\right\}. \tag{26}$$

Considering the given estimate (24), it is certain that inequality (23) holds.

Now, the following numerical series, consisting of the coefficients of unknowns A_n, B_n of the system (16), are considered:

$$\sum_{n=0}^{\infty}\left(\left|a_{1,n,k}\right|+\left|a_{2,n,k}\right|+\left|b_{1,n,k}\right|+\left|b_{2,n,k}\right|\right). \tag{27}$$

If series (27) is a convergent series, then the sum of the series depends on the parameter k. Therefore, denoting the sum of series (27) by S_k gives the following sequence:

$$\{S_k\} \ , \ k = 2,3,4,... \tag{28}$$

The next step is studying the convergence of sequence (28). For each summand of the common term of series (27), determined by formulas (19), (20), (21), and (22) separately, the following estimates could be obtained:

$$\left|a_{1,n,k}\right| \le \frac{e^4}{D_1+D_2}(P_{n-3}+\frac{D_2}{e^4}uP_{n-2})P_{k-1}; \ \left|a_{2,n,k}\right| \le \frac{e^4}{D_1+D_2}\frac{h_1}{2l}(P_{n-2}+\frac{D_2}{e^4}uP_{n-2})P_{k-1} \tag{29}$$

$$\left|b_{1,n,k}\right| \le \frac{e^4}{D_1+D_2}(P_{n-2}+\frac{D_2}{e^4}uP_{n-2})P_{k-1}; \ \left|b_{2,n,k}\right| \le \frac{(1-\nu_1^2)e}{h_1E_1}P_{n-1}+\frac{h_1e^3}{2(D_1+D_2)}(P_{n-1}+\frac{D_2}{e}uP_{n-1})P_{k-1}. \tag{30}$$

In obtaining these estimates, integration by parts in integrals (19), (20), (21), and (22) is applied. Then, the Cauchy–Bunyakovsky inequality and the following notation is used:

$$u = \max\left|u_1(x), u_2(x), u_3(x), u_4(x)\right| \quad , -1 \le x \le 1 \tag{31}$$

$$\left|b_{1,n,k}\right| \le \frac{e^4}{D_1 + D_2}(P_{n-2} + \frac{D_2}{e^4} u P_{n-2}) P_{k-1}; P_{n-2} = \frac{1}{4n(n-1)}\left\|P_{n-1}^{(3/2,3/2)}(x)\right\|^{1/2}; P_{n-3} = \frac{1}{8n(n-1)(n-2)}\left\|P_{n-3}^{(7/2,7/2)}(x)\right\|^{1/2}. \tag{32}$$

Substituting (29) and (30) into (27), and considering (28) gives the following:

$$S_k \le P_k \left\{ \frac{l^4}{D_1 + D_2} \sum_{n=3}^{\infty} P_{n-3} + \frac{D_2}{D_1 + D_2} u(3 + \frac{h_1}{2l}) \sum_{n=2}^{\infty} P_{n-2} + \left[\frac{(1-\nu_1^2) l}{h_1 E_1} + \frac{h_1 l^3}{2(D_1 + D_2)} \left(1 + \frac{D_2}{l^4} u\right) \right] \sum_{n=1}^{\infty} P_{n-1} \right\}. \tag{33}$$

Considering (31) and (32) in inequality (33) verifies that each series on the right-hand side of inequality (33) are convergent. Therefore, the following limit holds:

$$\lim_{k \to \infty} S_k = 0. \tag{34}$$

Since estimates (23) and (34) are true, then the infinite system of algebraic equations (16) is regular. Thus, the reduction method (23)–(34) can be applied to solve the system (16). As a result, the infinite system of algebraic equations (16) can be solved using the reduction method.

3.3. Test Case

The test case is the problem of bending a two-layer beam slab interacting with an elastic base when loaded with a uniformly distributed external load of the following form:

$$q_1(x) = q_1 = const, \qquad q_2(x) = q_2 = const.$$

Due to the symmetry of external loads, (6) and (7) will take the following form:

$$\left(\rho, \tau\right) = (1 - x^2)^{-1/2} \sum_{n=0}^{\infty} \left(A_{2n} T_{2n}(x), B_{2n+1} T_{2n+1}(x)\right), A_0 = 2(q_1 + q_2)/\pi. \tag{35}$$

Expressions (9) are written in the following form:

$$\begin{aligned}(V, U) = \pi\alpha_1 &\left[\left(-A_0, 0\right) \ln 2 + \sum_{n=1}^{\infty} \left(A_{2n} \frac{T_{2n}(x)}{2n}, B_{2n-1} \frac{T_{2n-1}(x)}{2n-1} \right) \right] + \\ +2\alpha_1 &\left[(0, -A_0) \arcsin x - \sum_{n=1}^{\infty} \left(B_{2n-1} \frac{U_{2n-1}(x)}{2n-1}, A_{2n} \frac{U_{2n}(x)}{2n} \right) \right].\end{aligned} \tag{36}$$

Beam slab deflections that satisfy the boundary conditions of the problem are written as:

$$\begin{aligned}(\overline{y}_1, \overline{y}_2) = \frac{l^4}{D_1 + D_2} &\left\{ (1,1)(q_1 + q_2) \left(\frac{x^4}{24} - \frac{x^2}{2} \right) - \sum_{n=1}^{\infty} A_{2n} \left[\left(\frac{D_2}{D_1}, 1 \right) F_{1,2n}(x) + \right.\right. \\ &\left.\left. + (1,-1) f_{2n}(x) \right] - \frac{h_1}{2l} \sum_{n=1}^{\infty} B_{2n-1} \left[\left(\frac{D_2}{D_1}, 1 \right) F_{2,2n}(x) + (1,-1) f'_{2n-1}(x) \right] \right\}.\end{aligned} \tag{37}$$

Here, $\overline{y}_1, \overline{y}_2$ are the relative deflections equal to $\overline{y}_1(x) = y_1(x) - y_1(0); \quad \overline{y}_2 = y_2(x) - y_2(0);$

$$\left(F_{1,2n}(x), F_{2,2n-1}(x)\right) = \left(\varphi_{5,2n}; \omega_{5,2n-1}\right) u_1(\alpha x) + \left(\varphi_{7,2n}; \omega_{7,2n-1}\right) u_3(\alpha x) + \frac{D_1}{l^4}\left(\varphi_{2n-1}(x); \omega_{2n-1}(x)\right); \tag{38}$$

$$\begin{aligned}\left(\varphi_{5,2n}, \varphi_{7,2n}\right) = \int_0^1 &\left\{ \left(u_4(\alpha); u_2(\alpha)\right) u_2[\alpha(1-s)] + \right. \\ &\left. +(2u_1(\alpha) - 2u_3(\alpha)) u_1[\alpha(1-s)] \right\} (1-s^2)^{-1/2} T_{2n}(s) ds;\end{aligned} \tag{39}$$

$$\left(\omega_{5,2n-1},\omega_{7,2n-1}\right)=-\frac{h_1}{2lD_1}\int_0^1\left\{\left(u_4(\alpha);u_2(\alpha)\right)u_2[\alpha(1-s)]+\right.$$
$$\left.+(2u_1(\alpha)-2u_3(\alpha))u_1[\alpha(1-s)]\right\}[(1-s^2)^{-1/2}T_{2n}(s)]'ds. \quad (40)$$

The horizontal displacement of the foot point of the beam slab, which satisfies the boundary conditions, is represented in the following form:

$$u_\tau=-h_1^2l^2/\left(12D_1\right)Tx-y'h_1/\left(2l\right). \quad (41)$$

Next, the system (16) is solved in correspondence to this example (41).

Here, the solution is restricted to eight equations relative to eight unknowns A_0, A_2, A_4, A_6, A_8, B_1, B_3, B_5, B_7.

The calculations are performed for the following values of the parameters of the slab and base [6]:

$$l=500\,cm,\ h_1=h_2=45\,cm,\ q_1=q_2=q,\ \nu_1=\nu_2=0.167$$
$$E_1=E_2=1.25\cdot10^5\ kg/cm^2,\ \nu_0=0.3,\ E_0=5\cdot10^2\ kg/cm^2$$

The modulus of elasticity and Poisson's coefficients obtained for slabs and elastic base correspond to concrete slabs and clay-sand soils, respectively.

Calculations will be performed separately according to the following numerical values of the stiffness coefficients of the filler $k\ [kPa]$: $24.52; 49.03; 73.55; 122.5; 171.6$.

These values correspond to elastic materials with very low, low, and moderate porosity in the order in which the filler layer stiffness coefficients are written [31].

Table 1 gives the calculation results. The analysis of the results obtained shows that an increase in the values of the stiffness coefficients of the filler *k* does not lead to a significant change in the numerical values of the algebraic equations. A change in the numerical values of the stiffness coefficients of the filler *k* does not lead to a significant change in the pressures in the base. Retaining three terms in the series in the expansion of the pressure in the base in terms of orthogonal Chebyshev polynomials is ensured by sufficient accuracy in determining the internal force factors in the beam slab.

Table 1. Numerical values of coefficients A_{2n}, B_{2n-1} for different values of stiffness coefficients of the filler k.

$k\,[kPa]$	24.52	49.03	73.55	122.5	171.6
A_0/q	1.273239	1.273239	1.273239	1.273239	1.273239
A_2/q	–0.308264	–0.306441	–0.304161	–0.302753	–0.301269
A_4/q	–0.041247	–0.040962	–0.037824	–0.033472	–0.031643
A_6/q	0.002193	0.002081	0.001973	0.001746	0.001385
A_8/q	0.0003427	0.0003165	0.0002761	0.0002472	0.0002169
B_1/q	0.817618	0.806977	0.793185	0.784676	0.778634
B_3/q	–0.427618	–0.421738	–0.415627	–0.410698	–0.407164
B_5/q	–0.044363	–0.041764	–0.037954	–0.031761	–0.030118
B_7/q	0.001394	0.001169	0.000875	0.000554	0.000347

The solutions to the system of algebraic equations presented in Table 1 differ from the quantities in the table presented in [6] by the characteristics of the equations and the numerical amount of the unknowns in it. From Table 1, it is possible to determine the numerical value of the unknowns of the system of algebraic equations in order to perform the project calculations with the required accuracy. In particular, for calculations with an accuracy of 0.01, it is enough that the number of unknowns in the system of algebraic equations is six.

Table 2 shows the highest values of bending moments $M_{1,\tau}$ and $M_{2,\tau}$ of the beam slabs (for $x = 0$). It shows the values of bending moments M_1 and M_2, obtained without considering the shear stresses given in [6].

Table 2. The highest values of bending moments in beam slabs.

$k\,[kPa]$	24.52	49.03	73.55	122.5	171.6
$M_{1,\tau}/(ql^2)$	0.025764	0.0241627	0.023866	0.023219	0.022761
$M_1/(ql^2)$	0.103563	0.103012	0.099073	0.098169	0.091864
$M_{2,\tau}/(ql^2)$	0.016191	0.016843	0.017384	0.018175	0.019568
$M_2/(ql^2)$	0.069591	0.071368	0.072437	0.074981	0.077576

The analysis of the results obtained allows to draw the following conclusions: with a decrease in the stiffness coefficients of the filler k, the bending moments in the first beam slab increase, and in the second beam slab, they decrease; with an increase in the stiffness of the filler k, the bending moments of the beam slab significantly approach each other; at the accepted values of the stiffness coefficients of the filler k, an account for the shear stresses in the base leads to a significant (up to 24%) decrease in the bending moments of the first and second beam slabs (Table 2).

4. Conclusions

1. A mathematical model was developed for the problem of bending of multilayer beam slabs interacting with an elastic half-space, taking into account shear stresses in the base.
2. An analytical method was proposed for solving the problem of bending of multilayer beam slabs interacting with an elastic half-space based on the approximation of Chebyshev polynomials.
3. The regularity of the infinite system of algebraic equations obtained by solving the problem was proved.
4. The required number of summands of the terms of the series was established with expanding the solution of the unknown pressure of the base into a series of Chebyshev polynomials.
5. The pattern of changes in the stiffness of the filler and their influence on the force factors in multilayer beam slabs was established.
6. A decrease in the force factors of multilayer beam slabs was established, considering shear stresses in the multi-layer plates.

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Information about the authors:

Mirziyod Mirsaidovich Mirsaidov, *Doctor of Technical Sciences*
ORCID: https://orcid.org/0000-0002-8907-7869
E-mail: mirsaidov1948@mail.ru

Nikolai Ivanovich Vatin, *Doctor of Technical Sciences*
ORCID: https://orcid.org/0000-0002-1196-8004
E-mail: vatin@mail.ru

Kazokboy Mamasoliev
ORCID: https://orcid.org/0000-0003-3371-5742
E-mail: q-mamasoliev@mail.ru

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