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Physical and mathematical model of a settling tank with thin-layer modules

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Abstract. The presence of fine impurities in industrial water complicates the operation of fine water purification systems. To solve this problem, thin-layer settling tanks have become widespread in industry due to their high consumption characteristics. Depending on the application and the consumer, the maximum permissible concentrations of dispersed particles can vary in wide ranges. Optimizing the operation of a thin-layer tank requires conducting field experiments in laboratory and industrial installations. Mathematical modeling allows to reduce the number of experiments. The aim of the work is to develop a mathematical proxy model of a settling tank with thin-layer modules, which for the first time considers the influence of the concentration of coagulant and alkali on the particle size distribution, as well as salinity on the particle deposition rate. This model is based on simplified laws of conservation of mass and momentum in the hydraulic approximation. To study the effect of the parameters of the settling tank and reagents on the concentration of dispersed particles at the outlet, experiments were planned and conducted on a laboratory installation in a wide range of changes in these parameters. The values of the concentration of dispersed particles at the outlet at different angles of inclination of the plates, their quantity and concentrations of coagulant, alkali, and salt were obtained. The simulation results are compared with experiments, and their satisfactory agreement with each other is shown with an average error of 3 %. Based on the sensitivity analysis, ranges of parameters of the settling tank and chemical reagents were determined, for which the mathematical model gives representative results. It was found that with an increase in the concentration of chemical reagents, the proportion of particles with sizes less than 100 μm in the stream decreases, which leads to an increase in the degree of water purification.

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1. *Introduction*

Currently, various types of settling tanks are used to effectively purify water from oil treatment facilities. These devices should be carefully selected for the composition of the incoming water, the types and concentration of pollutants – oil, suspended mineral particles and their fractional composition. The requirements of subsurface users for the degree of water purification for the reservoir pressure maintenance system are quite high, and the key parameter here is the permeability of the reservoir near the well, through which hundreds of cubic meters of water are injected. The concentration of suspended particles leads to gradual colmatation of the rock and directly affects the dynamics of reducing its permeability, which in turn increases the load on pumping and compressor equipment and, in extreme cases, can completely shut down the well. In this regard, the maximum concentration of suspended particles is regulated and should not exceed 50 mg/l[1], but in fact it should be even lower.

The mode of water injection into the pressure maintenance system, rock permeability, mineralization and density of water, and other parameters of the "above-ground equipment – fluid – reservoir" system are individual in each case, and the water treatment mode for pressure maintenance system also requires an individual approach: from a static tank for a low-flow well to a dynamic tank with a capacity of several hundred cubic meters per day. A similar approach is required by the design of settling tanks, the dimensions of which are limited by their transportability, primarily by rail transport, the requirements for ease of maintenance and a high degree of water purification with high productivity. The substantiation of the basic design of the settling tank and the effectiveness of its operation begins with the production of a laboratory sample that preserves the basic parameters of an industrial installation, and then testing its effectiveness on fluid models equivalent in composition and properties to the fluid in the field. Laboratory research is a mandatory and important component of research and development work [1], because it allows to avoid a lot of mistakes when designing a large-scale installation, save a lot of money and materials, test hypotheses and assumptions on a laboratory facility, which, unlike a full-scale installation, is easy to make changes, vary the initial and boundary parameters, easy to repair, etc. But probably the most important thing is that building a laboratory facility and testing it is relatively cheap.

Settling tanks with coalescent (thin-layer) modules are of particular interest for production and industrial implementation in field oil collection and treatment systems [2]. Such equipment is designed to purify water from mechanical impurities or oil products and has long proven itself for effective wastewater treatment. However, in order to adapt the technology for the oil industry, it is necessary, while maintaining the principle of operation, to review a number of features and materials used in the design, considering the requirements for the mode of operation of reservoir water treatment facilities and the quality of water treatment.

Thus, there is a need for a scientifically based description of the technology of separation of multiphase media using coalescer settling tanks, which includes both the creation of a laboratory installation for testing hypotheses, and a physical and mathematical model with a predictive function beyond laboratory tests, for example, in the case of scaling the installation to an industrial design, critically small or large flow rates, high concentration of suspended particles, and so on. Modeling becomes particularly relevant in the case of a significant number of influencing parameters and long-term experiments.

The existing models for calculating parameters in settling tanks with thin-layer modules are either based on a simplified hydraulic approach [3] using empirical relations [2, 4–8], or assume a detailed calculation of the multiphase flow in the settling tank using numerical methods [9–11] or commercial software [12–16]. The second approach allows to consider in more detail the nature of the flow in the device [12, 15] but requires significant time and financial costs, as well as long-term adjustment of the model to real data. The use of neural network modeling also requires a long learning process and a large sample of input information [17]. Therefore, the use of simplified modeling approaches is more justified [2]. Unfortunately, the existing models [2, 4–8] do not consider some of the effects observed in real experiments. In particular, simplified models do not consider the effect of the type and concentrations of reagents often used to increase the efficiency of the process [18, 19] on the particle size distribution, as well as mineralization of reservoir water on the particle deposition rate [20, 21].

The preparation of clean water is essential in various industries. Thus, water is used for heating rooms, cooling power plants, soil irrigation, and in many other cases [22, 23]. Since the oil and gas complex occupies a significant share in the Russian economy, the use of water for its purposes as a displacing agent in oil production should be considered [24]. As a rule, in this case, water previously extracted from the reservoir is used. Such water usually contains a large amount of various additional particles [25]. Injection of water with impurities into the reservoir can lead to contamination of the bottom-hole zone of wells and an increase in bottom-hole pressure to maintain the previous injection rates. Subsequently, this can lead to the formation of fractures in the formation [26]. Therefore, the relevant task is to clean water from impurities.

Therefore, the aim of the work is to create a physical and mathematical proxy-model of a settling tank with thin-layer modules, which for the first time considers the influence of the concentration of coagulant and alkali on the particle size distribution, as well as salinity on the particle deposition rate. Such

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[1] OST 39-225-88. Voda dlya zavodneniya neftyanyh plastov. Trebovaniya k kachestvu [Water for flooding oil reservoirs. Quality requirements]. Industry standard: Approved by Order of the Ministry of Petroleum and Industry No. 147 dated 03/28/88: introduced for the first time, date of introduction 07/01/90. Developed by Giprovostokneft of the Ministry of Oil and Gas Industry, BashNIPIneft, VNIISPTNEFT, TatNIPIneft, All-Union Oil and Gas Research Institute. Moscow: Minnefteprom, 1990. 10 p.

a model will be based on a simplified hydraulic approach using empirical relationships. To achieve this aim, the following tasks were formulated and solved: development of a physical and mathematical proxy-model for the operation of a thin-layer tank; design of experiments to determine the effect of the parameters of the tank and reagents on the concentration of dispersed particles at the outlet; assembly of a laboratory installation and experiments; validation of the developed proxy-model for the operation of a thin-layer tank; sensitivity analysis of the model.

2. Methods

2.1. Experimental Installation

The experimental installation should, on the one hand, recreate a model of reservoir water with a given mineralization and suspended mineral particles, and, on the other hand, purify this water from mineral impurities. Therefore, the first part of the installation should include a container designed for long-term experiments, an agitator to maintain the homogenization of the mixture, a pump for injection the mixture, a system for measuring and controlling flow, a system for maintaining water temperature, and so on. The second part of the installation is a "coalescer tank", which in laboratory design should allow visual observation of the processes of separation and purification of water but should repeat the basic design solutions of industrial settling tanks.

As a result of the design and preparation of design documentation, considering the limitations and requirements imposed on the laboratory sample, the laboratory installation scheme shown in Fig. 1 was developed and approved. The installation consists of the following main elements: a T500l tank, a propeller agitator for a T500 tank, a TSM.50M resistance thermometer, a two-channel TRM202 regulator meter, a Vortex FN-250A pump, a flow meter and a flow meter controller, a 0.8 kW fuel tank, an ESQ-A500 frequency converter, pipelines, shut-off valves, etc. The coalescer tank is made of 12 mm thick plexiglass sheet, its geometric parameters are: length – 1050 mm; width – 152 mm; height – 450 mm. The block of thin-layer modules includes plates and brackets for fixing the plates, which allow to vary the angle of inclination of the plates and their number within the range of 45° and 60°; number of plates: for 45° angle, the maximum number is 26 pcs., for 60° angle, the maximum number is 42 pcs. The material of the plates and brackets is St3 steel. Plate sizes for 45° angle are 495×151×2 mm, for an angle of 60° are 404×151×2 mm. The schematic diagram is shown in Fig. 1.

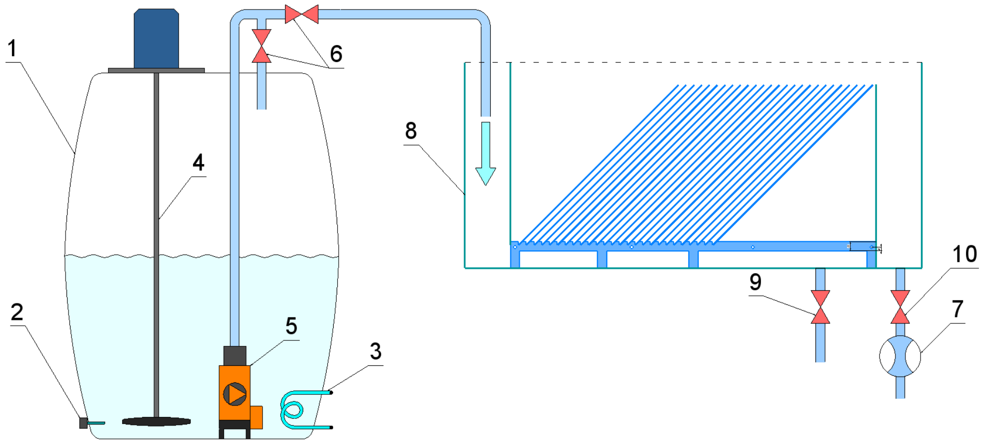

Figure 1. Schematic diagram of the laboratory installation: 1 – source water tank (500 l), 2 – temperature sensor, 3 – water heater, 4 – agitator, 5 – liquid pump, 6 – pressure control valves, 7 – flow meter, 8 – coalescer tank, 9 – sediment drain tap, 10 – tap for draining pure water.

The principle of operation of the installation is as follows. Process water is poured into the source water tank 1. The set temperature value is set on the controller of the temperature control system, the water is heated by the spiral heater 3, and the temperature is monitored using the temperature sensor 2. For uniform heating of the liquid, the top-mounted agitator 4 is switched on. If the experiment is carried out on mineralized water, the required amount of salt (NaCl) is poured into the water during the heating process to the value of the declared mineralization. To achieve concentrations of suspended solids, a combination of sand and clay of large and small fractions is poured into the container during mixing in a given mass ratio. If the experiment involved the use of chemical reagents to intensify purification, then aqueous solutions of these reagents were prepared in advance, and the solutions were added to the container based on the calculated concentration per volume of water in the container.

In parallel with the water preparation, the coalescer 8 was filled with the same 54 l of process water (the volume of the flow quenching zone and the zone of thin-layer modules), the water temperature was adjusted to the water temperature in the source water tank. Next, the liquid injection pump 5 is started and the required flow rate is set using the control valves 6. The flow rate is monitored both by means of a 7-meter flow meter and visually on a measuring scale in the clean water area. To measure the flow rate using a scale, it is necessary to periodically turn off the tap 10 and measure the filling volume of the pure water zone for a fixed time.

First, water with mechanical impurities enters the flow damping zone in the tank. There, the flow rate decreases and some of the mechanical impurities fall out, after which the liquid enters through the passage window into the zone of thin-layer modules and rises up. A block of thin-layer modules made of plates at a certain angle is installed in the flow path. With the correct selection of the operating mode, suspended particles, moving in the space between adjacent plates, should settle on the lower shelf, concentrate and slide down, accumulating in the lower part of the zone of thin-layer modules, and purified water overflows into the zone of clean water (see Fig. 2).

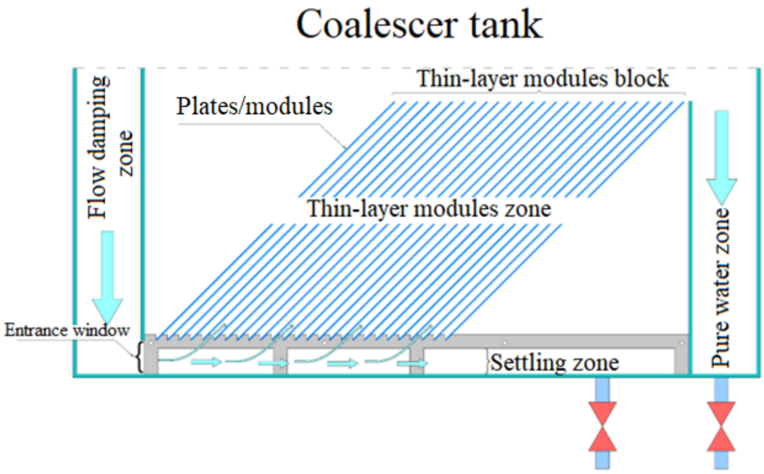

Figure 2. Scheme of the coalescer tank.

After injection of 100–130 l, it is assumed that the operation of the tank has reached a stationary mode, the water from the tank 1 has displaced all the original process water from the tank and enters the pure water zone. The last stage of the experiment involves sampling purified water from a pure water area. This stage is usually preceded by a control measurement of the flow rate on a measuring scale, after which the pump is turned off and 2 l of liquid are taken from the volume of liquid collected in the pure water zone to determine the degree of purification. The degree of water purification is assessed by filtering the selected samples through a blue-ribbon paper filter placed in a Buchner funnel mounted on a Bunsen flask connected to a vacuum pump. Based on the results of the difference in the mass of the filter before and after filtration, the efficiency of the coalescer sump was evaluated. The general view of the installation is shown in Fig. 3, the coalescer sump is shown in Fig. 4.

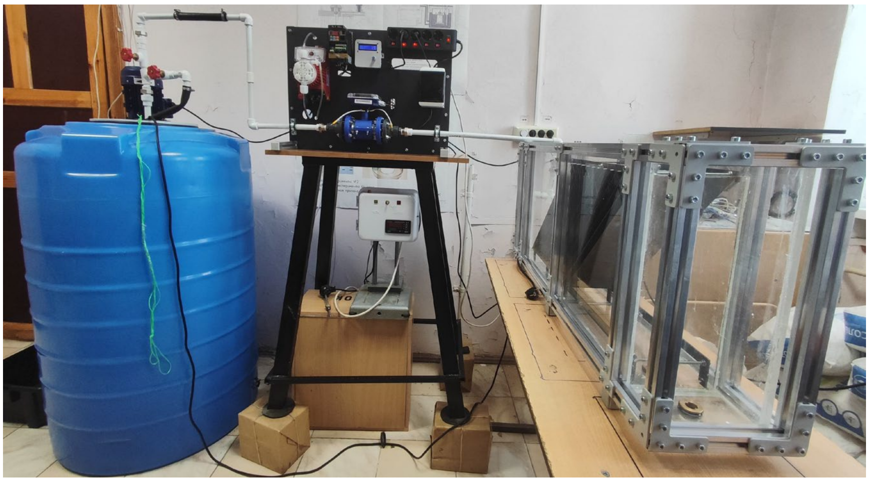

Figure 3. Assembled installation.

Figure 4. The final version of the coalescer tank.

2.2. Test Procedure

For testing, 200 l of process water is poured into the source water tank, and the temperature value is set on the temperature controller. The agitator turns on. To give the water the declared mineralization (200 g/l), NaCl salt is added to the water. A mixture of sand and clay in a 50/50 ratio and a total weight of 100 g is poured into the container. In the case of an experiment with chemical reagents, 0.9 l of NaOH alkali solution (100 g/l), 0.9 l of coagulant solution (100 g/l), and 0.2 l of flocculant solution (1 g/l) were added to 200 l of water. This sequence of addition of alkali and coagulant makes it possible to maintain the pH of the water in the range of 7.5–8.5. All reagents are pre-prepared in 1 l glass cups on a paddle mixer until they are completely dissolved in distilled water.

Depending on the experimental program, the shelves were placed at an angle of 45° in the amount of 13 or 26 pcs., or at an angle of 60° in the amount of 21 pcs.

2.3. Mathematical Model

The developed model is designed to calculate the concentration of substances at the outlet according to the known geometry of a settling tank with thin-layer modules and is based on a hydraulic approach using empirical ratios. The initial parameters of the model include: volumetric water flow Q_w, m³/h; the concentration of impurities at the tank inlet c_{in} mg/l; the proportion n_p of particles in the mixture with a radius of less than 100 µm, %; the length of the plate L_s, m; plate width B_s, m; plate inclination angle α, °; the distance between the plates in the perpendicular direction l_s, m; the height of the neutral layer under the module block h_n, m; the width of the hole at the inlet B_h, m; and its height L_h, m; the height of the settling zone H_1, m; water temperature, T, °C; water salinity n_s, ‰; device material, number of modules N, pcs.

Water inlet flow rate v_{win}, mm/s, is calculated considering the cross-sectional area of the hole at the inlet to the tank as:

$$v_{win} = \frac{1000 Q_w}{3600 B_h L_h}, \quad (1)$$

where the 3600 multiplier in the denominator considers the conversion of flow from m³/h to m³/s, the 1000 multiplier in the numerator considers the conversion of velocity from m/s to mm/s.

A uniform distribution of particles with sizes up to 100 µm is used for size values from 10 µm to 100 µm in increments of 10 µm, considering the proportion of n_p particles in the mixture with a radius of less than 100 µm. The particle size distribution over 100 µm is considered known, the total proportion of particles of all sizes is 100 %. For each range of radii with the particle index i, the arithmetic mean of the radius r_a, µm, is found. For particles over 1000 µm, it is found as 1500 µm. The particle fraction numbers are given in Table 1.

Table 1. Particle fractions index.

i	Size of particle, μm	*i*	Size of particle, μm
1	10	9	90
2	20	10	100
3	30	11	100–160
4	40	12	160–250
5	50	13	250–500
6	60	14	500–1000
7	70	15	Over 1000
8	80		

Based on the known values of the particle radius, the average mass value of the mass of one particle m_{ai}, mg, is calculated, considering the spherical shape of the particles:

$$m_{ai} = \frac{4}{3 \cdot 10^{12}} \pi r_{ai}^3 \rho_s, \tag{2}$$

where ρ_s is the particle density, which is equal to 2700 kg/m^3.

Inlet particle concentration with radius less than 100 μm c_{ins}, mg/l, is recalculated considering their share in the mixture according to the formula:

$$c_{ins} = \frac{n_p}{100} c_{in}. \tag{3}$$

Particle concentrations at the tank inlet in sizes from 10 μm to 100 μm with a step change in size of 10 μm are distributed uniformly, considering the value obtained from (3). For large particles, a uniform fraction distribution is also set, the proportion of the i-th fraction in the mixture n_{pi}, %, is calculated as:

$$n_{pi} = \frac{100\% - n_p}{5}, \quad i = 11, 12, \ldots, 15, \tag{4}$$

where the fraction of particles with sizes less than 100 μm n_p is substituted in %, index i is given in Table 1, number 5 is the number of isolated large fractions.

The particle concentrations at the tank inlet for the remaining fractions are calculated similarly to (3):

$$c_{ini} = \frac{n_{pi}}{100} c_{in}, \quad i = 11, 12, \ldots, 15, \tag{5}$$

where n_{pi}, %, is the fraction of particles of the i-th fraction in the mixture at the tank inlet.

In the pure water zone, the particle concentration of each fraction is assumed to be 0 if the angle between the direction of movement of the particle of the i-th fraction (i = 1, 2, ..., 10) and the direction of the diagonal between the plates β_i is positive, when the angle between the direction of movement of the particle and the direction of water flow α_{pwi} is greater than the angle between the direction of water flow and the direction of the diagonal between the plates γ, then the particle will sediment (see Fig. 5, where x is the horizontal coordinate axis). Otherwise, the particle will slip through the plates and get to the outlet of thin-layer modules zone, so the concentration of particles of such a fraction at the outlet will not change and will be equal to the concentration of particles of this fraction at the inlet. The resulting impurity concentration in the pure water zone c_{out} is found as the sum of the particle concentrations of all fractions in this zone.

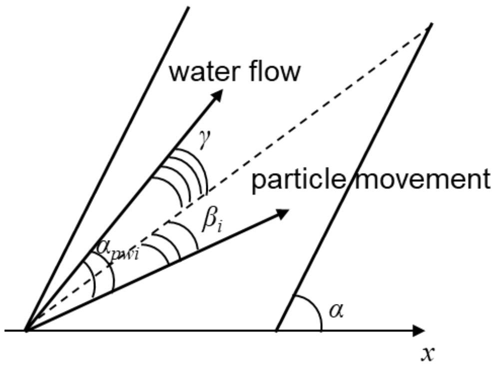

Figure 5. The case of a positive angle β*i* when the particles will sediment.

For different particle radii of less than 100 μm, described in Table 1, as well as for particles with a radius of r_0 = 1 μm, the deposition time t_i, s, and the sedimentation rate u_{si}, mm/s, are calculated. To do this, the radius is converted from mkm to m, the acceleration of gravity g = 9.81 m/s^2 is set, the density of water is ρ_1, kg/m^3, depending on the temperature T and salinity n_s, the density of sand is ρ_2, kg/m^3, the dynamic viscosity of water is μ, mPa·s, depending on the water temperature T. After that, the volume of the spherical particle V_i, m^3, is calculated:

$$V_i = \frac{4}{3}\pi r_{ai}^3, \tag{6}$$

where the particle radius is substituted in m.

The sedimentation rate, mm/s, is calculated based on Stokes' law:

$$u_{si} = 1000 \cdot \frac{2g\left(\rho_2 - \rho_1\right) r_{ai}^2}{9\mu_w}, \tag{7}$$

where the multiplier 1000 considers the conversion from m/s to mm/s, in formula (7) all values are substituted in SI.

To fulfill Stokes' law, laminar flow must be observed, so the Reynolds number must be less than 500. The Reynolds number itself for the i-th fraction is calculated as:

$$\mathrm{Re}_i = \frac{2\rho_1 u_{si} r_{ai}}{\mu_w}, \tag{8}$$

where all values are substituted in SI.

Sedimentation time is calculated as:

$$t_i = \frac{h_p}{u_{si}}, \tag{9}$$

where h_p is the height of particle sedimentation.

According to the known water flow velocity at the inlet v_{win}, mm/s, the plate inclination angle α, which is converted from degrees to radians, the real water flow velocity v_{tr}, mm/s, is calculated for movement in the module zone:

$$v_{tr} = \frac{v_{win}}{\cos\alpha}. \tag{10}$$

The angle between the direction of the water flow and the direction of the diagonal between the plater γ, °, is determined by the formula:

$$\gamma = \alpha - \varphi, \tag{11}$$

where φ, °, is the angle of the diagonal between the plates.

The angle between the direction of movement of the particle of the i-th fraction and the direction of the diagonal between the plates β_i (see Fig. 5), °, is calculated from geometric considerations:

$$\beta_i = \alpha_{pwi} - \gamma, \quad i = 1, 2, \ldots, 10, \tag{12}$$

where α_{pwi}, °, is the angle between the direction of movement of the particle of the i-th fraction (i = 1, 2, ..., 10) and the direction of the water flow.

If the angle (12) is positive, then the particle of the i-th fraction will sediment, if the angle is negative, then the particle will get to the outlet.

To verify the laminarity of the flow, the Reynolds number for the i-th fraction is calculated:

$$\mathrm{Re}_i = \frac{v_{resi} l_s}{1000 \nu_{cw}} \cdot 10^6, \tag{13}$$

where v_{resi} is the resulting velocity of a certain fraction particle, mm/s; v_{cw} is the kinematic viscosity of water, mm^2/s; the distance between plates in the perpendicular direction l_s is substituted in m; the multiplier 1000 in the denominator (13) considers the translation of v_{resi} from mm/s to m/s; the multiplier 10^6 in the numerator considers the conversion of v_{cw} from mm^2/s to m^2/s. If the Re_i < 500, then the particle flow of the i-th fraction is laminar, otherwise, it is turbulent.

Next, the water flow velocity at the outlet of the plate cross-sectional area v_s, mm/s, is calculated:

$$v_s = \frac{1000 Q_w}{3600 l_s B_s (N-1)}, \tag{14}$$

where the 3600 multiplier in the denominator considers the conversion of flow from m^3/h to m^3/s, the 1000 multiplier in the numerator is related to the conversion of velocity from m/s to mm/s.

Since the particle deposition velocity is perpendicular to the horizontal, and the true water flow velocity is directed along the plates inclined at an angle of α, °, the angle between u_{si} and v_{tr} is equal for any fraction (Fig. 6):

$$\alpha_v = 90^\circ + \alpha. \tag{15}$$

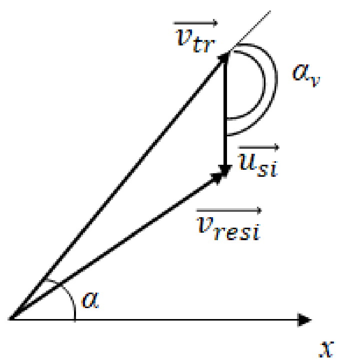

Figure 6. Calculation scheme of the resulting velocity vector.

This angle (15) can also be converted to radians. The direction of the resulting velocity is determined by adding vectors according to the rule of a triangle (Fig. 6) or a parallelogram, the modulus of the resulting velocity for the i-th fraction v_{resi} is calculated using the cosine theorem, considering that the angle between u_{si} and v_{tr} is adjacent to α_v:

$$v_{resi} = \sqrt{u_{si}^2 + v_{tr}^2 - 2u_{si}v_{tr}\cos\left(180^\circ - \alpha_v\right)}. \quad (16)$$

The angle of inclination of the resulting velocity to the horizontal axis $x\gamma_{resi}$, °, is calculated from geometric considerations:

$$\gamma_{resi} = \arccos\frac{v_{tr}\cos\alpha}{v_{resi}} = \arccos\frac{v_{tr}\sin\alpha_v}{v_{resi}}. \quad (17)$$

The resulting force is directed along the resulting velocity, so the angle δ_i, °, between this force and the sedimentation velocity is

$$\delta_i = 90^\circ + \gamma_{resi}, \quad (18)$$

where γ_{resi} is substituted in degrees.

Then the angle between the direction of motion of the particle of the i-th fraction (i = 1, 2, ..., 10) and the direction of the water flow α_{pwi}, °, is defined as:

$$\alpha_{pwi} = \alpha_v - \delta_i. \quad (19)$$

In addition, the coordinates of the vectors of the true water flow velocity, the particle deposition velocity, and the resulting velocity are determined. Horizontal axis is x, vertical axis is y. The origin of the vectors is placed at the origin. The end of the vector v_{tr} on the x axis corresponds to v_{win}, and on the y axis is defined as (in mm/s):

$$v_{try} = v_{tr}\sin\alpha = v_{tr}\sin\left(\alpha_v - 90^\circ\right). \quad (20)$$

The angle (in degrees) between the direction of v_{tr} vector and the *y* axis can be found as:

$$\delta_{try} = 90^\circ - \alpha. \quad (21)$$

The coordinate of the end of the u_{si} vector on the x axis is zero (the vector is perpendicular to this axis), and on the y axis it is equal to the u_{si} modulus taken with a minus sign. Accordingly, adding the coordinates of these vectors separately gives the coordinates of the end of the v_{resi} vector along these axes.

Finally, the angle between v_{resi} and the y-axis can be defined as:

$$\delta_{ryi} = \arccos\frac{v_{resiy}}{v_{resi}}, \quad (22)$$

where v_{resiy}, mm/s, is the coordinate of the v_{resi} vector end on the y-axis.

The overflow cross-sectional area between the flow damping zone and the thin-layer modules zone S_b, m², is defined as:

$$S_b = B_{sz}\left(H_1 + h_n\right), \quad (23)$$

where the width of the settling zone B_{sz}, m, is equal to the width of the modules B_s, m, increased by 0.2 m for the margin, all dimensions are substituted in m.

The area of one intermodular space S_1, m², is calculated as:

$$S_1 = B_s l_s. \quad (24)$$

Flow rate between two modules Q_{mod}, m³/h, is determined as:

$$Q_{\text{mod}} = \frac{3600 v_{tr} S_1}{1000}, \tag{25}$$

where the 3600 multiplier considers the conversion of flow rate from m³/s to m³/h, the 1000 multiplier in the denominator is associated with the conversion of velocity v_{tr}, mm/s, from mm/s to m/s.

Then, the total number of modules can be found using the formula:

$$N = \frac{Q_w}{Q_{\text{mod}}} + 1, \tag{26}$$

where the result is rounded up to the nearest integer.

The thickness of one module d_m, m, is known, as is the material density of the module ρ_m, kg/m³. From the geometry of the problem (Fig. 7), the horizontal distance between the plates b, m, is

$$b = \frac{l_s}{\sin\alpha}, \tag{27}$$

where l_s is substituted in m.

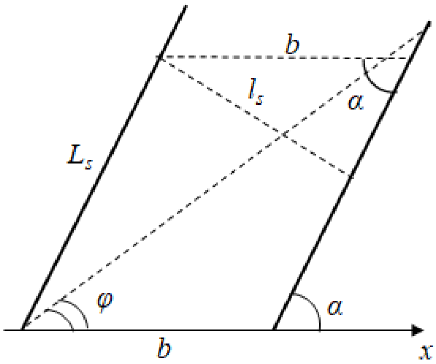

Figure 7. Calculation scheme of module dimensions.

Obviously, then the height of the particle sedimentation, h_p, mm, is

$$h_p = \frac{1000 l_s}{\cos\alpha}, \tag{28}$$

where l_s is substituted in m, the 1000 multiplier considers the conversion from m to mm.

By the cosine theorem, the diagonal distance between the plates is d, m, is

$$d = \sqrt{b^2 + L_s^2 + 2bL_s\cos\alpha}, \tag{29}$$

where all sizes are substituted in m.

The angle of inclination of the diagonal between the plates to the horizontal, φ, °, is also determined by the cosine theorem:

$$\varphi = \arccos\frac{b^2 + d^2 - L_s^2}{2bd}, \tag{30}$$

where all sizes are substituted in m.

As a result of applying the mathematical model, the concentration of impurities at the outlet of the tank is calculated, for which auxiliary parameters are determined using formulas (1)–(30).

3. Results and Discussion

To validate the correctness of the developed model, it was compared with 26 validation experiments. The installation parameters common to all experiments are shown in the Table 2. The sand density was ρ_2 = 1400 kg/m^3, the concentration of impurities at the tank inlet c_{in} = 500 mg/l.

Table 2. Installation parameters common to all experiments.

Parameter	Value	Parameter	Value
B_s, m	0.151	h_n, m	0.05
B_h, m	0.152	L_h, m	0.05
H_1, m	0.05		

The first series of five experiments was carried out at low flow rate on an installation with N = 13 plates located at an angle of α = 45° to the horizontal. The proportion of particles with sizes less than 100 µm n_p in the mixture is 100 %. The parameters of such an installation are: L_s = 0.495 m, l_s = 0.022 m, the initial experimental data and the results of calculating the concentration at the outlet of the installation in comparison with the experimental values are shown in Table 3, where c_{out} is the calculated concentration of particles at the outlet of the installation, c_{oute} is the concentration of particles at the outlet of the installation based on the experimental results, ε is an error.

Table 3. Experimental data for the first series and the results of determining the particle concentration at the outlet of the installation.

No. / Parameter	1	2	3	4	5
Q_w, l/min	0.305	0.124	0.303	0.561	0.300
n_s, ‰	0	0	200	100	0
T, °C	26	26	26	26	26
c_{out}, % from initial	20	10	3	10	7
c_{oute}, % from initial	17	12	10	7	6
ε, %	3	2	7	3	1

According to the results of the first series of experiments, it was possible to establish that the salinity of water affects the sedimentation rate. Excluding this fact, the error in determining the concentration of particles at the outlet exceeds 15 %. The approximation of the calculated parameters to compare them with the experimental results gave a refined dependence for the sedimentation rate at flow rates Q_w less than 0.09 m^3/h and salinity ns more than 1 %, when the sedimentation rate u_{si}, mm/s, is calculated from Stokes' law (7), considering the empirical correction for salinity:

$$u_{si} = 1000 \cdot \frac{2g\left(\rho_2 - \rho_1\right) r_{ai}^2}{9\mu_w} \sqrt{\frac{n_s}{10}}, \tag{31}$$

where the multiplier 1000 considers the conversion from m/s to mm/s, in formula (31), all values are substituted in SI, except for the salinity n_s, which is substituted in ‰, the multiplier 10 in the denominator converts the salinity from ‰ to %. In the absence of salinity of the water or at high Q_w flow rates of more than 0.09 m^3/h, the formula (7) remains valid for the sedimentation rate.

The effect of reagents in the first series of experiments was not researched. In most experiments, the error in determining the concentration of particles at the outlet was not more than 3 %. The disagreement in the results in the third experiment is due to the changing flow rate during its implementation.

The second series of four experiments (Nos. 6–9). It was carried out with higher flow rates on the same installation, although in the ninth experiment, the flow rate for verification was set lower, in addition, the effect of concentration of coagulants and flocculants was investigated. The initial experimental data and the results of calculating the concentration at the outlet of the installation in comparison with the experimental values are shown in Table 4, where V_c is the volume of coagulant solution (100 g/l), V_n is the volume of alkali solution (100 g/l), and V_f is the volume of flocculant solution (1 g/l). All volumes are given for 200 l of water. The proportion of particles with sizes less than 100 μm n_p in the mixture in the absence of coagulant is 100 %.

Table 4. Experimental data for the second series and the results of determining the particle concentration at the outlet of the installation.

No. / Parameter	6	7	8	9
Q_w, l/min	4.300	4.430	4.560	0.561
n_s, ‰	0	200	200	0
V_c, l	0	0	0.9	0.5
V_n, l	0	0	0.9	0.5
V_f, l	0	0	0.2	0.2
T, °C	26	26	26	26
c_{out}, % from initial	20	20	7	8
c_{oute}, % from initial	24	21	57	8
ε, %	4	1	50	0

According to the results of the second series of experiments, it was found that at high flow rates, salinity does not affect the sedimentation rate due to the rapid flow through the installation. To account for the effect of the concentration of coagulants and flocculants on the fraction of particles with a size of less than 100 μm in the flow, this fraction was manually adjusted so that the calculated value of the impurity concentration at the outlet corresponded to the experimental one. After the third series of experiments, these values were adjusted so that for all identical volumes of chemical reagents, the error in determining the calculated impurity concentration at the outlet compared to the actual one was minimal. The data in Table 4 are already given considering this adjustment. In most experiments, the error in determining the concentration of particles at the outlet was not more than 4 %. The discrepancy in the results in the eighth experiment is due to the clumping of particles and their removal by the flow.

The third series of experiments consisted of varying various parameters of the settling tank (the angle of inclination of the plates and their number, flow rate and salinity of water) to refine the parameters of the model. It consisted of the following 17 experiments: Nos. 10–26. For an installation with a different number of plates or angle of inclination than in the first two series of experiments, the parameters are shown in Table 5, the initial experimental data are shown in Table 6, and the results of calculating the concentration at the outlet of the installation in comparison with the experimental values are shown in Table 7.

Table 5. Installation parameters with a different number of plates or angle of inclination.

Parameter	Value for 26 plates	Value for 21 plates
α, °	45	60
L_s, m	0.495	0.404
l_s, m	0.01	0.0223

Table 6. Experimental data for the third series.

No.	Q_w, l/min	n_s, ‰	V_c, l	V_n, l	V_f, l	N, pcs.	α, °	T, °C
10	3.730	0	0	0	0	26	45	26
11	1.660	0	0	0	0	26	45	26
12	3.500	200	0	0	0	26	45	26
13	2.500	200	0	0	0	26	45	29
14	1.650	0	0	0	0	26	45	29
15	3.730	200	0	0	0	26	45	26
16	2.210	200	0.9	0.9	0.2	26	45	26
17	3.460	200	0.9	0.9	0.2	26	45	25
18	4.430	200	0.9	0.9	0.2	13	45	23
19	4.390	200	0.45	0.45	0.2	13	45	23
20	4.290	200	0.6	0.6	0.2	13	45	30
21	1.540	200	0.45	0.45	0.2	21	60	28
22	3.870	200	0.6	0.6	0.2	26	45	28
23	3.970	0	0.9	0.9	0.2	26	45	26
24	1.870	0	0	0	0	21	60	26
25	1.940	0	0.9	0.9	0.2	21	60	26
26	1.540	200	0.45	0.45	0.2	21	60	26

Table 7. The results of determining the particle concentration at the outlet of the installation for the third series of experiments.

№	c_{out}, % from initial	c_{oute}, % from initial	ε, %
10	10	17	7
11	10	15	5
12	20	19	1
13	20	24	4
14	10	15	5
15	20	21	1
16	6	8	2
17	7	30	23
18	14	34	20
19	16	29	13
20	13	10	3
21	16	14	2
22	6	8	2
23	3	2	1
24	20	19	1
25	6	3	3
26	16	14	2

From the experiments in this series, the effect of coagulant and alkali concentrations on the proportion of particles with sizes less than 100 μm in the mixture n_p, %, was established. To do this, set the volume of coagulant (equal to the volume of alkali) in l per 200 l of water V_c, then, the proportion of particles with sizes less than 100 μm in the mixture n_p, %, is determined from the empirical correlation:

$$n_p = 103.7V_c^2 - 162.2V_c + 92.0. \quad (32)$$

In the absence of the addition of coagulant, the proportion of particles with sizes less than 100 μm n_p in the mixture is 100 %.

The correlation coefficient for dependence (32) exceeds 0.99.

In most experiments, the error in determining the concentration of particles at the outlet was not more than 5 %. The disagreement in the results in the tenth experiment is due to a probable change in water flow during the experiment, in the seventeenth, eighteenth, and nineteenth experiments is due to the flushing of clumped particles by a stream.

The average error in determining the concentration of particles at the outlet, with the exception of the eighth, seventeenth, eighteenth, and nineteenth experiments, in which clumped particles are removed by a flow, according to the modified model is 3 %.

Verification of the developed proxy-model was carried out by comparing the calculated drop in the concentration of dispersed particles at the outlet of the tank 1 hour after the start of its operation with the data from the work of Kovalev et al. [1]. As a result of this comparison, according to the data of the article [1], the concentration drops by 1.62 times, according to the developed model by 1.67 times, which is an error of 3 %.

4. Conclusions

A laboratory installation of a coalescer settling tank has been developed. Experimental data were obtained on the degree of purification in a design with a number of plates of 26 and 13 pcs. for an angle of inclination of 45° and with a number of plates of 21 pcs. for an angle of inclination and 60°.

It is shown that the lower the flow rate in the coalescer tank, the better the water purification. The effect of water mineralization and the addition of auxiliary chemical reagents for the precipitation of suspended particles on the degree of purification has been established.

A modified mathematical model of a thin-layer settling tank is proposed, which for the first time considers the effect of the concentration of coagulant and alkali on the particle size distribution, as well as salinity on the particle sedimentation rate.

It was found that with an increase in the concentration of chemical reagents, the proportion of particles with sizes less than 100 μm in the flow decreases, which leads to an increase in water purification.

It is shown that the modified model reproduces laboratory results with an average error of 3 % with a change in flow rate from 0.124 l/min to 4.560 l/min, water temperature from 23 °C to 30 °C, number of plates from 13 to 26, plate inclination angles of 45° and 60°, volume of coagulant and alkali from 0.45 l to 0.9 l of each reagent per 200 l of water and the volume of flocculant of 0.2 l per 200 of water, salinities from 0 ‰ to 200 ‰ and are recommended for use in calculating the impurity concentration at the outlet for these parameter ranges, therefore, the model is validated.

A certificate of registration of a computer program was obtained for the developed model[2].

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